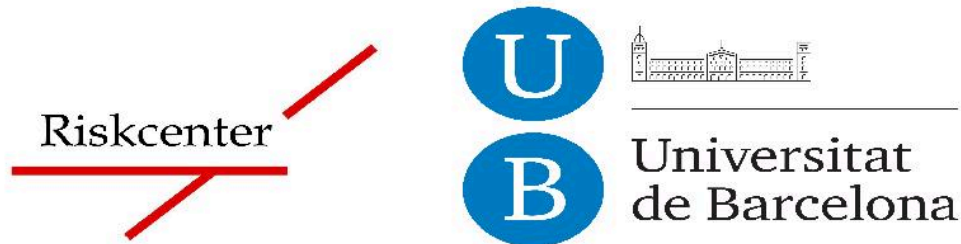


# Fundamentals of Cost and Risk that Matter to Pension Savers and Life Annuitants

Catherine Donnelly<sup>1</sup>, Montserrat Guillén<sup>2</sup> and  
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The Pension Research Council, USA, April 30, 2015

# How much wealth is available at a certain time horizon?

Previous work:

Donnelly, Guillén, and Nielsen (2013 and 2015) **Insurance Mathematics and Economics**

Gerrard, Guillén, Nielsen, and Pérez-Marín (2014) **The Scientific World Journal**

Guillén, Nielsen, Pérez-Marín, and Petersen (2013) **Scandinavian Actuarial Journal**

## Objective

Design transparent strategies for long-term investment of savings for retirement

## 1 Introduction and scope

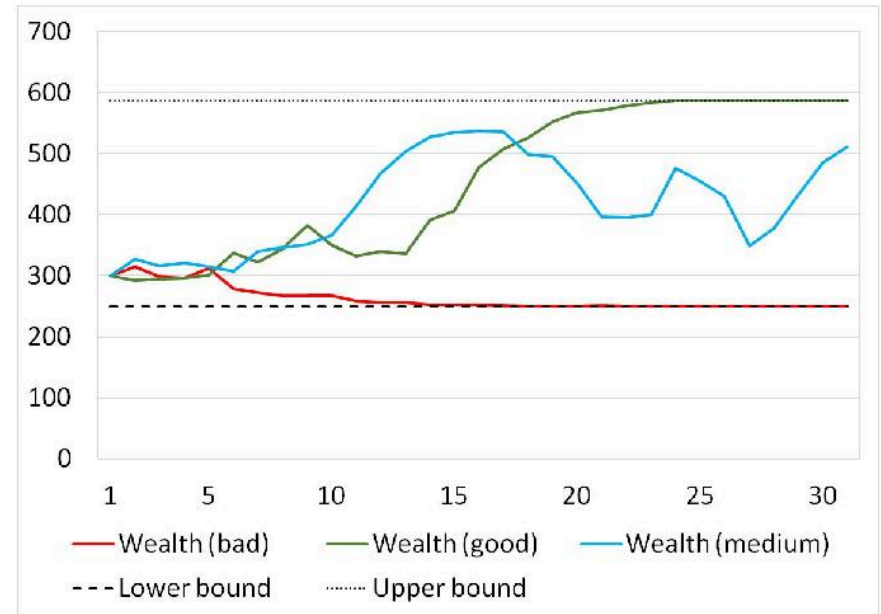
## 2 Main results

## 3 Numerical results

# Motivation

- Fees
  - ✓ return lost with portfolio administration
  - ✓ lack of transparency
- Duration
  - ✓ longevity
  - ✓ time-horizon
- Return with risk-adjustment
  - ✓ choosing a measure of performance
  - ✓ measuring risk and return with quantiles
- Smoothing
  - ✓ risky assets are volatile
  - ✓ aversion to loose savings

# Sample trajectories over 30 years



How can we control risk?

# Establishing bounds on terminal wealth

- Market model
- Investor
- Problem with an upper bound (Proposition 1: an optimal investment strategy)
- Problem with a lower and an upper bound (Proposition 2: an optimal investment strategy)

## Solution

Proportion of wealth to be invested in the risky asset at  $t$

$$\pi^\theta(t) := A [1 - \Phi(d_+(t, P(t); G) - \Phi(-d_+(t, P(t); F))] P(t).$$

$X^{\pi^\theta}(t) - \pi^\theta(t)$  is invested in the risk-free bond, in which  $P(t) = (z_0 + g(0)) Z(t)$  and the function  $d_+$  depends on the bounds and time horizon.

Details can be found in two working papers:

<http://www.pensionresearchcouncil.org/publications/document.php?file=1264>

<http://www.ub.edu/riskcenter/research/WP/UBriskcenterWP201502.pdf>



# Quantiles of terminal wealth

Table 1: Distribution of the wealth for various choices of the lower and upper bounds after 30 years and an initial investment of 300 units

Tolerance level (%)	No bounds	Only lower bound equal to 250	Only upper bound equal to 587.10	Both lower and upper bound equal
1	82.09	250.00	100.86	250.00
5	146.08	250.00	179.48	250.00
10	198.62	250.00	244.03	250.00
20	288.14	250.00	354.02	250.00
30	376.80	250.00	462.95	291.56
50	587.10	387.57	587.10	454.28
70	914.77	603.88	587.10	587.10
90	1735.38	1145.59	587.10	587.10
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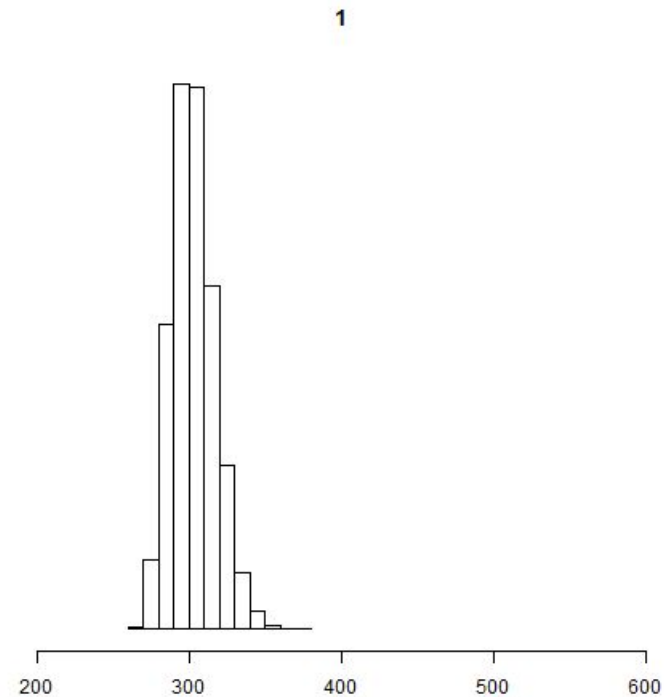
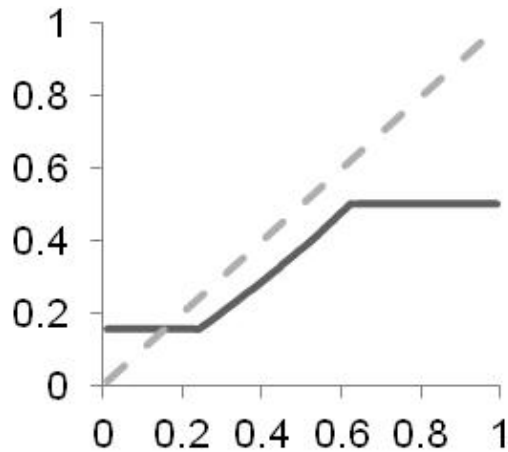
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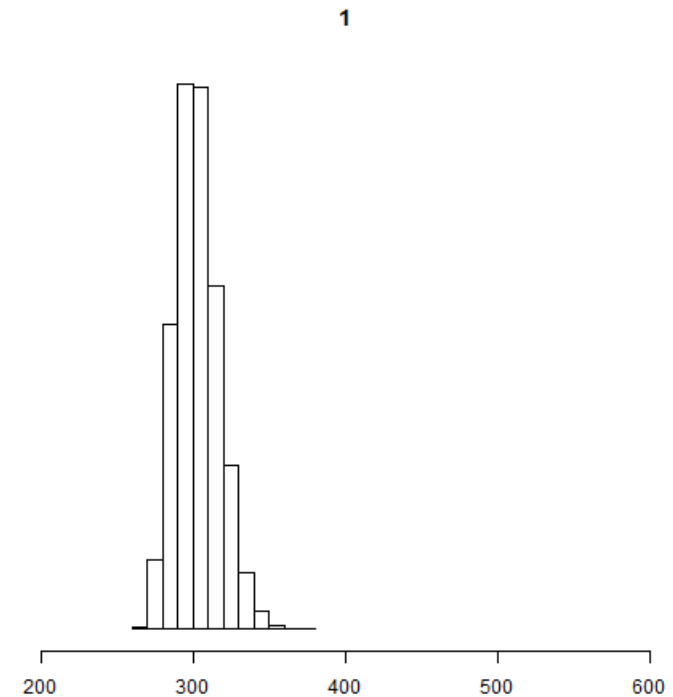
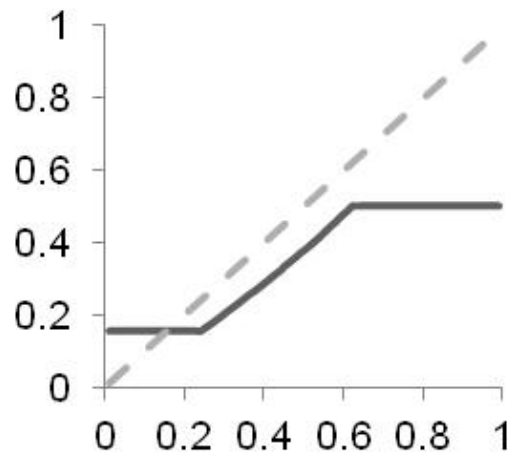
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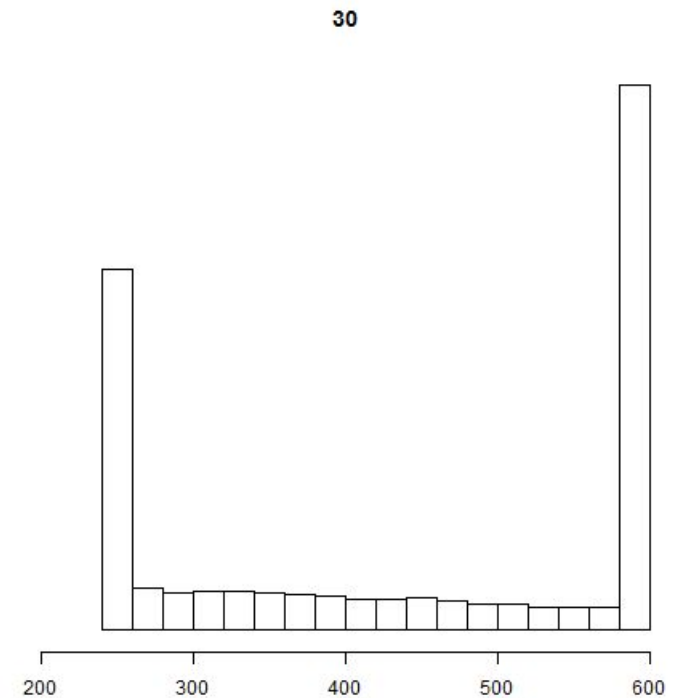
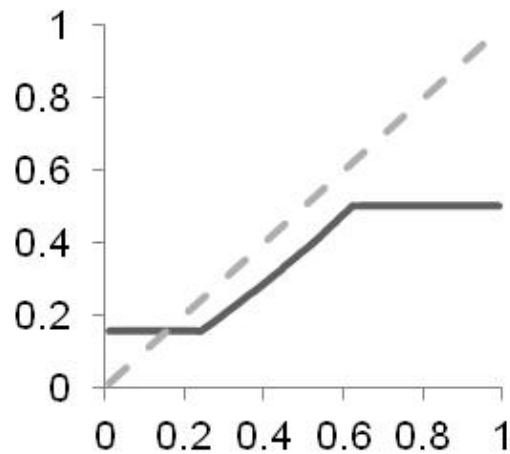
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- 4 Smoothing without pooling.

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- The extend of uncertainty using bounds on the desired income is easy to communicate and transparent.
- The visualization of quantiles shows how investment strategies work in the extremes.
- Our proposed mechanism does not need high managerial fees to pay for the fund manager's ability to reduce risk, but rather to improve risk-adjusted return.





Work in progress  
[www.ub.edu/riskcenter](http://www.ub.edu/riskcenter)