Putting the Pension Back in 401(k) Plans: Optimal versus Default Longevity Income Annuities

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Abstract

Most retirees take payouts from their defined contribution pensions as lump sums, but the US Treasury recently moved to encourage firms and individuals to convert some of the $15 trillion in plan balances into longevity income annuities paying lifetime benefits from age 85 onward. We evaluate the welfare implications of this reform using a calibrated lifecycle consumption and portfolio choice model embodying realistic institutional considerations. We show that defaulting a fixed fraction of workers’ 401(k) assets over a dollar threshold is a cost-effective and appealing way to enhance retirement security, enhancing welfare by up to 20% of retiree plan accruals.

Keywords: life cycle saving; household finance, longevity risk; 401(k) plans; retirement

JEL Codes: G11, G22, D14, D91

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1. Introduction

Much has been written on the theoretical economic appeal of annuities as important financial instruments to which private households should allocate their retirement assets, but in practice, few people purchase them (c.f., Benartzi et al., 2011; Davidoff et al., 2005; Inkmann et al., 2011 and originally Yaari 1965). Prior explanations of this phenomenon, which is often referred to as the “annuity puzzle,” have pointed to factors such as costs/loadings, retiree bequest motives, liquidity needs, and behavioral reasons including product complexity.¹

Yet one important explanation hitherto not examined pertains to a key institutional rule discouraging annuitization in employer-based 401(k) defined contribution plans, the standard tax-qualified mechanism in which private sector workers save for retirement in the U.S. and now worth $15 trillion (ICI 2016). Specifically, until 2014, US tax rules required retirees to withdraw from their retirement accounts following the so-called “Required Minimum Distribution” (RMD) rule each year from age 70.5 onward. The RMD was computed such that the sum of annual payouts was expected to exhaust the retiree’s 401(k) balance by the end of his life (IRS 2012b). Even if a retiree did buy an annuity with plan assets, the RMD was still calculated taking into account the value of his annuity. This had the unappealing consequence that the retiree might find himself needing to withdraw an amount in excess of his liquid assets (excluding the annuity value) and be forced to pay a 50% excise tax (Iwry 2014). Moreover, on the supply side, in this regulatory regime plan sponsors took on a significant fiduciary risk if they were to encourage financially-inexperienced workers to convert some of their accumulated 401(k) assets into life annuities requiring large RMD payouts. As a result, it is not surprising

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¹ Previtero (2014) provided evidence of a strong negative correlation between stock market returns and annuitization, suggesting that naïve beliefs and extrapolation from past returns drove behavior. Peijnenburg et al. (2016) showed that the low voluntary annuitization rates remain puzzling even after including such behavioral factors in more sophisticated lifecycle models.
that few 401(k) retirement plans in the U.S. offered access to lifelong income payments helping retirees cover the decumulation or drawdown phase of the lifecycle.\(^2\)

This paper contributes to the literature by evaluating how a recent change in regulatory policy by the U.S. Department of the Treasury corrects this institutional bias by providing “more options for putting the pension back” into private sector defined contribution plans (Iwry 2014). Specifically, the Treasury amended the required minimum distribution regulations for 401(k) plan (and also for non-profit firms’ 403(b) plans as well as Individual Retirement Accounts or IRAs) “to provide a measure of additional flexibility consistent with the statutory RMD provisions” (Iwry 2014).\(^3\) The eligible instruments must be deferred annuities, also referred to as longevity income annuities (LIAs), that begin payouts not later than age 85 and cost less than 25% of the retiree’s account balance (up to a limit).\(^4\) Under these conditions, the retiree’s annuity is no longer counted in determining his RMD. The policy change therefore relaxes the RMD requirements that had effectively precluded the offering of longevity annuities in the 401(k) contexts.

As we show below, this reform makes annuitization far more appealing for plan sponsors and for households making retirement portfolio allocation decisions. Such instruments provide a low-cost way to hedge the risk of outliving one’s assets, which is a key risk facing older people unable to return to work and confronting high healthcare costs. For example, the expected remaining lifetime for a 65-year-old US female is about 21 years (using general population statistics; Arias 2016). Yet there is substantial variability – about nine years – around

\(^2\) Benartzi et al. (2011) note that only about one-fifth of U.S. defined contribution plans currently offer annuities as a payout option; a small survey of 22 plan record-keepers by the US GAO (2016) concluded that few plans currently offer participants ways to “help them secure lifetime income in retirement.” Most innovation in the DC arena over the last decade has instead focused on the accumulation phase, with the introduction of products to attract saving including life cycle or target date funds and the widespread adoption of automatic 401(k) enrollment and automatic escalation of contributions (c.f. Gomes et al. 2008; Poterba et al. 2007). Some countries including Germany require retirees to convert a part of their accumulated tax-qualified retirement assets into a longevity annuity beginning at age 85 (see Horneff et al. 2014).

\(^3\) Treasury had originally proposed these amendments to the regulations two years earlier, referring to the new longevity annuities as “qualifying longevity annuity contracts” (or “QLACS”; see US Department of the Treasury 2014).

\(^4\) As suggested by Gale et al. (2008).
this mean, implying that individuals’ uncertainty about the length of their lifetimes can restrain retirement consumption and reduce lifetime well-being. Even in the current low interest rate environment, a deferred single life annuity purchased at age 65 by a man (woman) costing $10,000 provides an annual benefit flow from age 85 onward of $4,830 ($3,866) per year for life. This results from the investment returns earned over the 20 years prior to the withdrawal start date, plus the accumulated survival credits resulting from premiums paid by those who die earlier than expected being shared with those who survive in the annuitant pool.

In what follows, we build a realistic lifecycle model that matches data on 401(k) balances, which we use to quantify the potential impact of this new policy for a range of retiree types differentiated by sex, educational level, and preferences. Most importantly, and distinct from prior research, we do so while accounting for real-world income tax rules, Social Security contribution and benefit rules, and the Required Minimum Distribution (RMD) regulations. This model is then used to determine how much participants would optimally elect to annuitize given the opportunity to do so under the new RMD rules, when they face income, spending, and capital market shocks, and where they are also subject to uncertainty about their lifespans. In this realistic setting, we evaluate how much better off participants would be if their options included LIAs in the payout menu, versus without access to them. We also investigate how such products could be implemented as a default solution analogous to how Target Date Funds (TDFs) have been adopted during the accumulation phase. Specifically, we measure the potential improvements in well-being if a plan sponsor were to default a certain percentage of retirees’ assets (over a certain threshold) into a deferred annuity, taking into account mortality heterogeneity by education and sex.

5 Quotes available August 2016 on https://www.immediateannuities.com/  
6 The 2006 Pension Protection Act allowed plan sponsors to offer Target Date Funds as qualified default investment alternatives in participant-directed individual account plans (US DOL nd). A 2014 Treasury/IRS Administrative Guidance letter (IRS 2014) made clear that annuities – including deferred income annuities – could be a 401(k) default option.  
7 For instance Iwry (2014) discussed a case where the retiree could convert 15% of his plan assets into a deferred annuity. Iwry and Turner (2009) explored two approaches to make deferred income annuities the default payout
To preview our findings, we show that introducing a longevity income annuity would be quite attractive for most DC plan participants. Specifically, older individuals would optimally commit 8-15% of their plan balances at age 65 to a LIA which began paying out at age 85. When participants can select their own optimal annuitization rates, welfare increases by 5-20% of average retirement plan accruals as of age 66 (assuming average mortality rates), compared to not having access to LIAs. If, instead, plan sponsors were to default participants into deferred annuities using 10% of their plan assets, this would reduce retiree wellbeing only slightly compared to the optimum. Not surprisingly, results are less positive for those with substantially higher mortality vis a vis population averages: for such individuals, using a fixed percentage default rule generates lower welfare since annuity prices based on average mortality rates are too high. Converting retirement assets into a longevity annuity only for those having at least $65,000 in their retirement accounts overcomes this problem. Accordingly, we conclude that including well-designed LIA defaults in DC plans yields quite positive consequences for 401(k)-covered workers.8

Our research connects to and extends several strands of the literature on lifecycle consumption and portfolio choice initiated by Merton (1969). Various authors have extended these models by incorporating new sources of uncertainty (e.g., labor income risk, interest rate risk, mortality risk, or health risk), or nonfinancial assets such as housing, life insurance, and annuities.9 Yet little research to date has focused on how the institutional environments shape lifecycle financial decision making, especially key tax rules and requirements regarding retirement asset distribution. Love (2007) and Gomes et al. (2009) included tax-deferred 401(k) retirement accounts in a lifecycle model to study the impact of these on workers’ participation

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8 Moreover, our findings also apply to 403(b) and Individual Retirement Account payouts, since the RMD rules for these accounts are similar to those for 401(k) plans.

9 See for instance Cocco (2005); Cocco and Gomes (2012); Cocco et al. (2005); Fagereng et al. (2017) Gomes and Michaelides (2005); Inkmann et al. (2011); Kojien et al. (2016); Hubener et al. (2016); Kim et al. (2016); and Viceira (2001).
in the stock market. We extend that framework by incorporating crucially important additional features of taxation including progressive federal income taxes, Medicare taxes, Social Security taxes, and RMD rules regarding 401(k) withdrawals. We also include a realistic representation of Social Security benefits which depend on lifetime earnings, and we include the opportunity to buy a longevity income annuity at retirement. The careful incorporation of such institutional features in lifecycle model is of key importance in evaluating the impact of the policy reform.

In what follows, we describe our life cycle model and explain how we use it to study optimal consumption, investment, and annuitization decisions. In addition, we report the welfare implications of gaining access to in-plan LIAs. Sensitivity analyses illustrate how results vary across a range of parameters including uninsurable labor income profiles, sex, mortality assumptions, and preferences. Next, we discuss the impact of alternative default rules for retirement asset annuitization. A final section concludes.

2. Deferred longevity income annuities in a life cycle model: Methodology

Our discrete time dynamic portfolio and consumption model posits an individual who decides over his life cycle how much to consume optimally and how much to invest in stocks, bonds, and annuities. We model utility as depending on consumption and bequests, while constraints include a realistic characterization of income profiles, taxes, and the opportunity to invest in a 401(k)-type tax-qualified retirement plan (up to a limit). At retirement (assumed here to be age 66), the individual determines how much of his retirement account he wishes to convert to a deferred longevity income annuity, with the remainder held in liquid stocks and bonds. We also take into account the Required Minimum Distribution rules relevant to the US 401(k) setting, as well as a realistic formulation of Social Security benefits. In a subsequent section, we provide additional robustness analysis on different preferences and mortality heterogeneity across educational categories.

a. Preferences.
We build a discrete-time dynamic consumption and portfolio choice model for utility-maximizing investors over the life cycle. The individual’s decision period starts at \( t = 1 \) (age of 25) and ends at \( T = 76 \) (age 100); accordingly, each period corresponds to a year. The individual’s subjective probability of survival from time \( t \) until \( t + 1 \) is denoted by \( p^t_{t+1} \). Preferences at time \( t \) are specified by a time-separable CRRA utility function defined over current consumption, \( C_t \). The parameter \( \rho \) represents the coefficient of relative risk aversion and \( \beta \) is the time preference rate. Then the recursive definition of the corresponding value function is given by:

\[
J_t = \frac{(C_t)^{1-\rho}}{1-\rho} + \beta E_t \left( p^t_{t+1} J_{t+1} + (1 - p^t_{t+1}) b \frac{(Q_{t+1})^{1-\rho}}{1-\rho} \right),
\]

where terminal utility is \( J_T = \frac{(C_T)^{1-\rho}}{1-\rho} + \beta E_T \left( b \frac{(Q_{t+1})^{1-\rho}}{1-\rho} \right) \). The parameter \( b \) measures the strength of the bequest motive \( Q_t \), i.e. the utility from leaving financial wealth to the next generation in case the individual have died. In our base case we set the parameter \( b = 0 \), while in sensitivity analysis we allow it to be positive.

**b. The Budget Constraint during the Work Life.**

While working, the individual has the opportunity to invest a part \( (A_t) \) of his uncertain pre-tax salary \( Y_t \) (to an annual limit of $18,000)\(^{10}\) in a tax-qualified retirement plan held in stocks \( S_t \) and bonds \( B_t \):

\[
X_t = C_t + S_t + B_t + A_t.
\]

Here \( X_t \) is cash on hand after tax, \( C_t \) denotes consumption, and \( C_t, A_t, S_t, B_t \geq 0 \). One year later, his cash on hand is given by the value of his stocks having earned an uncertain gross return \( R_t \), bonds having earned riskless return of \( R_f \), labor income \( Y_{t+1} \) reduced by housing costs

\(^{10}\) The $18,000 limit was the legal limit on tax-deferred contributions to 401(k) plans in 2016, and if permitted by the plan, employees age 50+ can make additional 401(k) catch-up contributions of $6,000 per year.
$h_t$ modeled as a percentage of labor income (as in Love 2010), and withdrawals ($W_t$) from his 401(k) plan:\textsuperscript{11}

$$X_{t+1} = S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - Tax_{t+1} - Y_{t+1} d_w$$

During his work life, the individual also pays taxes, which reduce cash on hand available for consumption and investment.\textsuperscript{12} First, labor income is reduced by 11.65% ($d_w$), which is the sum of the Medicare (1.45%), city/state (4%), and Social Security (6.2%) taxes. In addition, the worker also must pay income taxes ($Tax_{t+1}$) according to US federal progressive tax system rules (IRS 2012b).

The individual may save in a tax-qualified 401(k) plan only during the working period, while non-pension saving in bonds and stocks is allowed over the entire life cycle. The exogenously-determined labor income process is $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$ with a deterministic trend $f(t)$, permanent income component $P_{t+1} = P_t \cdot N_{t+1}$ and transitory shock $U_{t+1}$.

Prior to retirement, his retirement plan assets are invested in bonds which earn the risk-free pre-tax return ($R_f$), and risky stocks paying an uncertain pre-tax return ($R_t$). The total value ($L_{t+1}$) of his 401(k) assets at time $t + 1$ is therefore determined by his previous period’s value, minus any withdrawals ($W_t \leq L_t$), plus additional contributions ($A_t$), and returns from stocks and bonds:

$$L_{t+1} = \omega_t^s (L_t - W_t + A_t) R_{t+1} + (1 - \omega_t^s) (L_t - W_t + A_t) R_f, \text{ for } t < K$$

His retirement plan assets are invested in a Target Date Fund with a relative stock exposure that declines according to age following the popular “Age – 100” rule ($\omega_t^s = (100 - Age)/100$).\textsuperscript{13}

\textsuperscript{11} Withdrawals before age 59 1/2 result in a 10% penalty tax.
\textsuperscript{12} For more details, see Appendix B.
\textsuperscript{13} This approach satisfies the rules for a Qualified Default Investment Alternative (QDIA) as per the US Department of Labor regulations (US DOL 2006). See also Malkiel (1996) and Kim et al. (2016).
The year before he retires at age 65 (K − 1), the individual determines how much of his 401(k) assets (LIA_{K−1}) he will switch to a deferred longevity income annuity with income benefits starting at age 85. Accordingly, the LIA income stream (PA) is determined as follows:

\[ PA = \frac{LIA_{K−1}}{\bar{a}_\tau}, \]  

(5)

where \( \bar{a}_\tau = \prod_{u=K}^{K+20} p_u a^{\sum_{s=0}^{100-(\tau−1)}(\prod_{i=\tau}^{s} p_i)R_f^{-(s+20)}} \) is the annuity factor transforming his lump sum into a payment stream from age 85. The amount used to buy the LIA reduces the value of his 401(k) assets invested in stocks and bonds, so the subsequent 401(k) payments are as follows:

\[
L_K = \omega_{K−1}^s (L_{K−1} - W_{K−1} + A_{K−1} - LIA_{K−1})R_K \\
+ (1 - \omega_{K−1}^s) (L_{K−1} - W_{K−1} + A_{K−1} - LIA_{K−1})R_f
\]  

(6)

c. The Budget Constraint in Retirement.

During retirement, the individual saves in stocks and bonds and consumes what remains:

\[ X_t = C_t + S_t + B_t \]  

(7)

Cash on hand for the next period evolves as follows:

\[
X_{t+1} = \begin{cases} 
S_t R_{t+1} + B_t R_f + Y_t(1-h_t) + W_t - Tax_{t+1} - Y_{t+1}d_r & K \leq t < \tau \\
S_t R_{t+1} + B_t R_f + Y_t(1-h_t) + W_t - Tax_{t+1} + PA - Y_{t+1}d_r & t \geq \tau
\end{cases}
\]  

(8)

where the LIA pays constant lifelong benefits (PA) from age 85 (\( \tau \)) onwards. At retirement, the worker has access to Social Security benefits determined by his Primary Insurance Amount (PIA) which is a function of his average lifetime (35 best years of) earnings.\(^{14}\) His Social Security payments (\( Y_{t+1} \)) in retirement (\( t \geq K \)) are given by:

\[ Y_{t+1} = PIA_t \cdot \epsilon_{t+1} \]  

(9)

\(^{14}\) The Social Security benefit formula is a piece-wise linear function of the Average Indexed Monthly Earnings and providing a replacement rate of 90% up to a first bend point, 32% between the first and a second bend point, and 15% above that.
where $\epsilon_t$ is a lognormally-distributed transitory shock $\ln(\epsilon_t) \sim N(-0.5\sigma_\epsilon^2, \sigma_\epsilon^2)$ with a mean of one which reflects out-of-pocket medical and other expenditure shocks (as in Love 2010).\footnote{The transitory variances assumed are $\sigma^2_\epsilon = 0.0784$ for high school and less than high school graduates, and $\sigma^2_\epsilon = 0.0767$ for college graduates (as in Love 2010).}

According to the new US Treasury rules, the present value of the LIA is excluded when determining the retiree’s RMD. However, LIA benefit payments from age 85 onward are subject to income taxes. During retirement, Social Security benefits are taxed (up to certain limits)\footnote{For detail on how we treat Social Security benefit taxation see Appendix B. Due to quite generous allowances, not many individuals pay income taxes on their Social Security benefits.} at the individual federal income tax rate as well as the city/state/Medicare tax rate. Payouts from the 401(k) plan are given by:

$$L_{t+1} = \omega_t (L_t - W_t) R_{t+1} + (1 - \omega_t) (L_t - W_t) R_f, \quad for \ t < K . \quad (10)$$

Moreover, the RMD rules require that 401(k) participants take a minimum withdrawal from their plans from age 70.5 onwards, defined as a specified age-dependent percentage ($m_t$) of plan assets, or else they must pay a substantial tax penalty. Accordingly, to avoid the excise penalty, plan payouts are set so $mL_t \leq W_t < L_t$.

3. **Model calibration**

Survival rates entering into the utility function are taken from the US Population Life Table (Arias 2010). For annuity pricing, we use the US Annuity 2000 mortality table provided by the Society of Actuaries (SOA nd). Annuity survival rates are higher than those for the general population because they take into account adverse selection among annuity purchasers.\footnote{The implied loads using the annuity table are about 15-20%; see Finkelstein and Poterba (2004)} Social Security old age benefits are based on the 35 best years of income and the bend points as of 2013 (US SSA nd). Accordingly, the annual Primary Insurance Amounts (or the unreduced Social Security benefits) equal 90 percent of (12 times) the first $791 of average indexed monthly earnings, plus 32 percent of average indexed monthly earnings over $791 and...
through $4,768, plus 15 percent of average indexed monthly earnings over $4,768. Required Minimum Distributions from 401(k) plans are based on life expectancy using the IRS Uniform Lifetime Table (IRS 2012b). In line with US rules, federal income taxes are calculated based on the household’s taxable income, six income tax brackets, and the corresponding marginal tax rates for each tax bracket (for details see Appendix B).

Our financial market parameterizations include a risk-free interest rate of 1% and an equity risk premium of 4% with a return volatility of 18%. The labor income process during the work life has both a permanent and transitory component, with uncorrelated and normally distributed shocks as $\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)$ and $\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)$. Following Hubener et al. (2016), we estimate the deterministic component of the wage rate process $w_t^i$ along with the variances of the permanent and transitory wage shocks $N_t^i$ and $U_t^i$ using the 1975–2013 waves of the PSID. These are estimated separately by sex for three education levels: high school dropouts, high school graduates, and those with at least some college (<HS, HS, Coll+). Wages rates are converted into yearly income by assuming a 40-hour workweek and 52 weeks of employment per year. Results for the six subgroups appear in Figure 1, where, for the three different educational groups, panel A reports the expected income profiles for females, and panel B for males. For all cases, the labor income pattern follows the typical hump-shaped profile in expectation. At age 66, on retirement, the worker receives a combined income stream from his 401(k) pension and Social Security benefits, and from age 85 on, payments from longevity income annuities.

*Figure 1*

We use dynamic stochastic programming to solve this optimization problem. There are five state variables: wealth ($X_t$), the total value of the individual’s fund accounts ($L_t$), payments

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18 For more on the Social Security formula see [https://www.ssa.gov/oact/cola/piaformula.html](https://www.ssa.gov/oact/cola/piaformula.html). A similar approach is taken by Hubener et al. (2016).

19 Dollar values are all reported in $2013.

20 More details on parameters are provided in Appendix A.
from the LIA \((PA)\), permanent income \((P_t)\), and time \((t)\).\(^{21}\) We also compute individual consumption and welfare gains under alternative scenarios using our modeling approach.

The values of the preference parameters for the six subgroups are selected so that the model generates 401(k) wealth profiles consistent with empirical evidence. Specifically, we calibrate the model to data from the Employee Benefit Research Institute (EBRI 2014) which reported 401(k) account balances for 7.5 million plan participants in five age groups (20-29, 30-39, 40-49, 50-59, and 60-69) in 2012. To generate 401(k) simulated balances, we first solve the lifecycle model where the agents have no access to longevity income annuities, and we generate 100,000 lifecycles using optimal feedback controls for each of the six subgroups (male/female with <HS, HS, and Coll+ education). We then aggregate the subgroups to obtain national median values using weights from the National Center on Education Statistics (2012).\(^{22}\) Finally, to compare our results to the EBRI (2014) data, we construct average account levels for each of the five age subgroups. We repeat this procedure for several sets of preference parameters. We find that a coefficient of relative risk aversion \(\rho\) of 5 and a time discount rate \(\beta\) of 0.96 are the parameters that closely match simulated model outcomes to empirical evidence on 401(k) balances.\(^{23}\) Figure 2 displays simulated and empirical data for the five age groups, and it shows that our simulated outcomes are remarkably close to the empirically-observed 401(k) account values.

\textit{Figure 2}

4. Results and discussion of the baseline case

\(^{21}\) For discretization, we split the five dimensional state space by using a \(30(X)\times20(L)\times10(PA)\times8(P)\times76(t)\) grid size. For each grid point we calculate the optimal policy and the value function.

\(^{22}\) Specifically, the weights are 50.7% female (and 62% with Coll+, 30% with HS, and 8% with <HS education), and 49.3% male (and 60% with Coll+, 30% HS and 10% <HS education).

\(^{23}\) Interestingly, these parameters are also in line with those used in prior work on life-cycle portfolio choice. See for instance Brown (2001).
In this section, we describe the average optimal life cycle patterns for labor income, consumption, assets held inside and outside tax-qualified retirement plans, and income generated from 401(k) plans based on simulated data for the US population having access to 401(k) plans. As described above, for each of the six subgroups (male/female by three educational levels), we use optimal feedback controls of our lifecycle model to generate 100,000 simulated lifecycle reflecting uncertain stocks returns and labor income shocks. To obtain national median values, we aggregate the simulated life cycle patterns of the subgroups assuming 50.7% are female and 49.3% are male. Moreover, 62% of the females are in the Coll+ group, 30% in the HS group, and 8% in the <HS group, while 60% of the males are Coll+, 30% HS, and 10% <HS (as per National Center on Education Statistics 2012). Based on this procedure, we then construct and compare two scenarios. With the old RMD rules (prior to the 2014 reform), this results in a situation where no LIA is available. With the new RMD rules, workers at age 65 can convert some of their 401(k) account assets into LIAs that begin paying benefits from age 85. Subsequent sensitivity analysis compares results for people with different lifetime income profiles, different mortality assumptions, and preferences. A final subsection provides an analysis of welfare gains when people have access to longevity income annuities based on the new RMD-rules.

4.1 Profiles for consumption, wealth, and annuity for the full population

Panel A of Figure 3 reports average optimal life cycle patterns for the full population where individuals lack access to the LIA, while Panel B shows what happens when the same people have the option to buy annuities from their 401(k) accounts at age 65. Initially, people work full-time and, by age 25, earn an annual pre-tax income of $30,800. The average worker saves from his gross earnings up to a maximum of $18,000 per year (as per current law) in his tax-qualified 401(k) account. By age 65, retirement plan assets peak at $205,785 (in expectation). The average consumption pattern (solid line) is slightly hump-shaped. Workers begin withdrawing from their 401(k) accounts starting around age 60 (red dotted line) when
they no longer incur the 10% penalty tax.\textsuperscript{24} This is in line with the empirical evidence showing a modest rate and size of pre-retirement withdrawals from 401(k) plans (Poterba et al. 2000). On retiring, the individual boosts his plan withdrawals substantially to compensate for the fact that his Social Security income is far below his pre-retirement labor income. The gray line represents the average amount of financial assets (stocks and bonds) held outside the tax-qualified retirement plan. These are held mainly as precautionary saving to buffer uninsurable labor income risk during the work life, and to cover out-of-pocket medical expenses in retirement.

\textit{Figure 3}

Panel B of Figure 3 displays the average life cycle profile when the same worker now has access to the LIA under the new RMD regime. As before, the pre-tax annual earnings at age 25 amount to $30,800 (dashed-dotted line). But now, the employee has the opportunity to purchase the LIA so he can save 1.6\% less in his 401(k) plan: $202,427 as of age 65 (in expectation) instead of $205,785. Thereafter, the worker reallocates $26,615 from his 401(k) account to the LIA, at which point no taxes are payable. Withdrawals from the 401(k) plan (red dotted line) start at age 60, and, on average, the retiree exhausts that account by age 85. Thereafter, the LIA pays an annual benefit of $7,050 (worth 39.3\% of the Social Security benefit) for the rest of his life. During the work life, the average amount of assets held outside the tax-qualified retirement plans is the same as without having access to the LIA, but in retirement, precautionary savings are lower. Also of interest is the fact that the individual having access to the LIA consumes more, in expectation, compared to when he lacks access, particularly after age 85. This is because the individual is insured against running out of money in old age.

\textsuperscript{24} Before age 59.5, the individual pays 10\% penalty for each withdrawal from a 401(k) plan.
Figure 4 displays the difference in consumption with and without access to the LIA. The x-axis represents the individual’s age, and the y-axis the consumption difference (in $000). We depict these in percentiles (95%; 5%) using a fan chart, where differences are measured for each of the 100,000 simulation paths. Darker areas represent higher probability masses, and the solid line represents the expectation. Results show that, prior to age 85, consumption differences are small: the median difference is only $2 at age 50. But by age 85, the retiree with the LIA can consume about $1,000 more per year on average, and $2,500 more by age 95. There is also heterogeneity in the outcomes, such that at age 50, the difference is only -$2 for the bottom quarter of the sample, while it is $8 for the 75th percentile. The heterogeneity in outcomes increases substantially after age 65: for instance, at age 95, the difference is $1,000 for the 25th percentile, but $5,700 for the 75th quantile.

Overall, we conclude that the opportunity to purchase a longevity income annuity provides individuals with the potential to save less yet consume substantially more, particularly at older ages.

4.2 Other Comparisons

In this section, we report results for other educational groups by sex. In addition, we explore the sensitivity of our results to different mortality assumptions, add a bequest motive, and evaluate what happens if the LIA has an earlier start age.

Differences by Sex and Educational Attainment. Table 1 shows how results differ for men and women with other educational levels, and hence labor earnings, patterns. To this end, we show retirement plan assets over the life cycle for women and men in the three educational brackets of interest here, namely high school dropouts, high school graduates, and the Coll+ group. Panel A reports outcomes when individuals lack access to the LIA, and Panel B shows asset values when they have access. Panel C provides average amounts used to purchase the LIA when available, along with the resulting lifelong benefits payable from age 85.
Since the Coll+ female earns more than her female high school dropout counterparts, she also saves more in her 401(k) plan over her life cycle. For example, without a LIA, by age 55-64, the average Coll+ woman with no LIA access saves $233,340 in her 401(k) account, over four times the $52,470 held by the High School dropout, and double the $114,850 of the High School graduate. With a LIA, the best-educated woman saves slightly less in her retirement account (around $3,000 less), while the HS graduate is not much affected. Interestingly, the least-educated female optimally saves slightly more (4%) in her 401(k) account when she can access the LIA. A similar pattern obtains for the three cases of male savers depicted. As the Coll+ male earns more than the Coll+ female, he accumulates more in his 401(k) account, on the order of $274,380 with no LIA. This is 80% more than the male HS graduate ($151,980), and over three times the $85,090 of the HS dropout. Once access to the LIA is available, the best-educated man needs to save $10,310 less, while the HS graduate changes behavior very little (as with the females). Again, the male HS dropout saves slightly more.

With the LIA, all groups of women and men withdraw more and retain less in their defined contribution plans post-retirement, compared to those lacking access. For instance, the Coll+ woman without the LIA keeps an average of $167,600 in her retirement plan between ages 65-74, or 22% more than with the LIA where she retains only $130,920 in investible assets. Similarly, the best-educated male age 65-74 without the LIA keeps 24% more ($186,700) than the $141,660 in his retirement account with the LIA. A similar pattern obtains for the other two educational groups by sex. With or without the LIA, the two less-educated men and women have very little remaining in their 401(k) plans close to the ends of their lives, though they have more without the annuity than with. At very old ages, 85-94, the most educated people having no access to the LIA still hold about $25,000 in their 401(k) accounts, while with the annuity, they have virtually nothing.
The reason for this difference is that those with LIAs use a substantial portion of their retirement assets to purchase longevity annuities which generate a yearly lifelong income. Panel C in Table 1 shows that the Coll+ women optimally use about $34,750 of their 401(k) assets to purchase their deferred annuity, and even the HS group buys annuities using $11,640 of their retirement accounts. The HS dropout group buys the least, not surprisingly in view of the redistributive nature of the Social Security system. They spend only $3,050 on the deferred income product. Men have similar patterns to women, though their shorter life expectancies motivate the least-educated to devote only $8,300 to LIAs.

From age 85 onwards, both groups having LIAs enjoy additional income compared to the non-LIA group. For instance, the 85-year old Coll+ woman receives an annual LIA payment for life averaging $7,790, while the female HS graduate receives $2,610 per year. The HS dropout receives the least given her small purchase, paying out only $680 per annum. For men, the optimal LIA purchase at 66 generates an annual benefit of $11,100 for the Coll+, $5,210 for the HS graduate, and a still relatively high annual benefit of $2,510 for the HS dropout. In other words, the LIA pays a reasonably appealing benefit for those earning middle/high incomes during their work lives. They are smaller, on net, for those who earned only what HS dropouts did over their lifetimes.

Impact of Alternative Mortality Assumptions, Payout Dates, and a Bequest Motive. Thus far, we have assumed that the LIAs are priced using age- and sex-specific annuitant tables. Yet it is also of interest to explore how the demand for LIAs varies with alternative mortality assumptions, including pricing for individuals with higher mortality rates as well as unisex pricing. We also consider a scenario where the LIA starts paying out younger, at age 80 instead of age 85. Finally, we show what happens if the worker has a bequest motive.

Taking into account alternative mortality assumptions is interesting for two reasons. First, recent studies report widening mortality differentials by education, raising questions about whether the least-educated will benefit much from longevity annuities. For instance,
Kreuger et al. (2015) report that male high school dropouts average 23% excess mortality and females 32%, compared to high school graduates. By contrast, those with a college degree live longer: men average a 6% lower mortality rate, and women 8%. Though only 10% of Americans have less than a high school degree (Ryan and Bauman 2016) and they comprise only 8% of the over-age 25 workforce (US DOL 2016), this group is more likely to be poor. Second, employer-provided retirement accounts in the US are required to use unisex life tables to compute 401(k) payouts (Turner and McCarthy 2013). While men’s lower survival rates may make LIAs less attractive to men than to women, it has not yet been determined how men’s welfare gains from accessing LIA products relate to women’s. Accordingly, in what follows, we present results for those persons anticipating shorter lifespans.

Table 2 presents results for each of these alternative scenarios. In Column 1, we report the impact of having the LIA priced using a unisex mortality table, as would be true in the US company retirement plan context. Columns 2 and 3 show results when annuities for high school dropouts of both sexes are priced using higher mortality (as in Kreuger et al. 2015). In Column 4 reports the impact of assuming a shorter deferral period: that is, here, the LIA begins paying out at age 80 instead of age 85. The last column depicts outcomes for females (Coll+) with a bequest motive.

Table 2 here

Results show that when the LIAs modeled are priced using the higher mortality rates for male and female high school dropouts, this makes them less appealing for both groups. For instance, the female HS dropout buys a much smaller LIA at age 65 – spending only $1,401 versus $3,050 in Table 1 – and hence it pays out much less ($320 versus $680 per year). The male HS dropout also spends less on the LIA, allocating only $5,330 to the deferred product versus $8,300; this lower LIA results in an income stream of only $1,610 per annum instead of $2,510. In general, using age/education group mortality tables does not completely erase the demand for LIAs, but it does diminish it substantially.
Turning next to the impact of using a unisex instead of a female mortality table to price the LIA, we find that this has little effect on outcomes. In other words, Coll+ women would devote almost as much money to longevity income annuities, regardless of whether sex-specific or unisex annuity life tables are used to price them. Further analysis will indicate how results change across other groups.

In Column 4 we report what happens when an earlier LIA payout is permitted, that is, at age 80 instead of age 85. Now the Coll+ woman saves slightly less in her 401(k) account as of age 55-64 ($2,000 less) than when she could only access the LIA at age 85, namely $228,970. The earlier starting age is attractive, so at retirement she will optimally allocate $60,910 to the LIA, almost double than in the Coll+ ($34,750). Her annual income payment will now be $7,830 at age 80+, $40 more per year than the $7,790 under the LIA payable at age 85.

Finally, we turn to the case where individual has a (strong) bequest motive, solving the model with a bequest parameter of $b = 4$ (as in Love 2010) in the value function. Results appear in the final column of table 2 for a female with average mortality and a college education. Compared to the result without a bequest motive (Table 1, column 3), her 401(k) assets are similar during the work life. Not surprisingly, however, during retirement the individual wanting to leave a bequest draws down her assets more slowly as to leave an inheritance in the event she dies. For example, the retiree having access to LIAs and a bequest motive holds an average of $21,800 in her retirement account at age 85-94, versus only $1,850 without a bequest motive. Yet the amount she optimally coverts into a lifelong annuity at age 65 differs only slightly, $ 29,810 (with the bequest motive) versus $34,750 (without). Hence, we conclude that

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25 Bernheim (1991) and, more recently, Inkmann and Michaelides (2012), have suggested that US and UK households’ life insurance demand is compatible with a bequest motive, and Bernheim et al. (1985) report that many older persons indicate that they desire to leave bequests. Nevertheless, evidence regarding the strength of the bequest motive is mixed. Hurd (1989) estimates an almost-zero intentional bequest preference and concludes that, in the US at least, most households leave only accidental bequests.
the existence of a bequest motive produces higher savings in retirement accounts at advanced ages, but it has little impact on the demand for LIAs.

4.3 Welfare Analysis

We next discuss the welfare gains when people have access to longevity income annuities by comparing two workers, both age 66. Each behaves optimally before and after retirement, but the first has the opportunity to buy LIAs at age 65, while the second does not. Since people are risk averse, it is not surprising that the utility level of those having access to LIAs at age 66 is generally higher than those without. We also compute the additional 401(k) wealth needed to compensate those lacking LIAs, to make them as well off as those having the products. Formally, we find the additional asset \((\mathit{wg})\) that would need to be deposited in the 401(k) accounts of individuals lacking access to LIA, so their utility would be equivalent to that with access to the LIA product. This is defined as follows:

\[
E[\text{with}\_\text{LIA}(X_t, L_t, P_A t, P_t, t)] = E[\text{without}\_\text{LIA}(X_t, L_t + \mathit{wg}, P_t, t)].
\] (12)

Table 3 provides the results. For the Coll+ female, access to the LIA enhances welfare by a value equivalent to $13,120 (first row). In this circumstance, she optimally devotes 15% of her 401(k) account to the deferred lifetime income annuity. If unisex mortality tables were required (second row), the optimal fraction of her account devoted to the LIA would change only trivially, and the welfare gain is actually higher due to the fact that, on average, women benefit from the use of unisex tables. If the LIA product initiated payouts from age 80 instead of age 85 (third row), more retirement money would be devoted to this product (26.7% of the account value) and the woman’s welfare gain would amount to 17% ($15,802).

Table 3 here

The next few rows of the table report results for different educational groups by sex. Among women, we see that welfare is enhanced by having access to the LIA product, though the gain of $6,280 for the HS graduates still exceeds that for HS dropouts (regardless of whether population or higher mortality rates are used). For men, we see that the gain for the Coll+ group
is substantial when LIAs are available, on the order of $35,837 as of age 66. Smaller results obtain for the less-educated, though even HS dropouts with the lower survival probabilities still benefit more than women, on average. Gains are still positive, though small, if the least-educated group has higher mortality as shown.

In sum, in our framework, both women and men benefit from access to longevity income annuities. While workers anticipating lower lifetime earnings and lower longevity do benefit proportionately less than the Coll+ group, all subsets examined gain from having access to the LIA when they can optimally allocate their retirement assets to these accounts.

5 How Might a Default Solution for the Longevity Annuity Work?

Thus far, our findings imply that a majority of 401(k) plan participants would benefit from having access to a longevity income annuity based on the new RMD rules implemented by the Treasury in 2014. Nevertheless, some people might still be unwilling or unable to commit to an LIA even if it were sensibly priced (as here).26 For this reason, a plan sponsor could potentially implement a payout default, wherein a portion of the retiring workers’ retirement plan assets would be used at age 65 to automatically purchase deferred lifetime payouts. Such a default would accomplish the goal of “putting the pension back” into the retirement plan.

One policy option along these lines would be for an employer to default a fixed fraction of retirees’ 401(k) accounts – say 10% – into a LIA when they turn age 65. This fixed fraction approach is compatible in spirit with the optimal default rates depicted in Table 3, where most retirees would find such a default amount appealing. Yet some very low-earners might optimally save so little in their 401(k) accounts that defaulting them into a LIA might not be practical. Accordingly, an alternative would be to default 10% of savers’ 401(k) accounts only

26 For instance, Brown et al. (2017) showed that people find annuitization decisions complex, particularly for the least financially literate.
when participants had accumulated some minimum amount such as $65,000 in their plans. In this *fixed fraction* + *threshold* scenario, the LIA default is implemented when the worker’s 401(k) account equals or exceeds the threshold. Of course, the 10% deferred annuitization rate will still be below what some would desire in terms of the optimum, and higher for others. Our question is, how would welfare effects change for these default deferred payout options?

Our analysis of the two different default approaches appears in Table 4. The next-to-last column reports welfare gains assuming the 10% default applied to everyone, while the last column assumes that retirees are defaulted into LIAs only if their retirement accounts exceed $65,000. In both cases, 10% of the assets invested by default would go to a LIA payable at age 85.

*Table 4 here*

For the base case Coll+ female, we see that her welfare gain from the *fixed fraction* default comes to $12,810, just slightly ($310) lower than the gain in the fully optimal case in Table 3. She still benefits under the fixed fraction approach when a unisex mortality table is used, but it provides 12% lower welfare gain than in the full optimality case (or $1,827 less than the $15,384 amount in Table 3). Welfare gains for the *fixed fraction* + *threshold* approach are comparable for the Coll+ woman. Accordingly, older educated women would likely favor LIAs beginning at age 85, under both the *fixed fraction* and the *fixed fraction* + *threshold* approaches.

Turning to the less-educated women, it is not surprising to learn that welfare gains are smaller for both default options. For instance, requiring the less-educated to annuitize a *fixed fraction* (10%) of their 401(k) wealth reduces utility for the HS graduates using sex-specific mortality tables by 13% (i.e., from $6,280 to $5,467), and by more, 41.5%, for HS dropouts.

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27 This appears to be a reasonable threshold in that workers in their 60’s with at least five years on the job averaged $68,800 or more in their 401(k) plans, as of 2014 (Vanderhei et al. 2016). The same source found that workers in their 60s who earned $40-$60,000 per year averaged $96,400 in their 401(k) accounts; those earning $60-$80,000 per year averaged $151,800; and those earning $80-$100,000 held an average of $223,640 in these retirement accounts.
(i.e., from $2,204 to $1287). If mortality rates for HS dropouts were 34% higher, as noted above, these least-educated women would actually be worse off under the fixed fraction approach. For such individuals, the *fixed fraction + threshold* would be more appealing, as those with very low incomes and low savings would be exempted from buying LIAs. In fact, HS graduates do just about as well under this second policy option as in the optimum.

Regarding results for men, we see that the default 10% LIA has little negative impact on their welfare. This is primarily due to their higher lifetime earnings, allowing them to save more, as well as lower survival rates. For instance, the Coll+ male’s welfare gain in the optimum is $35,837 (Table 3) and just slightly less, $33,032, under the *fixed fraction* option. The *fixed fraction + threshold* default is likewise not very consequential for the best-educated male, with welfare declining only 8% compared to the optimum. Less-educated males experience only slightly smaller welfare gains with both default policies; indeed, if they are permitted to avoid annuitization when they have less than $65,000 in their retirement accounts, benefits are quite close to the optimum welfare levels across the board.

Finally, we repeat our welfare analysis for the default solutions assuming that the LIAs are priced using a unisex instead of a sex-specific mortality table. If a retiree retains his tax qualified retirement assets with his former company during the decumulation phase, the annuity must be priced using a unisex table. Alternatively, a retiree can transfer his 401(k) plan assets to an individual retirement account (IRA) offered by a private-sector financial institution, which is allowed to use sex-specific mortality tables to price annuities offered outside the plan. Table 5 depicts results for the various subgroups when LIA’s are priced using a unisex table. For men (women), not surprisingly, the welfare gains of such the default solutions decreases (increases) compared to the situation with sex-specific annuity pricing (see Table 4). Yet the welfare gain is still remarkably high for workers having Coll+ and High School education. Even for female high school dropouts, the simple default solution based on a 10%-fixed percentage rule produces a small welfare cost ($-465) (assuming mortality rate 34% above average). The fixed-
percentage rule plus an asset threshold of $ 65,000 overcomes this problem since the welfare
gains are again positive ($558). Overall, introducing the asset threshold generally yields welfare
gains compared to the situation without the asset threshold.

Table 5 here

In sum, this section has shown that requiring workers to devote a fixed fraction of their
401(k) accounts to longevity income annuities starting at age 85, and additionally, limiting the
requirement to savers having at least $65,000 in their retirement accounts, does not place undue
hardships on older men or women across the board. Moreover, this approach offers a way for
retirees to enhance their lifetime consumption, protect against running out of money in old age,
and enjoy greater utility levels than without the LIAs.

6 Conclusion and Implications

We have examined the potential impact of a recent effort to “put the pension back” into
defined contribution plans. This is a concern to the extent that financially-inexperienced
consumers may do a poor job handling investment and longevity risk in their self-directed
retirement accounts.28

This important change in Treasury regulations has dramatically reversed a deep-seated
institutional bias against including annuities in US private-sector pensions, by permitting
retirees to purchase a deferred lifetime income annuity using a portion of their plan assets
without negative tax consequences.29 We show that this development can reverse the traditional
reluctance to annuitize in the context of a realistic and richly-specified life cycle model which
takes into account stochastic capital market returns, labor income streams, and mortality, as
well as taxes, Social Security benefits, and RMD rules for 401(k) plans. We show that both
women and men benefit in expectation from the LIAs, and even lower-paid and less-educated

28 For a review of the impact of financial illiteracy on economic behavior see Lusardi and Mitchell (2014).
29 Similar suggestions have been made in the context of state-sponsored retirement plans for the non-pensioned,
now under development in 28 states (e.g., Iwry and Turner 2009; IRS 2014).
individuals stand to gain from this innovation. Moreover, we conclude that plan sponsors wishing to integrate a deferred lifetime annuity as a default in their plans can do so to a meaningful extent by converting as little as 10% of retiree plan assets, and particularly if the default is implemented for workers having plan assets over a reasonable threshold.

Financial institutions, insurance companies, and mutual fund companies are increasingly focused on helping Baby Boomers build retirement security, so this research should interest those seeking to guide this generation as it determines how to manage 401(k) plan assets into retirement. Similar recommendations are likewise relevant to the management of Individual Retirement Accounts as these too are subject to the RMD rules and relevant tax considerations described above. Additionally, regulators concerned with enhancing retirement security will find useful the default LIA mechanism described here, to help protect retirees from running out of money in old age. Certainly not least, our results confirm that those seeking to explain household saving and portfolio allocation patterns can benefit by incorporating influential and highly important institutional features of the financial environment into their models.
References


Iwry, J. M. (2014). Excerpted Remarks of J. Mark Iwry, Senior Advisor to the Secretary of the Treasury and Deputy Assistant Secretary for Retirement and Health Policy. For the *Insured Retirement Institute*, July 1.


Figure 1: Estimated average income profiles for females and males

Panel A. Female expected income profiles

Panel B. Male expected income profiles

Note: The average income profiles are based on our wage rate regressions from PSID data (see Appendix A for details), assuming a 40 hour work-week and 52 weeks of employment per year. Educational groupings are less than High School, High School graduate, and at least some college (<HS, HS, +Coll). Source: Authors’ calculations.
Figure 2: Simulated versus empirical 401(k) median account values

Note: The figure compares empirical 401(k) account balances across the US population with our model simulations where workers lack access to LIAs. Model simulations are based on median 401(k) levels generated for 100,000 simulated lifecycles for each of six subgroups of employees (male/female by three education groups, <HS, HS, and Coll+). Model parameters include risk aversion $\rho = 5$; time preference $\beta = 0.96$; retirement age 66; risk-free interest rate 1%; mean stock return 5%; and stock return volatility 18%. For parameters for labor income profiles see Table A1. Values for the full population are generated using education subgroups fractions from the National Center on Education Statistics (2012); see text. Empirical account balance data are taken from the Employee Benefit Research Institute (2014); age groups referred to as 20s, 30s, 40s, 50s, and 60s denote average values for persons age 20-29, 30-39, 40-49, 50-59, and 60-69. Source: Authors’ calculations
Figure 3: Life cycle profiles without vs with access to a Longevity Income Annuity (LIA)

Panel A. No lifetime income annuity available (Pre-2014 RMD rules)
Panel B. With longevity income annuity (Post RMD change)

Note: These two figures show expected values from 100,000 simulated lifecycles for the US population having access to 401(k) plans. Panel A shows average consumption, wealth, withdrawals, and income (work, pension, and LIA benefits if any) without and Panel B with access to longevity income annuities. Model parameters include risk aversion $\rho = 5$; time preference $\beta = 0.96$; labor income risk; retirement age 66; risk-free interest rate 1%; mean stock return 5%; stock return volatility 18%. Source: Authors’ calculations.
Figure 4: Consumption differences over the life cycle with versus without access to the Longevity Income Annuity (LIA)

Note: Distribution (95%; 5%) of consumption differences for 100,000 life-cycles of average US workers with 401(k) plans, with and without access to LIAs starting benefits at age 85. Men and women in three educational groups are modeled, namely those with Coll+, HS, <HS. Darker areas represent higher probability mass. For parameter, values see Figure 1. Source: Authors’ calculations.
Table 1: Life cycle patterns of 401(k) accumulations ($000) by sex and education groupings: Without and with access to Longevity Income Annuities (LIA)

<table>
<thead>
<tr>
<th></th>
<th>Female &lt;HS</th>
<th>Female HS</th>
<th>Female Coll+</th>
<th>Male &lt;HS</th>
<th>Male HS</th>
<th>Male Coll+</th>
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<tbody>
<tr>
<td><strong>A: 401(k) account ($000) without access to LIA</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Age 25-34</td>
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<td>9.95</td>
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<td><strong>B: 401(k) account ($000) with access to LIA</strong></td>
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</tbody>
</table>

**Note:** Expected values in $2013 based on 100,000 simulated life cycles; we report average values over 10-year age bands. Model calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; labor income risk (see Table A1); retirement age 66; Social Security benefits are computed as described in the text with bend points as of 2013; LIA refers to annuitized 401(k) assets paying lifelong annuity benefits from age 85 on; minimum required withdrawals from 401(k) plans are based on life expectancy using the IRS-Uniform Lifetime Table 2013; for taxes, 401(k) plans available in tax-qualified account, taxation as described in Appendix B; risk-free interest rate 1%; mean stock return 5%; stock return volatility 18%. Source: Authors’ calculations.
Table 2: Life cycle patterns of 401(k) accumulations ($000) by sex and education groupings: Without and with access to Longevity Income Annuities (LIA) using alternative assumptions on mortality, preferences, and deferring time

<table>
<thead>
<tr>
<th></th>
<th>Female Coll+ LIA w/ unisex mort</th>
<th>Male &lt;HS; mort. +25%</th>
<th>Female &lt;HS; mort. +34%</th>
<th>Female Coll+ LIA @80</th>
<th>Female Coll+ w/ Bequest</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A: 401(k) account</strong>&lt;sup&gt;($000)&lt;/sup&gt; <strong>without access to LIA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 25-34</td>
<td>42.80</td>
<td>17.53</td>
<td>10.31</td>
<td>42.80</td>
<td>30.98</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>118.99</td>
<td>39.62</td>
<td>23.54</td>
<td>118.99</td>
<td>113.28</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>187.97</td>
<td>60.63</td>
<td>36.25</td>
<td>187.97</td>
<td>189.33</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>233.34</td>
<td>78.25</td>
<td>48.51</td>
<td>233.34</td>
<td>245.88</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>167.60</td>
<td>45.71</td>
<td>24.20</td>
<td>167.60</td>
<td>188.15</td>
</tr>
<tr>
<td>Age 75-84</td>
<td>78.35</td>
<td>11.41</td>
<td>3.96</td>
<td>78.35</td>
<td>98.96</td>
</tr>
<tr>
<td>Age 85-94</td>
<td>22.37</td>
<td>1.42</td>
<td>0.33</td>
<td>22.37</td>
<td>40.34</td>
</tr>
<tr>
<td><strong>B: 401(k) account</strong>&lt;sup&gt;($000)&lt;/sup&gt; <strong>with access to LIA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 25-34</td>
<td>42.93</td>
<td>17.28</td>
<td>9.79</td>
<td>42.82</td>
<td>31.00</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>117.83</td>
<td>38.76</td>
<td>23.42</td>
<td>117.29</td>
<td>112.77</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>184.52</td>
<td>60.19</td>
<td>36.17</td>
<td>185.05</td>
<td>188.50</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>227.09</td>
<td>78.85</td>
<td>48.48</td>
<td>228.97</td>
<td>243.22</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>129.87</td>
<td>41.85</td>
<td>23.18</td>
<td>99.90</td>
<td>154.62</td>
</tr>
<tr>
<td>Age 75-84</td>
<td>35.03</td>
<td>7.51</td>
<td>2.97</td>
<td>13.96</td>
<td>62.69</td>
</tr>
<tr>
<td>Age 85-94</td>
<td>1.44</td>
<td>0.22</td>
<td>0.11</td>
<td>1.30</td>
<td>21.80</td>
</tr>
<tr>
<td><strong>C: LIA purchased at age 65 (000)</strong></td>
<td>32.89</td>
<td>5.33</td>
<td>1.41</td>
<td>60.91</td>
<td>29.81</td>
</tr>
<tr>
<td><strong>D: LIA Payout p.a.($000)</strong></td>
<td>8.45</td>
<td>1.61</td>
<td>0.32</td>
<td>7.83</td>
<td>6.68</td>
</tr>
</tbody>
</table>

**Note:** First column reports results for a female Coll+ participant without and with access to the LIA available at age 85, priced with unisex mortality tables. Second (third) columns refer to a male (female) high school dropout without and with access to the LIA available at age 85, assuming higher sex-specific mortality (see text). Fourth column refers to female Coll+ participant without and with access to the LIA available at age 80, priced with female mortality tables. Final column female Coll+ participant without and with access to the LIA available at age 85, priced with female mortality tables, and including a bequest motive b=4 (see text). Source: Authors’ calculations.
Table 3: Welfare gains and ratio of 401(k) devoted to annuity at age 66 without and with access to Longevity Income Annuities (LIA): Optimal annuitization outcomes

<table>
<thead>
<tr>
<th>Case</th>
<th>Education</th>
<th>Alternative specifications</th>
<th>Optimal LIA Ratio (%)</th>
<th>Welfare Gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female age 66</td>
<td>Coll+</td>
<td>LIA sex specific</td>
<td>15.04</td>
<td>13,120</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td>LIA unisex mortality</td>
<td>14.48</td>
<td>15,384</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>LIA at age 80</td>
<td>26.72</td>
<td>15,802</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Bequest</td>
<td>12.10</td>
<td>12,968</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +34%</td>
<td>9.79</td>
<td>6,280</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +25%</td>
<td>5.27</td>
<td>2,204</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +34%</td>
<td>2.64</td>
<td>424</td>
</tr>
</tbody>
</table>

Male age 66

<table>
<thead>
<tr>
<th>Education</th>
<th>Alternative specifications</th>
<th>Optimal LIA Ratio (%)</th>
<th>Welfare Gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coll+</td>
<td>LIA sex specific</td>
<td>14.26</td>
<td>35,837</td>
</tr>
<tr>
<td>High School</td>
<td>LIA unisex mortality</td>
<td>11.32</td>
<td>13,999</td>
</tr>
<tr>
<td>&lt; High School</td>
<td>LIA at age 80</td>
<td>8.94</td>
<td>5,696</td>
</tr>
<tr>
<td>&lt; High School</td>
<td>Bequest</td>
<td>6.28</td>
<td>2,764</td>
</tr>
</tbody>
</table>

Note: See notes to Table 1. LIA Ratio (%) refers to the fraction of the individual’s 401(k) plan assets used to purchase the LIA at age 65. Welfare Gain ($) refers to the retiree’s additional utility value from having access to the LIA versus no access at age 66. Source: Authors’ calculations.

Table 4: Welfare gains at age 66 without and with access to default Longevity Income Annuities (LIA): Two default solutions

<table>
<thead>
<tr>
<th>Case</th>
<th>Education</th>
<th>Alternative specifications</th>
<th>Welfare gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female age 66</td>
<td>Coll+</td>
<td>LIA sex specific</td>
<td>10% fixed fraction default (No min assets)</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td>LIA unisex mortality</td>
<td>12,810</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td>LIA at age 80</td>
<td>5,467</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td>Bequest</td>
<td>1,287</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td>Mortality +34%</td>
<td>-1,149</td>
</tr>
</tbody>
</table>

Male age 66

<table>
<thead>
<tr>
<th>Education</th>
<th>Alternative specifications</th>
<th>Welfare gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coll+</td>
<td>LIA sex specific</td>
<td>33,032</td>
</tr>
<tr>
<td>High school</td>
<td>LIA unisex mortality</td>
<td>13,245</td>
</tr>
<tr>
<td>&lt; High School</td>
<td>LIA at age 80</td>
<td>5,208</td>
</tr>
<tr>
<td>&lt; High School</td>
<td>Bequest</td>
<td>1,840</td>
</tr>
<tr>
<td>&lt; High School</td>
<td>Mortality +25%</td>
<td>32,938</td>
</tr>
<tr>
<td>&lt; High School</td>
<td>Mortality +34%</td>
<td>5,393</td>
</tr>
</tbody>
</table>

Notes: In the case of the fixed fraction default approach, 10% of retirees’ 401(k) accounts are converted into a LIA when they turn age 65. In this fixed fraction + threshold default approach, 10% of assets are converted into longevity income annuities only when the worker’s 401(k) account equals or exceeds the threshold of $65,000. See notes to Tables 1 and 3. Source: Authors’ calculations.
Table 5: Welfare gains at age 66 without and with access to default Longevity Income Annuities (LIA): Two default solutions with unisex pricing of LIA

<table>
<thead>
<tr>
<th>Case</th>
<th>Education</th>
<th>Alternative specifications</th>
<th>10% fixed fraction default (No min assets)</th>
<th>10% fixed fraction + threshold default (Min $ 65K assets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female age 66</td>
<td>Coll+</td>
<td></td>
<td>13,557</td>
<td>13,521</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td></td>
<td>7,557</td>
<td>7,796</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td></td>
<td>3,643</td>
<td>4,403</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td>Mortality +34%</td>
<td>-465</td>
<td>558</td>
</tr>
<tr>
<td>Male age 66</td>
<td>Coll+</td>
<td></td>
<td>28,451</td>
<td>28,445</td>
</tr>
<tr>
<td></td>
<td>High school</td>
<td></td>
<td>10,644</td>
<td>10,787</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td></td>
<td>4,007</td>
<td>4,481</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +25%</td>
<td>421</td>
<td>1,317</td>
</tr>
</tbody>
</table>

Notes: In the case of the fixed fraction default approach, 10% of retirees’ 401(k) accounts are converted into a LIA when they turn age 65. In the fixed fraction + threshold default approach, the 10% of assets are converted into longevity income annuities only when the worker’s 401(k) account equals or exceeds the threshold of $65,000. See notes to Tables 1 and 3. Source: Authors’ calculations.
Appendix A: Wage rate estimation

We calibrated the wage rate process using the Panel Study of Income Dynamics (PSID) 1975-2013 from age 25 to 69. During the work life, the individual’s labor income profile has deterministic, permanent, and transitory components. The shocks are uncorrelated and normally distributed according to \( \ln(N_t) \sim N(-0.5\sigma_N^2, \sigma_N^2) \) and \( \ln(U_t) \sim N(-0.5\sigma_U^2, \sigma_U^2) \). The wage rate values are expressed in $2013. These are estimated separately by sex and by educational level. The educational groupings are: less than High School (<HS), High School graduate (HS), and those with at least some college (Coll+). Extreme observations below $5 per hour and above the 99th percentile are dropped.

We use a second order polynomial in age and dummies for employment status. The regression function is:

\[
\ln(w_{i,y}) = \beta_1 \ast \text{age}_{i,y} + \beta_2 \ast \text{age}_{i,y}^2 + \beta_5 \ast \text{ES}_{i,y} + \beta_{\text{waves}} \ast \text{wave dummies},
\]

where \( \ln(w_{i,y}) \) is the natural log of wage at time \( y \) for individual \( i \), age is the age of the individual divided by 100, \( \text{ES} \) is the employment status of the individual, and wave dummies control for year-specific shocks. For employment status we include three groups depending on work hours per week as follows: part-time worker (\( \leq 20 \) hours), full-time worker (\( < 20 \& \leq 40 \) hours) and over-time worker (\( < 40 \) hours). OLS regression results for the wage rate process equations appear in Table A1.

To estimate the variances of the permanent and transitory components, we follow Carroll and Samwick (1997) and Hubener et al. (2016). We calculate the difference of the observed log wage and our regression results, and we take the difference of these differences across different lengths of time \( d \). For individual \( i \), the residual is:

\[
r_{i,d} = \sum_{s=0}^{d-1} (N_{t+s}) + U_{i,t+d} - U_{i,t}
\]

We then regress the \( v_{id} = r_{i,d}^2 \) on the lengths of time \( d \) between waves and a constant:

\[
v_{id} = \beta_1 \cdot d + \beta_2 \cdot 2 + e_{id},
\]

where the variance of the permanent factor \( \sigma_N^2 = \beta_1 \) and the \( \sigma_U^2 = \beta_2 \) represents the variance of the transitory shocks.
### Table A1: Regression results for wage rate

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Male &lt;HS</th>
<th>Male HS</th>
<th>Male +Coll</th>
<th>Female &lt;HS</th>
<th>Female HS</th>
<th>Female +Coll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age/100</td>
<td>3.146***</td>
<td>6.098***</td>
<td>9.117***</td>
<td>1.253***</td>
<td>2.820***</td>
<td>4.646***</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.050)</td>
<td>(0.073)</td>
<td>(0.109)</td>
<td>(0.047)</td>
<td>(0.075)</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
<td>(0.063)</td>
<td>(0.093)</td>
<td>(0.131)</td>
<td>(0.061)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>Part-time work</td>
<td>-0.110***</td>
<td>-0.159***</td>
<td>-0.086***</td>
<td>-0.088***</td>
<td>-0.127***</td>
<td>-0.088***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Over-time work</td>
<td>0.004</td>
<td>0.049***</td>
<td>0.095***</td>
<td>0.017***</td>
<td>0.075***</td>
<td>0.106***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.006)</td>
<td>(0.002)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.929***</td>
<td>1.468***</td>
<td>1.073***</td>
<td>2.068***</td>
<td>1.968***</td>
<td>1.950***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.011)</td>
<td>(0.015)</td>
<td>(0.028)</td>
<td>(0.01)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Observations</td>
<td>49,083</td>
<td>315,685</td>
<td>270,352</td>
<td>31,651</td>
<td>279,375</td>
<td>207,640</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.068</td>
<td>0.102</td>
<td>0.147</td>
<td>0.033</td>
<td>0.044</td>
<td>0.093</td>
</tr>
</tbody>
</table>

| Permanent   | 0.009*** | 0.013*** | 0.019***  | 0.008***   | 0.013***   | 0.0189***    |
|             | (0.0005) | (0.0002) | (0.0003)  | (0.0006)   | (0.0002)   | (0.0003)     |
| Transitory  | 0.028*** | 0.031*** | 0.041***  | 0.023***   | 0.028***   | 0.040***     |
|             | (0.001)  | (0.0006) | (0.0009)  | (0.0015)   | (0.0006)   | (0.001)      |
| Observations | 28,548   | 170,469  | 131,836   | 20,884     | 170,735    | 114,700      |
| R-squared   | 0.214   | 0.279    | 0.301     | 0.157      | 0.252      | 0.266        |

**Notes:** Regression results for the natural logarithm of wage rates are based on information in the Panel Study of Income Dynamics (PSID) for persons age 25-69 in waves 1975-2013. Independent variables include age and age-squared, and dummies for part time work (≤20 hours per week) and overtime work (≥ 40 hours per week). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Source: Authors’ calculations.

### Appendix B: 401(k) plans tax-qualified pension account

We integrate a US-type progressive tax system into our model to explore the impact of having access to a qualified (tax-sheltered) pension account of the EET type.\(^{30}\) Here the worker must pay taxes on labor income and on capital gains from investments in bonds and stocks. During the working life, he invests \(A_t\) in the tax-qualified pension account, which reduces taxable income up to an annual maximum amount \(D_t=$18,000. Correspondingly, withdrawals \(W_t\) from the tax-qualified account increase taxable income. Finally, the worker’s taxable

\(^{30}\) That is, contributions and investment earnings in the account are tax exempt (E), while payouts are taxed (T).
income is reduced by a general standardized deduction $GD$. For a single person, this deduction amounted to $5,950 per year. Consequently, taxable income in working age is given by:

$$Y_{t+1}^{\text{tax}} = \max\{\max(S_t \cdot (R_t + 1) + B_t \cdot (R_f - 1) + 0) + Y_{t+1}(1-h_t) + W_t - \min(A_t; D_t) - GD; 0\} \quad \text{(B1)}$$

For Social Security ($Y_{t+1}$) taxation up to age 66, we use the following rules: when combined income$^{31}$ is between $25,000 and $34,000 (over $34,000), 50% (85%) of benefits are taxed.$^{32}$

In line with US rules for federal income taxes, our progressive tax system has six income tax brackets (IRS 2012a). These brackets $i = 1, \ldots, 6$ are defined by a lower and an upper bound of taxable income $Y_{t+1}^{\text{tax}} \in [lb_i, ub_i]$ and determine a marginal tax rate $r_i^{\text{tax}}$. For the year 2012, the marginal taxes rates for a single household are 10% from $0$ to $8700, 15% from $8701 to $35,350, 25% from $35,351 to 85,659, 28% from $85,651 to $178,650, 33% from $178,651 to $388,350, and 35% above $388,350 (see IRS 2012a). Based on these tax brackets, the dollar amount of taxes payable is given by:$^{33}$

$$\text{Tax}_{t+1}(Y_{t+1}^{\text{tax}}) = (Y_{t+1}^{\text{tax}} - lb_6) \cdot 1_{[Y_{t+1}^{\text{tax}} \geq lb_6]} \cdot r_6^{\text{tax}} + (Y_{t+1}^{\text{tax}} - lb_5) \cdot 1_{[lb_6 > Y_{t+1}^{\text{tax}} \geq lb_5]} \cdot (ub_5 - lb_5) \cdot 1_{[Y_{t+1}^{\text{tax}} \geq lb_5]} \cdot r_5^{\text{tax}} + (Y_{t+1}^{\text{tax}} - lb_4) \cdot 1_{[lb_5 > Y_{t+1}^{\text{tax}} \geq lb_4]} \cdot (ub_4 - lb_4) \cdot 1_{[Y_{t+1}^{\text{tax}} \geq lb_4]} \cdot r_4^{\text{tax}} + (Y_{t+1}^{\text{tax}} - lb_3) \cdot 1_{[lb_4 > Y_{t+1}^{\text{tax}} \geq lb_3]} \cdot (ub_3 - lb_3) \cdot 1_{[Y_{t+1}^{\text{tax}} \geq lb_3]} \cdot r_3^{\text{tax}} + (Y_{t+1}^{\text{tax}} - lb_2) \cdot 1_{[lb_3 > Y_{t+1}^{\text{tax}} \geq lb_2]} \cdot (ub_2 - lb_2) \cdot 1_{[Y_{t+1}^{\text{tax}} \geq lb_2]} \cdot r_2^{\text{tax}} + (Y_{t+1}^{\text{tax}} - lb_1) \cdot 1_{[lb_2 > Y_{t+1}^{\text{tax}} \geq lb_1]} \cdot (ub_1 - lb_1) \cdot 1_{[Y_{t+1}^{\text{tax}} \geq lb_1]} \cdot r_1^{\text{tax}}, \quad \text{B2}$$

where, for $A \subseteq X$, the indicator function $1_A : \{0, 1\}$ is defined as:

$$1_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases} \quad \text{(B3)}$$

In line with US regulation, the individual must pay an additional penalty tax of 10% on early withdrawals prior to age 59 ½ ($t = 36$):

$$\text{Tax}_{t+1}(Y_{t+1}^{\text{tax}}) = \begin{cases} \text{Tax}_{t+1}(Y_{t+1}^{\text{tax}}) & t \geq 36 \\ \text{Tax}_{t+1}(Y_{t+1}^{\text{tax}}) + 0.1W_t & t < 36 \end{cases} \quad \text{(B4)}$$

$^{31}$ Combined income is sum of adjusted gross income, nontaxable interest, and half of his Social Security benefits.

$^{32}$ See https://www.ssa.gov/planners/taxes.html

$^{33}$ Here we assume that capital gains are taxed at the same rate as labor income, so we abstract from the possibility that long-term investments may be taxed at a lower rate.
Appendix C: Population mortality tables differentiated by education and sex

Research has shown that lower-educated individuals have lower life expectancies than better-educated individuals. This is relevant to the debate over whether and which workers need annuitization. To explore the impact of this difference in mortality rates by educational levels, we follow Kreuger et al. (2015) who calculated mortality rates by education and sex ($M_{sex}^{education}$) as below:

$$M_{male}^{average} = 0.1M_{male}^{<HS} + 0.3M_{male}^{HS} + 0.6M_{male}^{Coll+}$$

$$= 0.1(M_{male}^{HS} \cdot 1.23) + 0.3M_{male}^{HS} + 0.6(M_{male}^{HS} \cdot 0.94)$$

$$= 0.987 \cdot M_{male}^{HS}$$

Next we calculate the mortality for a male with a HS degree as follows:

$$M_{male}^{HS} = \frac{M_{male}^{average}}{0.987}$$

And mortality for a male high school dropout or with Coll+ level education is as follows:

$$M_{male}^{<HS} = \frac{M_{male}^{average}}{0.987} \cdot 1.23$$

$$M_{male}^{Coll+} = \frac{M_{male}^{average}}{0.987} \cdot 0.94$$

Analogously, we calculate for females with different levels of education the following:

$$M_{female}^{<HS} = \frac{M_{female}^{average}}{0.984} \cdot 1.32$$

$$M_{female}^{HS} = \frac{M_{female}^{average}}{0.984}$$

$$M_{female}^{Coll+} = \frac{M_{female}^{average}}{0.984} \cdot 0.92$$

We price the annuity as before using average annuitant mortality tables.