Putting the Pension Back in 401(k) Plans: Optimal versus Default Longevity Income Annuities

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Abstract

Most defined contribution pension plans pay benefits as lump sums, yet the US Treasury has recently encouraged firms to protect retirees from outliving their assets by converting a portion of their plan balances into longevity income annuities (LIA). These are deferred annuities which initiate payouts not later than age 85 and continue for life, and they provide an effective way to hedge systematic (individual) longevity risk for a relatively low price. Using a life cycle portfolio framework, we measure the welfare improvements from including LIAs in the menu of plan payout choices, accounting for mortality heterogeneity by education and sex. We find that introducing a longevity income annuity to the plan menu is attractive for most DC plan participants who optimally commit 8-15\% of their plan balances at age 65 to a LIA that starts paying out at age 85. Optimal annuitization boosts welfare by 5-20\% of average retirement plan accruals at age 66 (assuming average mortality rates), compared to not having access to the LIA. We also compare the optimal LIA allocation versus two default options that plan sponsors could implement. We conclude that an approach where a fixed fraction over a dollar threshold is invested in LIAs will be preferred by most to the status quo, while enhancing welfare for the majority of workers.

Keywords: dynamic portfolio choice; longevity risk; variable annuity; retirement income

JEL Codes: G11, G22, H55, J26, J32

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In the US workplace, defined contribution (DC) plans have become the norm as the primary tax-qualified mechanism helping private sector workers save for retirement. Yet most 401(k) plans do not currently offer access to lifelong income payments to cover the decumulation or drawdown phase of the lifecycle.¹ This is a concern to the extent that financially inexperienced consumers may do a poor job handling investment and longevity risk in their self-directed retirement accounts.² To correct this problem, the US Department of the Treasury recently launched an initiative to provide firms and employers “more options for putting the pension back” into private sector defined contribution plans (Iwry 2014). This was accompanied by an adjustment in the tax rules governing retirement plans that facilitated lifelong payouts - not only in 401(k) plans, but also in Individual Retirement Accounts (IRAs) and 403(b) tax-sheltered annuities for employees of nonprofit employers, by converting retirement assets into longevity income annuities (LIAs).³ These are deferred life annuities that start payouts at an advanced age (e.g., age 85) and continue for life. Such instruments provide a low-cost way to hedge the risk of outliving one’s assets.

This paper develops a realistic life cycle model to quantify the potential impact of this new policy for a range of retiree types, differentiated by sex, educational level, and preferences. Taking account of real-world income tax rules, Social Security contribution and benefit rules, and the RMD regulations, we first evaluate how much participants will optimally elect to

¹ Benartzi, Previtero, and Thaler (2011) note that only about one-fifth of U.S. defined contribution plans currently offer annuities as a payout option; a small survey of 22 plan record-keepers by the US GAO (2016: 13) concluded that few plans currently offer participants ways to “help them secure lifetime income in retirement.” Most innovation in the DC arena over the last decade has instead focused on the accumulation phase, with the introduction of products to attract saving including life cycle or target date funds and the widespread adoption of automatic 401(k) enrollment and automatic escalation of contributions (c.f. Gomes, Kotlikoff and Viceira 2008; Poterba et al. 2007). Some countries including Germany require retirees to convert a part of their accumulated tax-qualified retirement assets into a longevity annuity beginning at age 85 (see Maurer and Somova 2009 and Dus et al. 2005).
² For a review of the impact of financial illiteracy on economic behavior see Lusardi and Mitchell (2015).
³ This was originally suggested by Gale et al. (2008).
annuitize given the opportunity to do so, when they face income, spending, and capital market shocks, and where they also are subject to uncertainty about their lifespans. In such an environment, we assess how much better off they would be if their options included LIAs in the payout menu, versus without access to LIAs. Next, we compare this case with what would happen if the plan sponsor were to default a certain percentage of retiree assets into a deferred annuity. And finally, we compare the retiree’s optimal allocation to LIAs versus a default option, taking into account mortality heterogeneity by education and sex. In this paper, we use a life cycle framework to explore the impact of including longevity income annuities in the menu of payout choices. We measure the potential improvements in well-being resulting from this reform, and our results indicate how the demand for these annuity products varies with participant characteristics such as educational levels and mortality experience, while taking into account both labor income and capital market risk. We also investigate how such products can be implemented as a default solution analogous to how Target Date Funds (TDFs) have been adopted during the accumulation phase. Most importantly, we present the anticipated welfare implications of incorporating such products in retirement plans, taking into account realistic income taxation and required minimum distribution rules.

It is worth noting that it is quite inexpensive to protect against running out of money with a deferred annuity. Even in the current low interest rate environment, a deferred single life annuity purchased at age 65 for a male (female) costing $10,000 can generate an annual benefit flow from age 85 onward of $4,830 ($3,866) per year for life. This results from the investment returns earned over the 20 years prior to the withdrawal start date, plus the accumulated survival

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4 For instance Iwry (2014) illustrated the case where the retiree converts 15% of her plan assets to the deferred annuity. Iwry and Turner (2009) explored two approaches to make deferred income annuities the default payout approach in 401(k) plans. A US Department of Labor letter to Mark Iwry (US DOL 2014) explicitly permitted plan sponsors to include annuity contracts as fixed income investments in a 401(k) plan.
5 The 2006 Pension Protection Act allowed plan sponsors to offer Target Date Funds as qualified default investment alternatives in participant-directed individual account plans (US DOL nd). A 2014 Treasury/IRS Administrative Guidance letter (IRS 2014) made clear that annuities – including deferred income annuities – could be a 401(k) default option.
6 Quotes available August 2016 on https://www.immediateannuities.com/
credits resulting from premiums paid by those who die earlier than expected being shared with those who survive.

Much has been written on the economic appeal of annuities in a household context, yet in practice few people purchase them (Brown et al. 2001; Mitchell et al. 2011). Explanations point to factors such as product costs/loadings, retiree bequest motives and/or liquidity needs, and behavioral factors including complexity. Yet one important reason not examined to date has to do with institutional factors discouraging annuitization in 401(k) plans. Specifically, until 2014, US tax rules required retirees to withdraw from their retirement accounts the so-called “Required Minimum Distribution” (RMD) amount each year from age 70.5 onward, where the RMD was computed so that the sum of annual payouts was expected to exhaust the retiree’s 401(k) balance by the end of her life (IRS 2012b). If a retiree did purchase an annuity with her plan assets, her RMD was still calculated taking into account the value of her annuity. This had the unappealing consequence that the retiree might find herself needing to withdraw an amount in excess of her liquid assets (excluding the annuity value) and be forced to pay a 50% excise tax (Iwry 2014).

In 2014, the US Treasury decided to permit and, for the first time, encourage the offering of longevity annuities within the more than $14 trillion US 401(k) and IRA markets “by amending the required minimum distribution regulations…to provide a measure of additional flexibility consistent with the statutory RMD provisions” (Iwry 2014). Approved deferred annuities thus had to begin payouts not later than age 85 and cost less than 25% of the retiree’s account balance (up to a limit). Under these conditions, the retiree’s annuity would no longer be counted in determining her RMD. This policy change therefore relaxed the RMD requirements that had effectively precluded the offering of longevity annuities in the 401(k)

7The discrepancy between the appeal in theoretical models (see originally Yaari 1965, and more recently Davidoff et al. 2005) and the low annuity take-up rates of households is also referred to as the ‘annuity puzzle’ (see, e.g., Inkmann et al., 2011).
8 Treasury had originally proposed these amendments to the regulations two years earlier, referring to the new longevity annuities as “qualifying longevity annuity contracts” (or “QLACS”); see US Treasury 2014).
and IRA contexts. This is important because outliving one’s assets is one of the most important risks people face, an especially critical matter in old age when one generally cannot return to work and when healthcare costs may be large. For example, the expected remaining lifetime for a 65-year-old US female is about 21 years using the general population statistics (Arias 2016). Yet there is considerable volatility around the mean (around nine years), implying that individuals’ uncertainty about the length of their lifetimes will drive retirement consumption and thus lifetime well-being.

To explore the policy, we develop a realistic life cycle model to quantify the potential impact of this new policy for a range of retiree types, differentiated by sex, educational level, and preferences. We take account of real-world income tax rules, Social Security contribution and benefit rules, and the RMD regulations discussed above. Our analysis first evaluates how much participants will optimally elect to annuitize given the opportunity to do so, when they face income, spending, and capital market shocks, and where they also are subject to uncertainty about their lifespans. In such an environment, we then assess how much better off they would be if their options included LIAs in the payout menu, versus without access to them. Next, we compare this case with what would happen if the plan sponsor were to default a certain percentage of retiree assets into a deferred annuity. And finally, we compare the retiree’s optimal allocation to LIAs versus a default option, taking into account mortality heterogeneity by education and sex.

To preview our findings, we show that introducing a longevity income annuity to the plan menu is quite attractive to the majority of DC plan participants. Overall, older individuals would optimally commit 8-15% of their plan balances at age 65 to a LIA which begins payouts at age 85. When participants can select their own optimal annuitization rates, welfare increases by 5-20% of average retirement plan accruals as of age 66 (assuming average mortality rates)

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9 For instance Iwry (2014) illustrated a case where the retiree converted 15% of her plan assets to the deferred annuity.
compared to not having access to LIAs. If, instead, plan sponsors were to default participants into deferred annuities using 10% of their retirement age plan assets, this would reduce retiree wellbeing only slightly compared to the optimum. Results are less positive for those with substantially higher mortality vis a vis population averages; for such individuals, using a fixed percentage default rule generates lower welfare since annuity prices based on average mortality rates are too high. Converting retirement assets into a longevity annuity only for those having over $65,000 in their retirement accounts overcomes this problem. Accordingly, we conclude that including well-designed LIA defaults in DC plans yields quite positive consequences for 401(k)-covered workers. Moreover, our findings also apply to Individual Retirement Account payout designs, since the RMD rules for these accounts are nearly the same as those for 401(k) plans.

In what follows, we describe our life cycle model and explain how we use it to study optimal consumption, investment, and annuitization decisions. The model includes a realistic formulation of US income tax rules, required minimum distribution rules for 401(k)-plans, payroll taxes for Social Security benefits, and rules for claiming retirement benefits. In addition, we report the possible welfare implications of having access to LIAs. Sensitivity analyses illustrate how results vary across a range of parameters including uninsurable labor income profiles, sex, and preferences. Next, we discuss the impact of alternative default rules for retirement asset annuitization. A final section concludes.

**Deferred Longevity Income Annuities in a Life Cycle Model: Methodology**

Our dynamic portfolio and consumption model time posits an individual who decides over her life cycle how much to consume optimally and how much to invest in stocks, bonds, and annuities.\(^\text{10}\) We model utility as depending on consumption, while constraints include a

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\(^\text{10}\) Comparable life cycle models are devised in the work by Cocco and Gomes (2012), Kim, Maurer, and Mitchell (2016), Horneff et al. (2015), Hubener, Maurer, and Mitchell (2015), and Maurer et al. (2013).
realistic characterization of income profiles, taxes, and the opportunity to invest (to a limit) in a 401(k)-type tax-qualified retirement plan. At retirement (set here at age 66), the individual determines how much of her retirement account she wishes to convert to a deferred longevity income annuity, as well as how much she will retain in liquid stocks and bonds. We also take into account the Required Minimum Distribution rules relevant to the US 401(k) setting, as well as a realistic formulation of Social Security benefits. In a subsequent section, we provide additional robustness analysis on different preferences and mortality heterogeneity across educational categories.

Preferences. We build a discrete-time dynamic consumption and portfolio choice model for utility-maximizing investors over the life cycle. The individual’s decision period starts at $t = 1$ (age of 25) and ends at $T = 76$ (age 100); accordingly, each period corresponds to a year. The individual’s subjective probability of survival from time $t$ until $t + 1$ is denoted by $p_t$. Preferences at time $t$ are specified by a time-separable CRRA utility function defined over current consumption, $C_t$. The parameter $\rho$ represents the coefficient of relative risk aversion and $\beta$ is the time preference rate. Then the recursive definition of the corresponding value function is given by:

$$J_t = \frac{(C_t)^{1-\rho}}{1-\rho} + \beta E_t(p_t J_{t+1}),$$

(1)

where terminal utility is $J_T = \frac{(C_T)^{1-\rho}}{1-\rho}$.

The Budget Constraint during the Working Life. While working, the individual has the opportunity to invest a part ($A_t$) of her uncertain pre-tax salary $Y_t$ (to an annual limit of $18,000)$\textsuperscript{11} in a tax-qualified retirement plan held in stocks $S_t$ and bonds $B_t$:

$$X_t = C_t + S_t + B_t + A_t.$$  

(2)

\textsuperscript{11} The $18,000 limit was the legal limit on tax-deferred contributions to 401(k) plans in 2016, and if permitted by the plan, employees age 50+ can make additional 401(k) catch-up contributions of $6,000 per year.
Here $X_t$ is cash on hand after tax, $C_t$ denotes consumption, and $C_t, A_t, S_t, B_t \geq 0$. One year later, her cash on hand is given by the value of her stocks having earned an uncertain gross return $R_t$, bonds having earned riskless return of $R_f$, labor income $Y_{t+1}$ reduced by housing costs $h_t$ modeled as a percentage of labor income (as in Love 2010), and withdrawals ($W_t$) from her 401(k) plan:\footnote{Withdrawals before age 59 1/2 result in a 10% penalty tax.}

$$X_{t+1} = S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - Tax_{t+1} - Y_{t+1} d_w$$  \hfill (3)

During her working life, the individual also pays taxes, which reduce her cash on hand available for consumption and investment.\footnote{For more details, see Appendix B.} First, labor income is reduced by 11.65\% ($d_w$), which is the sum of the Medicare (1.45\%), city/state (4\%), and Social Security (6.2\%) taxes. In addition, the worker also must pay income taxes ($Tax_{t+1}$) according to US federal progressive tax system rules (IRS 2012b).

The individual may save in her tax-qualified 401(k) plan only during her working period, but non-pension saving in bonds and stocks is allowed over her entire life cycle. The exogenously-determined labor income process is $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$ with a deterministic trend, $f(t)$, permanent income component, $P_{t+1} = P_t \cdot N_{t+1}$, and transitory shock $U_{t+1}$.

Prior to retirement, her retirement plan assets are invested in bonds which earn the risk-free pre-tax return ($R_f$), and risky stocks paying an uncertain pre-tax return ($R_t$). The total value ($L_{t+1}$) of her 401(k) assets at time $t + 1$ is therefore determined by her previous period’s value, minus any withdrawals ($W_t \leq L_t$), plus additional contributions ($A_t$), and returns from stocks and bonds:

$$L_{t+1} = \omega_t^2 (L_t - W_t + A_t) R_{t+1} + (1 - \omega_t^5) (L_t - W_t + A_t) R_f, \quad \text{for} \ t < K$$  \hfill (4)
Her retirement plan assets are invested in a Target Date Fund with a relative stock exposure that declines according to her age, following the popular “Age – 100” rule ($\omega_t^s = (100 - Age)/100$).

The year before she retires, at age 65 ($K - 1$), the individual can determine how much of her 401(k) assets ($LIA_{K-1}$) she will switch to a deferred longevity income annuity with income benefits starting at age 85. Accordingly, her LIA income stream ($PA$) is determined as follows:

$$PA = \frac{LIA_{K-1}}{\bar{a}_\tau}, \quad (5)$$

where $\bar{a}_\tau = \prod_{u=K}^{K+20} p_u^a \sum_{s=0}^{100-(\tau-1)} (\prod_{i=\tau}^{\tau+s} p_i^a) R_f^-(s+20)$ is the annuity factor transforming her lump sum into a payment stream from age 85. The amount she uses to buy the LIA reduces the value of her 401(k) assets invested in stocks and bonds, so the subsequent 401(k) payments are as follows:

$$L_K = \omega_{K-1}^s (L_{K-1} - W_{K-1} + A_{K-1} - LIA_{K-1}) R_K$$

$$+ (1 - \omega_{K-1}^s) (L_{K-1} - W_{K-1} + A_{K-1} - LIA_{K-1}) R_f \quad (6)$$

**The Budget Constraint in Retirement.** During retirement, the individual saves in stocks and bonds and consumes what remains:

$$X_t = C_t + S_t + B_t \quad (7)$$

Cash on hand for the next period evolves as follows:

$$X_{t+1} = \begin{cases} 
S_t R_{t+1} + B_t R_f + Y_K (1 - h_t) + W_t - Tax_{t+1} - Y_{t+1} d_r & K \leq t < \tau \\
S_t R_{t+1} + B_t R_f + Y_K (1 - h_t) + W_t - Tax_{t+1} + PA - Y_{t+1} d_r & t \geq \tau 
\end{cases} \quad (8)$$

where the LIA pays constant lifelong benefits ($PA$) from age 85 ($\tau$) onwards. At retirement, the worker has access to Social Security benefits determined by her Primary Insurance Amount.

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14 This approach satisfies the rules for a Qualified Default Investment Alternative (QDIA) as per the US Department of Labor regulations (US DOL 2006). See also Malkiel (1996) and Kim, Maurer, and Mitchell (2016).
(PIA) which is a function of her average lifetime (35 best years of) earnings.\textsuperscript{15} Her Social Security payments \((Y_{t+1})\) in retirement \((t \geq K)\) are given by:

\[
Y_{t+1} = PIA_t \cdot \varepsilon_{t+1}
\]

where \(\varepsilon_t\) is a lognormally distributed transitory shock \(\ln(\varepsilon_t) \sim N(-0.5\sigma^2_\varepsilon, \sigma^2_\varepsilon)\) with a mean of one which reflects out-of-pocket medical and other expenditure shocks (as in Love 2010).\textsuperscript{16}

According to the 2014 US Treasury rules, the present value of the LIA is excluded when determining the retiree’s RMD. However, benefit payments from the LIA from age 85 are subject to income taxes. During retirement, Social Security benefits are taxed (up to certain limits)\textsuperscript{17} at the individual federal income tax rate as well as the city/state tax rate. Payouts from the 401(k) plan are given by:

\[
L_{t+1} = \omega_t^s(L_t - W_t)R_{t+1} + (1 - \omega_t^s)(L_t - W_t)R_f, \quad \text{for } t < K .
\]

Moreover, the RMD rules require that 401(k) participants take minimum withdrawals from their plans from age 70.5 onwards, defined as a specified age-dependent percentage \((m_t)\) of plan assets, or else they must pay a substantial tax penalty. Accordingly, to avoid the excise penalty, plan payouts must be set so that \(mL_t \leq W_t < L_t\).

**Model Calibration**: Our base case parameters are consistent with those used in prior work on life-cycle portfolio choice.\textsuperscript{18} For the utility function, we set the coefficient of relative risk aversion \(\rho\) to 5, and the time discount rate \(\beta\) is 0.96. Survival rates entering into the utility function are for the US Population Life Table (from Arias 2010). For annuity pricing, we use the US Annuity 2000 mortality table provided by the Society of Actuaries (SOA nd). Annuity

\textsuperscript{15} The Social Security benefit formula is a piece-wise linear function of the Average Indexed Monthly Earnings and providing a replacement rate of 90\% up to a first bend point, 32\% between the first and a second bend point, and 15\% above that. Details are provided in Appendix C.

\textsuperscript{16} The transitory variances assumed are \(\sigma^2_\varepsilon = 0.0784\) for high school and less than high school graduates, and \(\sigma^2_\varepsilon = 0.0767\) for college graduates (as in Love 2010).

\textsuperscript{17} For detail on how we treat Social Security benefit taxation see Appendix B. Due to quite generous allowances, not many individuals pay income taxes on their Social Security benefits.

\textsuperscript{18} See for instance Brown et al. (2001).
survival rates are higher than those for the general population, because they take into account adverse selection among annuity purchasers.\textsuperscript{19} Social Security old age benefits are based on the 35 best years of income and the bend points as of 2013 (SSA nd). Thus the annual Primary Insurance Amounts (or the unreduced Social Security benefits) equal 90 percent of (12 times) the first $791 of average indexed monthly earnings, plus 32 percent of average indexed monthly earnings over $791 and through $4,768, plus 15 percent of average indexed monthly earnings over $4,768.\textsuperscript{20} Required minimum distributions (RMD) from 401(k)-plans are based on life expectancy using the IRS Uniform Lifetime Table 2013. Federal income taxes are calculated using the tax-brackets given for the year 2013 (for details see Appendix B).

Our financial market parameterizations include a risk-free interest rate of 1\% and an equity risk premium of 4\% with a return volatility of 18\%. The labor income process during the working life has both a permanent and transitory component, with uncorrelated and normally distributed shocks as \(\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)\) and \(\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)\). Following Hubener et al. (2016), we estimate the deterministic component of the wage rate process \(w_{s,t}^i\) along with the variances of the permanent and transitory wage shocks \(N_t^i\) and \(U_t^i\) using the 1975–2013 waves of the PSID.\textsuperscript{21} These are estimated separately by sex and for three education levels: for high school dropouts, high school graduates, and those with at least some college (<HS, HS, Coll+).\textsuperscript{22} Wages rates are converted into yearly income by assuming a 40 hour work week and 52 weeks of employment per year. Results appear in Figure 1, where panel A reports for the three different educational groups the expected income profiles for females and panel B for males, respectively. For all cases, the labor income pattern follows the typical hump-shaped profile in expectation. At age 66, on retirement, the worker receives a combined income stream

\textsuperscript{19} The implied loads using the annuity table are about 15-20\%; see Finkelstein and Poterba (2004)
\textsuperscript{20} For more on the Social Security formula see \url{https://www.ssa.gov/oact/cola/piaformula.html}. A similar approach is taken by Hubener et al. (2015).
\textsuperscript{21} Dollar values are all reported in $2013.
\textsuperscript{22} More details are provided in Appendix A.
from her 401(k) pension, Social Security benefits, and from age 85 on, payments from longevity
income annuities.

**Figure 1**

We use dynamic stochastic programming to solve this optimization problem. For the base case, we have five state variables: wealth \(X_t\), the total value of the individual’s fund accounts \(L_t\), payments from the LIA \(PA\), permanent income \(P_t\), and time \(t\).\(^{23}\) We also compute the individual’s consumption and welfare gains under alternative scenarios using our modeling approach.

**Results: Base Case**

This section describes the individual’s optimal demand for stocks, bonds, consumption, and saving in 401(k) plans over her life cycle; our base case focuses on the college-educated female. We construct and compare two scenarios. In the first scenario, no LIA is available, while in the second scenario, at age 65 the individual can convert some of her 401(k) account assets to the LIA that begins paying benefits as of age 85. Subsequent sensitivity analysis compares results for people with different lifetime income profiles.

Figure 2 displays outcomes for the base case, where expected values are based on 100,000 simulated life cycles. Panel A reports life cycle patterns where the individual lacks access to the LIA, while Panel B presents the alternative where she does have the option to buy additional annuities at age 25. Initially, the individual works full-time and earns an annual pre-tax income of $35,000 at age 25. She saves in the tax-qualified 401(k) account from her gross salary up to a maximum of $18,000 per year (as per current law), such that at age 65, her retirement plan assets peak at $234,416 (in expectation). Her consumption pattern (solid line) is slightly hump-shaped. She begins withdrawing from her 401(k) beginning around age 60 (red

\(^{23}\) For discretization, we split the five dimensional state space by using a \(30\times 20\times 10\times 8\times 76\) grid size. For each grid point we calculate the optimal policy and the value function.
dotted line), when this is feasible without the 10%-penalty tax.24 After retiring at age 66, she boosts her withdrawals substantially to compensate for the fact that her Social Security income stream is far below her pre-retirement labor income.

*Figure 2*

Panel B of Figure 2 displays the life cycle profile when the same worker now has access to the LIA. As before, her pre-tax annual earnings at age 25 are $35,000 (dashed-dotted line). But now she has the opportunity to purchase an LIA, so she needs to save somewhat less in the 401(k) plan: $231,000 as of age 65 (in expectation). Thereafter, she reallocates $34,745 from her 401(k) account to the LIA, at which point no taxes are payable. She also withdraws from her 401(k) plan (red dotted line) starting at age 60, and she exhausts that account by age 85. From age 85 onward, her LIA pays her an annual $7,789 (worth 42% of her Social Security benefit) for the rest of her life. Also of interest is the fact that the individual having the LIA consumes more, in expectation, compared to when she lacks access, particularly after age 85. This is because she is insured against running out of money in old age.

Figure 3 displays the difference in consumption between the two cases, with and without access to the LIA. The x-axis represents the individual’s age, and the y-axis the consumption difference (in $000). We depict these in percentiles (99%; 1%) using a fan chart, where differences are measured for each of the 100,000 simulation paths. Darker areas represent higher probability masses, and the solid line represents the expectation. Results show that, prior to age 85, consumption differences are small: the mean is only $3 at age 50. But by age 85, the retiree with the longevity income annuity is able to consume about $1,000 more per year, and $6,000 more by age 99. There is also heterogeneity in the outcomes, such that at age 25, the difference is only $150 for the bottom quarter, while it is $1,400 for the 75th percentile. At age 99, the difference is $96 for the 25th percentile, but $9,680 for the 75th quantile.

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24 Before age 59.5, the individual pays 10% penalty for each withdrawal from a 401(k) plan.
Overall, we conclude that the opportunity to purchase a longevity income annuity provides individuals with substantially higher consumption levels, particularly at older ages.

Comparisons with Other Groups

In this section we report results for other educational groups, and for men as well as women. In addition we provide an analysis of different mortality assumptions and for a LIA with an earlier start age.

Differences by Sex and Educational Attainment. Table 1 shows how results vary for men and women having different educational, and hence labor earnings, patterns. To this end, we show retirement plan assets over the life cycle for women and men in the three educational brackets of interest: namely, high school dropouts, high school graduates, and the Coll+ group. Panel A reports outcomes when individuals lack access to the LIA, and Panel B shows asset values when they have access. Panel C provides average amounts used to purchase the LIA when available, along with the resulting lifelong benefits payable from age 85.

Since the Coll+ female earns more than her female counterparts, she also saves more in her 401(k) plan over her life cycle. For example, without a LIA, by age 55-64, the average Coll+ woman with no LIA access saves $233,340 in her 401(k) account, over four times the $52,470 held by the High School dropout, and double the $114,850 of the High School graduate. With a LIA, the best-educated woman saves slightly less in her retirement account (around $3,000 less), while the HS graduate is not much affected. Interestingly, the least-educated female optimally saves slightly more (4%) in her 401(k) account when she can access the LIA. A similar pattern obtains for the three cases of male savers depicted. As the Coll+ male earns more than the Coll+ female, he accumulates more in his 401(k) account, on the order of $274,380 with no LIA. This is 80% more than the male HS graduate ($151,980), and over
three times the $85,090 of the HS dropout. Once access to the LIA is available, the best-educated man needs to save $10,310 less, while the HS graduate changes behavior very little (as with the females). Again, the male HS dropout saves slightly more.

With the LIA, all groups of women and men withdraw more and retain less in their retirement plans post-retirement, compared to those without access to lifelong benefits. For instance, the Coll+ woman with having an LIA keep an average of $165,390 in her retirement plan between ages 65-74, or 24% more than with the LIA when she retains only $129,230 in investible assets. Similarly, the best-educated male age 65-74 retains $138,880 in his retirement account with the LIA, whereas without it he keeps 23% more ($181,870). A comparable pattern applies to the other two educational groups of both sexes. With or without the LIA, the two less-educated men and women have very little left in their 401(k) plans close to the ends of their lives, though they have more without the annuity than with it. At very old ages, 85-94, the most educated people with no access to the LIA still hold about $15,000 in their 401(k) accounts, while, with the annuity, they have virtually nothing.

The reason for this difference is that those with LIA access use a substantial portion of their retirement assets to purchase longevity annuities which generate a yearly lifelong income. Row C in Table 1 shows that the Coll+ women optimally use about $34,750 of their 401(k) plans to purchase their deferred annuity, and even the HS group buys annuities using $11,640. The HS dropout group buys the least, spending only $3,050 on the deferred income product. This is not surprising given the redistributive nature of the Social Security system. Men have similar patterns, though their shorter life expectancy motivates the least-educated to devote some $8,300 to LIAs.

From age 85 onwards, both groups with LIAs enjoy additional income, compared to the non-LIA group. From age 85, the Coll+ women receive an annual LIA payment for life of $7,790, while the HS women are paid $2,610 per year. The HS dropout receives the least given her small purchase, paying out only $680 per annum. For men, the optimal LIA purchase at 66
generates an annual benefit of $11,100 for the Coll+, $5,210 for HS grads, and a still relatively high annual benefit of $2,510 for HS dropouts. In other words, the LIA pays a reasonably appealing benefit for those earning middle/high incomes during their work-lives. They are smaller, on net, for those who earned only what HS dropouts did over their lifetimes.

Impact of Alternative Mortality Assumptions and Payout Dates. Thus far we have assumed that the LIAs are priced using relevant age and sex annuitant tables. Yet it is also of interest to explore how the demand for LIAs varies with alternative mortality assumptions, including pricing for individuals with higher mortality rates, as well as unisex pricing. We also consider a scenario where the LIA starts paying out at age 80, instead of age 85.

Taking into account alternative mortality assumptions is interesting for two reasons. First, recent studies report widening mortality differentials by education, which raises questions about whether the least-educated will benefit much from longevity annuities. For instance, Kreuger et al. (2015) report that male high school dropouts average 23% excess mortality and females 32%, compared to high school graduates. By comparison, those with a college degree live longer: men average a 6% lower mortality rate, and women 8%. Though only 10% of Americans have less than a high school degree (Ryan and Bauman 2016) and they comprise only 8% of the over-age 25 workforce (US DOL 2016), this group is more likely to be poor. Second, employer-provided retirement accounts in the US are required to use unisex life tables to compute 401(k) payouts (Turner and McCarthy 2013). While men’s lower survival rates may make LIAs less attractive to men than to women, it has not yet been determined how men’s welfare gains from accessing LIA products compare to women’s. Accordingly, in what follows, we compare our results for the base case to those for persons anticipating shorter lifespans.

Table 2 presents results for each of these alternative scenarios. The first column replicates outcomes for the base case female (Coll+). In Column 2 we report the impact of having the LIA priced using a unisex mortality table, as would be true in the US company retirement plan context. Columns 3 and 4 show results when annuities for high school dropouts
of both sexes are priced using higher mortality (as in Kreuger et al. 2015). Column 5 reports the impact of assuming a shorter deferral period: that is, the LIA begins paying out at age 80 instead of age 85.

*Table 2 here*

Results show that when the LIAs modeled are priced using the higher mortality rates for male and female high school dropouts, this makes them less appealing for both groups. For instance, the female HS dropout buys a much smaller LIA at age 65 – spending only $1,401 versus $3,050 in Table 1 – which pays out much less ($320 per year versus $680 per year). The male HS dropout also spends less on the LIA, allocating only $5,330 to the deferred product versus $8,300; this lower LIA pays only $1,610 per annum instead of $2,510. In general, using age/education group mortality tables does not completely erase the demand for LIAs, but it does diminish it substantially.

Turning next to the impact of using a unisex instead of a female mortality table to price the LIA, we find that this has little effect on outcomes. In other words, Coll+ women are almost as well off, and would devote almost as much money to longevity income annuities, regardless of whether sex-specific or unisex annuity life tables are used to price the LIA. Further analysis will indicate how results change across other groups.

In Column 5 we report what happens if an earlier LIA payout is permitted, that is, at age 80 instead of age 85. Now the Coll+ woman saves slightly less ($2,000 less) than in the base case, namely $228,970 in her 401(k) account as of age 55-64. The earlier starting age is attractive, so at retirement she will optimally allocate $32,970 to the LIA, just a little less than in the base case ($34,750). Her annual income payment will now be $8,470 at age 85+, 8% more than the $7,790 under the LIA payable at age 85. In other words, having access to the longevity payout slightly earlier does not change results dramatically.
Welfare Analysis

We next report the results of a welfare analysis comparing access to longevity income annuities versus no access. To calculate the welfare gains of having access to LIAs at retirement versus not, we compare the situation at age 66 for two sets of workers. Both behave optimally before and after retirement, but the first has the opportunity to buy LIAs at age 65 while the second does not. Since people are risk averse, it is not surprising that the utility level of those having access to LIAs at age 66 is generally higher than those without. We also compute the equivalent increase in the 401(k) wealth needed for those lacking the LIAs, to be as well off as those with the products. Formally, we find the additional asset \((w_A)\) that would need to be deposited in the 401(k) accounts of individuals lacking access to LIA, so their utility would be equivalent to that with access to the LIA product.\(^{25}\) This is defined as follows:

\[
E_{\text{LIA}}[J(X_t, L_t, P_{A}, P_t, t)] = E_{\text{without LIA}}[J(X_t, L_t + w_A, P_t, t)].
\] (12)

Table 3 shows results. For the Coll+ female, access to the LIA enhances welfare by a value equivalent to $13,120. In this circumstance, she optimally devotes 15% of her 401(k) account to the deferred lifetime income annuity. If unisex tables were required, the fraction of her account devoted to the LIA would change only trivially, and the welfare gain is actually higher due to the fact that, on average, women benefit from unisex tables due to longer lifespans. If the LIA product started payouts at age 80 instead of age 85, more retirement money would be devoted to this product (26.7% of the account value) and the welfare gain would rise by 17% (to $15,802).

Table 3 here

The next few rows of the table report results for different educational groups by sex. Among women, we see that welfare is enhanced by having access to the LIA product, though the gain of $6,280 for the HS graduates exceeds that for HS dropouts (whether population or

---

\(^{25}\) The value of \(w_A\) could be negative if \(E_{\text{LIA}}[J(X_t, L_t, P_{A}, P_t, t)] < E_{\text{without LIA}}[J(X_t, L_t + w_A, P_t, t)]\). This situation is ruled out in the optimal case.
higher mortality rates are used). For men, we see that the gain for the Coll+ group is substantial when LIAs are available, on the order of $35,837 as of age 66. Smaller results obtain for the less-educated, thought even HS dropouts with the lower survival probabilities still benefit more than women, on average.

In sum, in our framework, both women and men benefit from access to a longevity income annuity. While workers anticipating lower lifetime earnings and lower longevity do benefit proportionately less than the Coll+ group, all subsets examined gain from having access to the LIA when they can optimally allocate their retirement assets to these accounts.

How Might a Default Solution for Longevity Annuity Work?

Thus far, our findings imply that a majority of 401(k) plan participants would benefit from having access to a longevity income annuity. Nevertheless, some people might still be unwilling or unable to commit to an LIA, even if it were sensibly priced (as here).26 For this reason, a plan sponsor could potentially implement a payout default, wherein a portion of the individual’s retirement plan assets would be used to automatically purchase a deferred lifetime payout at age 65. In this way, such a default would accomplish the goal of “putting the pension back” into the retirement plan.

One policy option along these lines would be for an employer to default a fixed fraction – say 10% – of retirees’ 401(k) accounts into a LIA when they turn age 65. This fixed fraction approach is compatible in spirit with the optimal default rates depicted in Table 3, where most retirees would find such a default amount appealing. Yet some very low-earners might save so little in their 401(k) accounts that defaulting them into a LIA might not be practical. Accordingly, a second policy option would be to default 10% of savers’ 401(k) accounts only

---
26 For instance, Brown et al. (2016) show that people find annuitization decisions complex, particularly for the least financially literate.
when participants had accumulated some minimum amount such as $65,000 in their plans. In this second fixed fraction + threshold scenario, the LIA default is implemented when the worker’s 401(k) account equals or exceeds the threshold. Of course, the 10% deferred annuitization rate will still be below what some would desire in terms of the optimum, and higher for others. Our question is, how would welfare effects change, for such default deferred payout policies?

Our analysis of the two different default approaches appears in Table 4. The next-to-last column reports welfare gains assuming the 10% default applies to everyone, while the last column defaults people into LIAs only if their retirement accounts exceed $65,000. In both cases, 10% of the assets invested by default would go to a LIA payable at age 85.

Table 4 here

For the base case Coll+ female, we see that her welfare gain from the fixed fraction default comes to $12,810, just slightly ($310) lower than the gain in the fully optimal case in Table 3. She still benefits under the fixed fraction approach when a unisex mortality table is used, but it provides 23% lower welfare gain than in the full optimality case (or $3362 less than the $14,360 in Table 3). If the LIA were available from age 80, her welfare gain under the fixed fraction option would be just one-fourth as large as if she could buy an optimal level of LIA; in fact holding her to a 10% fraction makes her much less well off than allowing her to devote almost 27% to the LIA payable at age 80. Welfare gains for the fixed fraction + threshold approach are comparable for the Coll+ woman. Accordingly, older educated women would likely favor LIAs beginning at age 85, under both the fixed fraction and the fixed fraction + threshold approaches.

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27 This appears to be a reasonable threshold in that workers in their 60’s with at least five years on the job averaged $68,800 or more in their 401(k) plans, as of 2014 (Vanderhei et al. 2016). The same source found that workers in their 60s who earned $40-$60,000 per year averaged $96,400 in their 401(k) accounts; those earning $60-$80,000 per year averaged $151,800; and those earning $80-$100,000 held an average of $223,640 in these retirement accounts.
Turning to the less educated women, it is not surprising to learn that welfare gains are lower for both of the default options. For instance, requiring the less-educated to annuitize a fixed fraction (10%) of their 401(k) wealth reduces utility for the HS graduates using population mortality tables by 13% (i.e., from $6,280 to $5,467), and by more, 75%, for HS dropouts (i.e., from $5,270 to $1287). If mortality rates for HS dropouts were 34% higher, as noted above, these least-educated women would actually be worse off under the fixed fraction approach. For such individuals, the fixed fraction + threshold would be more appealing, as those with very low incomes and low savings would be exempted from buying LIAs. In fact, HS graduates do just about as well under this second policy option as in the optimum.

Regarding results for men, we see that the default 10% LIA has little negative impact on their welfare. This is primarily due to their higher lifetime earnings, allowing them to save more, as well as lower survival rates. For instance, the Coll+ male’s welfare gain in the optimum is $35,837 (Table 3) and just a bit less, $33,032, under the fixed fraction option. The fixed fraction + threshold default is likewise not very consequential for the best-educated male, with welfare declining only 8% compared to the optimum. Less-educated males experience only slightly smaller welfare gains with both default policies; indeed if they are permitted to avoid annuitization if they have less than $65,000 in their retirement accounts, benefits are quite close to the optimum welfare levels across the board.

Finally, we repeat our welfare analysis for the default solutions assuming that the LIAs are priced using a unisex table instead of a sex-specific mortality table. At retirement, workers can transfer the assets of their 401(k) company plans into an individual retirement account (IRA) offered by a private sector financial institution. In such a case, the private sector institution can use sex-specific mortality tables to price annuities offered inside the plan. Yet if the worker kept her tax qualified retirement assets with the company during the decumulation phase, the annuity must be priced using a unisex table. Table 5 depicts the results for the various education groups if LIA’s are priced using a unisex table. For men (women), not surprisingly,
the welfare gains of such the default solutions decreases (increases) compared to the situation with sex-specific annuity pricing (see Table 4). Yet the welfare gain is still remarkably high for workers having Coll+ and High School education. Again, the simple default solution based on a 10%-fixed percentage rule produces a small welfare cost ($ -479) for females with a high school education and mortality rate 34% higher than the average population. The fixed percentage rule plus an asset threshold of $ 65,000 overrides this problem, i.e. also for this group the welfare gains are positive ($555). Looking at other subgroups, the introduction of an asset threshold produces welfare gains compared to the situation without the asset threshold.

In sum, this section has shown that requiring workers to devote a fixed fraction of their 401(k) accounts to longevity income annuities starting at age 85, and additionally, limiting the requirement to savers having at least $65,000 in their retirement accounts, does not place undue hardships on older men or women across educational groups. Moreover, this approach offers a way for retirees to enhance their lifetime consumption, protect against running out of money in old age, and enjoy greater utility levels than without the LIAs.

Conclusions and Implications

This paper has examined the potential impact of a recent effort to “put the pension back” in 401(k) plans. This recent change in Treasury regulations reversed the traditional institutional bias against including annuities as retirement plan payouts in US private-sector pension regulation, and it now allows retirees to purchase a deferred lifetime income annuity using a portion of their plan assets. Similar suggestions have been the subject of discussion in the context of new state-sponsored retirement plans for the non-pensioned, now under development in 28 states (e.g., Iwry and Turner 2009; IRS 2014).

Our analysis contributes to the policy debate by using a richly-specified life cycle model to measure how much peoples’ wellbeing is enhanced by including LIAs in the retirement plan menu. We take into account stochastic capital market returns, labor income streams, and
mortality, and we also realistically model taxes, Social Security benefits, and 401(k) rules. What we find is that both women and men benefit in expectation from the LIAs, and even less-educated and lower-paid persons stand to gain from this innovation. Moreover, we show that plan sponsors wishing to integrate a deferred lifetime annuity as a default in their plans can do so to a meaningful extent, by converting as little as 10% or 15% of retiree plan assets, particularly if the default is implemented for workers having plan assets over a reasonable threshold.

Financial institutions, insurance companies, and mutual fund companies are increasingly focused on helping Baby Boomers manage their $18 trillion in assets during retirement, so this research will interest those seeking to guide this generation as it decides how to manage 401(k) plan assets into retirement. Similar recommendations are likewise relevant to the management of Individual Retirement Accounts, as these too are subject to the RMD rules and relevant tax considerations described above. Regulators concerned with enhancing retirement security will also find useful the default LIA mechanism described here, to help protect retirees from running out of money in old age.
References


Appendix A: Wage Rate Estimation

We calibrated the wage rate process using the Panel Study of Income Dynamics (PSID) 1975-2013 from age 25 to 69. During the working life, the individual’s labor income profile has deterministic, permanent, and transitory components. The shocks are uncorrelated and normally distributed according to $\ln(N_t) \sim N(-0.5\sigma^2_n, \sigma^2_n)$ and $\ln(U_t) \sim N(-0.5\sigma^2_u, \sigma^2_u)$. The wage rate values are expressed in $2013. These are estimated separately by sex and by educational level. The educational groupings are: less than High School (<HS), High School graduate (HS), and those with at least some college (Coll+). Extreme observations below $5 per hour and above the 99th percentile are dropped.

We use a second order polynomial in age and dummies for employment status. The regression function is:

$$\ln(w_{i,y}) = \beta_1 \cdot \text{age}_{i,y} + \beta_2 \cdot \text{age}^2_{i,y} + \beta_3 \cdot \text{ES}_{i,y} + \beta_{\text{waves}} \cdot \text{wave dummies}, \quad (A1)$$

where $\log(w_{i,y})$ is the natural log of wage at time $y$ for individual $i$, age is the age of the individual divided by 100, ES is the employment status of the individual, and wave dummies control for year-specific shocks. For employment status we include three groups depending on working hours per week as follows: part-time worker ($\leq 20$ hours), full-time worker ($< 20$ & $\leq 40$ hours) and over-time worker ($< 40$ hours). OLS regression results for the wage rate process equation appear in Table A1.

To estimate the variances of the permanent and transitory components, we follow Carroll and Samwick (1997) and Hubener at al. (2016). We calculate the difference of the observed log wage and our regression results, and we take the difference of these differences across different lengths of time $d$. For individual $i$, the residual is:

$$r_{i,d} = \sum_{s=0}^{d-1} (N_{t+s}) + U_{i,t+d} - U_{i,t}$$

$$\quad (A2)$$

We then regress the $v_{id} = \frac{r_{i,d}^2}{r_{i,d}}$ on the lengths of time $d$ between waves and a constant:

$$v_{id} = \beta_1 \cdot d + \beta_2 \cdot 2 + e_{id},$$

$$\quad (A3)$$

where the variance of the permanent factor $\sigma^2_N = \beta_1$ and the $\sigma^2_U = \beta_2$ represents the variance of the transitory shocks.
Table A1: Regression results for wage rate

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Male &lt;HS</th>
<th>Male HS</th>
<th>Male +Coll</th>
<th>Female &lt;HS</th>
<th>Female HS</th>
<th>Female +Coll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age¹/100</td>
<td>3.146*** (0.108)</td>
<td>6.098*** (0.0495)</td>
<td>9.117*** (0.0728)</td>
<td>1.253*** (0.109)</td>
<td>2.820*** (0.0472)</td>
<td>4.646*** (0.0750)</td>
</tr>
<tr>
<td>Age²/10000</td>
<td>-3.314*** (0.130)</td>
<td>-6.581*** (0.0633)</td>
<td>-9.388*** (0.0933)</td>
<td>-1.326*** (0.131)</td>
<td>-2.997*** (0.0608)</td>
<td>-4.886*** (0.0974)</td>
</tr>
<tr>
<td>Part-time work</td>
<td>-0.110*** (0.0196)</td>
<td>-0.159*** (0.009)</td>
<td>-0.086*** (0.0118)</td>
<td>-0.088*** (0.006)</td>
<td>-0.127*** (0.003)</td>
<td>-0.088*** (0.004)</td>
</tr>
<tr>
<td>Over-time work</td>
<td>0.00441 (0.004)</td>
<td>0.0494*** (0.0015)</td>
<td>0.0951*** (0.0018)</td>
<td>0.0171*** (0.0056)</td>
<td>0.0753*** (0.002)</td>
<td>0.106*** (0.003)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.929*** (0.032)</td>
<td>1.468*** (0.0111)</td>
<td>1.073*** (0.0151)</td>
<td>2.068*** (0.0284)</td>
<td>1.968*** (0.0101)</td>
<td>1.950*** (0.0151)</td>
</tr>
<tr>
<td>Observations</td>
<td>49,083</td>
<td>315,685</td>
<td>270,352</td>
<td>31,651</td>
<td>279,375</td>
<td>207,640</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.068</td>
<td>0.102</td>
<td>0.147</td>
<td>0.033</td>
<td>0.044</td>
<td>0.093</td>
</tr>
</tbody>
</table>

Permanent: 0.00907*** (0.0005) | 0.0133*** (0.0002) | 0.0188*** (0.0003) | 0.00747*** (0.0006) | 0.0128*** (0.0002) | 0.0188*** (0.0003) |

Transitory: 0.0276*** (0.001) | 0.0307*** (0.0006) | 0.0414*** (0.0009) | 0.0226*** (0.0015) | 0.0275*** (0.0006) | 0.0395*** (0.001) |

Observations | 28,548 | 170,469 | 131,836 | 20,884 | 170,735 | 114,700 |
| R-squared   | 0.214 | 0.279 | 0.301 | 0.157 | 0.252 | 0.266 |

Notes: Regression results for the natural logarithm of wage rates are based on information in the Panel Study of Income Dynamics (PSID) for persons age 25-69 in waves 1975-2013. Independent variables include age and age-squared, and dummies for part-time work (≤20 hours per week) and overtime work (≥ 40 hours per week). Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Source: Authors’ calculations.

Appendix B: 401(k) plans tax-qualified pension account

We integrate a US-type progressive tax system into our model to explore the impact of having access to a qualified (tax-sheltered) pension account of the EET type.²⁸ Here the household must pay taxes on labor income and on capital gains from investments in bonds and stocks. During the working life, it invests $\text{A}_t$ in the tax-qualified pension account, which reduces taxable income up to an annual maximum amount $\text{D}_t=\$18,000$. Correspondingly, withdrawals $\text{W}_t$ from the tax-qualified account increase taxable income. Finally, the household’s taxable income is reduced by a general standardized deduction $\text{GD}$. For a single

²⁸ That is, contributions and investment earnings in the account are tax exempt (E), while payouts are taxed (T).
household, this deduction amounted to $5,950 per year. Consequently, taxable income in working age is given by:

\[
Y_{t+1}^{tax} = \max[\max(S_t \cdot (R_{t+1} - 1) + B_t \cdot (R_f - 1); 0) + Y_{t+1}(1 - h_t) + W_t - \min(A_t; D_t) - GD; 0]
\]  

For Social Security \((Y_{t+1})\) taxation up to age 66, we use the following rules: when the individual combined income\(^{29}\) is between $25,000 and $34,000 (over $34,000), 50% (85%) of benefits are taxed.\(^{30}\)

In line with US rules for federal income taxes, our progressive tax system has six income tax brackets. These brackets \(i = 1, \ldots, 6\) are defined by a lower and an upper bound of taxable income, \(Y_{t+1}^{tax} \in [l_b, u_b]\), and determine a marginal tax rate, \(r_i^{tax}\). For the year 2012, the marginal taxes rates for a single household are 10% from $0 to $8700, 15% from $8701 to $35,350, 25% from $35,351 to 85,659, 28% from $85,651 to $178,650, 33% from $178,651 to $388,350, and 35% above $388,350 (see IRA 2012). Based on these tax brackets, the household’s dollar amount of taxes payable is given by:\(^{31}\)

\[
\text{Tax}_{t+1}(Y_{t+1}^{tax}) = (Y_{t+1}^{tax} - l_b_6) \cdot 1_{[Y_{t+1}^{tax} \leq l_b_6]} \cdot r_6^{tax} + \left( (Y_{t+1}^{tax} - l_b_5) \cdot 1_{[l_b_5 > Y_{t+1}^{tax} \leq l_b_3]} + (u_b_5 - l_b_5) \cdot 1_{[Y_{t+1}^{tax} > l_b_6]} \right) \cdot r_5^{tax} + \left( (Y_{t+1}^{tax} - l_b_4) \cdot 1_{[l_b_4 > Y_{t+1}^{tax} \leq l_b_3]} + (u_b_4 - l_b_4) \cdot 1_{[Y_{t+1}^{tax} > l_b_6]} \right) \cdot r_4^{tax} + \left( (Y_{t+1}^{tax} - l_b_3) \cdot 1_{[l_b_3 > Y_{t+1}^{tax} \leq l_b_2]} + (u_b_3 - l_b_3) \cdot 1_{[Y_{t+1}^{tax} > l_b_6]} \right) \cdot r_3^{tax} + \left( (Y_{t+1}^{tax} - l_b_2) \cdot 1_{[l_b_2 > Y_{t+1}^{tax} \leq l_b_1]} + (u_b_2 - l_b_2) \cdot 1_{[Y_{t+1}^{tax} > l_b_6]} \right) \cdot r_2^{tax} + \left( (Y_{t+1}^{tax} - l_b_1) \cdot 1_{[l_b_1 > Y_{t+1}^{tax} \leq l_b_1]} + (u_b_1 - l_b_1) \cdot 1_{[Y_{t+1}^{tax} > l_b_6]} \right) \cdot r_1^{tax},
\]

where, for \(A \subseteq X\), the indicator function \(1_A \rightarrow \{0, 1\}\) is defined as:

\[
1_A(x) = \begin{cases} 1 & | x \in A \\ 0 & | x \notin A \end{cases}
\]

In line with US regulation, the individual must pay an additional penalty tax of 10% on early withdrawals prior to age 59 ½ \((t = 36)\):

\(^{29}\) Combined income is sum of individual adjusted gross income, nontaxable interest and \(1/2\) of his Social Security benefits.

\(^{30}\) See US SSA at https://www.ssa.gov/planners/taxes.html

\(^{31}\) Here we assume that capital gains are taxed at the same rate as labor income, so we abstract from the possibility that long-term investments may be taxed at a lower rate.
\[ T_{x_{t+1}}(y_{t+1}^\text{tax}) = \begin{cases} T_{x_{t+1}}(y_{t+1}^\text{tax}) & t \geq 36 \\ T_{x_{t+1}}(y_{t+1}^\text{tax}) + 0.1W_t & t < 36. \end{cases} \]  

(B4)

Appendix C: Population mortality tables differentiated by education and sex

Research has shown that lower-educated individuals have lower life expectancies than better-educated individuals. This is relevant to the debate over whether and which workers need annuitization. To explore the impact of this difference in mortality rates by educational levels, we follow Kreuger et al. (2015) who calculated mortality rates by education and sex \((M_{\text{sex}}^{\text{education}})\) as below:

\[
M_{\text{average}}^{\text{male}} = 0.1M_{\text{male}}^{<\text{HS}} + 0.3M_{\text{male}}^{\text{HS}} + 0.6M_{\text{male}}^{\text{Coll}+} \tag{C1}
\]

\[
= 0.1(M_{\text{male}}^{\text{HS}} \cdot 1.23) + 0.3M_{\text{male}}^{\text{HS}} + 0.6(M_{\text{male}}^{\text{HS}} \cdot 0.94)
\]

\[
= 0.987 \cdot M_{\text{male}}^{\text{HS}}
\]

Next we calculate the mortality for a male with a HS degree as follows:

\[
M_{\text{male}}^{\text{HS}} = \frac{M_{\text{average}}^{\text{male}}}{0.987} \tag{C2}
\]

And mortality for a male high school dropout or with Coll+ level education is as follows:

\[
M_{\text{male}}^{<\text{HS}} = \frac{M_{\text{average}}^{\text{male}}}{0.987} \cdot 1.23 \tag{C3}
\]

\[
M_{\text{male}}^{\text{Coll}+} = \frac{M_{\text{average}}^{\text{male}}}{0.987} \cdot 0.94 \tag{C5}
\]

Analogously, we calculate for females with different levels of education the following:

\[
M_{\text{female}}^{<\text{HS}} = \frac{M_{\text{average}}^{\text{female}}}{0.984} \cdot 1.32 \tag{C6}
\]

\[
M_{\text{female}}^{\text{HS}} = \frac{M_{\text{average}}^{\text{female}}}{0.984} \tag{C7}
\]

\[
M_{\text{female}}^{\text{Coll}+} = \frac{M_{\text{average}}^{\text{female}}}{0.984} \cdot 0.92 \tag{C8}
\]

We price the annuity as before using average annuitant mortality tables.
Figure 1: Estimated Average Income Profiles for Female and Male:

Panel A. Expected Income Profiles Female

Panel B. Expected Income Profiles Male

Note: The average income profiles are based on our wage rate regression from PSID data (see Appendix A for details), assumes a 40 hour work-week, and 52 weeks of employment per year. Educational groupings are: less than High School, High School graduate, and at least some college (<HS, HS, Coll+). Source: Authors’ calculations.
Figure 2: Life Cycle Profiles Without vs With Access to a Longevity Income Annuity (LIA)

Panel A. No Lifetime Income Annuity Available

Panel B. With Lifetime Income Annuity

Note: These two figures show expected values from 100,000 simulated lifecycles for college+ females. Panel A shows average consumption, wealth, withdrawals, and income (work, pension, and LIA benefits if any) without and Panel B with access to longevity income annuities. Model parameters include the following: risk aversion $\rho = 5$; time preference $\beta = 0.96$; labor income risk ($\sigma_u = 0.0188$; $\sigma_n = 0.0395$); retirement age 66; risk-free interest rate 1%; mean stock return 5%; stock return volatility 18%. Source: Authors’ calculations.
Figure 3: Consumption Differences over the Life Cycle With versus Without Access to the Longevity Income Annuity (LIA)

Note: Distribution (99%; 1%) of consumption differences for 100,000 life-cycles with optimal feedback controls with and without access to longevity income annuities starting benefits at age 85. Darker areas represent higher probability mass. The solid line represents expected consumption differences. For parameter values see Table 1. Source: Authors’ calculations.
Table 1: Life Cycle Patterns of 401(k) Accumulations ($000) By Sex and Education Groupings: Without and With Access to Longevity Income Annuity (LIA) Product

<table>
<thead>
<tr>
<th></th>
<th>Female &lt;HS</th>
<th>Female HS</th>
<th>Base Case Female Coll+</th>
<th>Male &lt;HS</th>
<th>Male HS</th>
<th>Male Coll+</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A: 401(k) account ($000) without access to LIA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 25-34</td>
<td>12.78</td>
<td>20.83</td>
<td>42.80</td>
<td>17.03</td>
<td>28.05</td>
<td>35.30</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>29.94</td>
<td>60.47</td>
<td>118.99</td>
<td>44.30</td>
<td>75.37</td>
<td>120.73</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>40.81</td>
<td>90.95</td>
<td>187.97</td>
<td>65.23</td>
<td>120.53</td>
<td>210.19</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>52.47</td>
<td>114.85</td>
<td>233.34</td>
<td>85.09</td>
<td>151.98</td>
<td>274.38</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>26.91</td>
<td>75.74</td>
<td>165.39</td>
<td>52.54</td>
<td>98.29</td>
<td>181.87</td>
</tr>
<tr>
<td>Age 75-84</td>
<td>4.85</td>
<td>24.48</td>
<td>70.30</td>
<td>14.33</td>
<td>35.75</td>
<td>73.50</td>
</tr>
<tr>
<td>Age 85-94</td>
<td>0.40</td>
<td>3.64</td>
<td>14.98</td>
<td>1.63</td>
<td>5.42</td>
<td>15.58</td>
</tr>
<tr>
<td><strong>B: 401(k) account ($000) with access to LIA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 25-34</td>
<td>12.71</td>
<td>20.63</td>
<td>42.25</td>
<td>16.90</td>
<td>27.58</td>
<td>32.31</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>33.51</td>
<td>60.16</td>
<td>117.71</td>
<td>43.63</td>
<td>74.00</td>
<td>119.09</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>45.36</td>
<td>90.58</td>
<td>186.17</td>
<td>64.62</td>
<td>119.41</td>
<td>206.85</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>54.46</td>
<td>114.74</td>
<td>230.77</td>
<td>85.53</td>
<td>151.29</td>
<td>264.07</td>
</tr>
<tr>
<td>Age 65-74</td>
<td>25.12</td>
<td>64.39</td>
<td>129.23</td>
<td>45.73</td>
<td>81.77</td>
<td>138.88</td>
</tr>
<tr>
<td>Age 75-84</td>
<td>3.19</td>
<td>13.02</td>
<td>32.02</td>
<td>8.07</td>
<td>17.86</td>
<td>35.29</td>
</tr>
<tr>
<td>Age 85-94</td>
<td>0.05</td>
<td>0.14</td>
<td>0.64</td>
<td>0.10</td>
<td>0.21</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>C: LIA purchased at age 65 ($000)</strong></td>
<td>3.05</td>
<td>11.64</td>
<td>34.75</td>
<td>8.30</td>
<td>17.21</td>
<td>36.67</td>
</tr>
<tr>
<td><strong>D: LIA Payout p.a.($ 000)</strong></td>
<td>0.68</td>
<td>2.61</td>
<td>7.79</td>
<td>2.51</td>
<td>5.21</td>
<td>11.10</td>
</tr>
</tbody>
</table>

**Note:** Expected values in $2013 based on 100,000 simulated life cycles; we report average values over 10-year age bands. Base case calibration: risk aversion \( \rho = 5 \); time preference \( \beta = 0.96 \); labor income risk \( \sigma_u = 0.0188 \); \( \sigma_n = 0.0395 \); retirement age 66; Social Security benefits are computed as described in the text with bend points as of 2013; LIA refers to annuitized 401(k) assets paying lifelong annuity benefits from age 85 on; minimum required withdrawals from 401(k) plans are based on life expectancy using the IRS-Uniform Lifetime Table 2013; for taxes, 401(k) plans available in tax-qualified account, taxation as described in Appendix B; risk-free interest rate 1%; mean stock return 5%; stock return volatility 18%. Source: Authors’ calculations.
Table 2: Life Cycle Patterns of 401(k) Accumulations ($000) By Sex and Education Groupings: Without and With Access to Longevity Income Annuity (LIA) Product Using Alternative Mortality Assumptions

<table>
<thead>
<tr>
<th>Age</th>
<th>Base Case</th>
<th>Female Coll+</th>
<th>LIA w/ unisex mort</th>
<th>Male &lt;HS; mort.+25%</th>
<th>Female &lt;HS; mort. +34%</th>
<th>Female Coll+ LIA @80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A: 401(k) account ($000) without access to LIA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 25-34</td>
<td>42.80</td>
<td>42.80</td>
<td>17.53</td>
<td>10.31</td>
<td>42.80</td>
<td></td>
</tr>
<tr>
<td>Age 35-44</td>
<td>118.99</td>
<td>118.99</td>
<td>39.62</td>
<td>23.54</td>
<td>118.99</td>
<td></td>
</tr>
<tr>
<td>Age 45-54</td>
<td>187.97</td>
<td>187.97</td>
<td>60.63</td>
<td>36.25</td>
<td>187.97</td>
<td></td>
</tr>
<tr>
<td>Age 55-64</td>
<td>233.34</td>
<td>233.34</td>
<td>78.25</td>
<td>48.51</td>
<td>233.34</td>
<td></td>
</tr>
<tr>
<td>Age 65-74</td>
<td>165.39</td>
<td>165.39</td>
<td>45.41</td>
<td>24.08</td>
<td>165.39</td>
<td></td>
</tr>
<tr>
<td>Age 75-84</td>
<td>70.30</td>
<td>70.30</td>
<td>10.38</td>
<td>3.74</td>
<td>70.30</td>
<td></td>
</tr>
<tr>
<td>Age 85-94</td>
<td>14.98</td>
<td>14.98</td>
<td>0.74</td>
<td>0.20</td>
<td>14.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B: 401(k) account ($000) with access to LIA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 25-34</td>
<td>42.25</td>
<td>42.29</td>
<td>17.28</td>
<td>9.79</td>
<td>42.82</td>
<td></td>
</tr>
<tr>
<td>Age 35-44</td>
<td>117.71</td>
<td>117.80</td>
<td>38.76</td>
<td>23.42</td>
<td>117.29</td>
<td></td>
</tr>
<tr>
<td>Age 45-54</td>
<td>186.17</td>
<td>185.98</td>
<td>60.19</td>
<td>36.17</td>
<td>185.05</td>
<td></td>
</tr>
<tr>
<td>Age 55-64</td>
<td>230.77</td>
<td>230.62</td>
<td>78.85</td>
<td>48.48</td>
<td>228.97</td>
<td></td>
</tr>
<tr>
<td>Age 65-74</td>
<td>129.23</td>
<td>129.66</td>
<td>41.59</td>
<td>23.06</td>
<td>98.74</td>
<td></td>
</tr>
<tr>
<td>Age 75-84</td>
<td>32.02</td>
<td>31.50</td>
<td>6.80</td>
<td>2.82</td>
<td>11.76</td>
<td></td>
</tr>
<tr>
<td>Age 85-94</td>
<td>0.64</td>
<td>0.52</td>
<td>0.06</td>
<td>0.05</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C: LIA purchased at age 65 (000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>34.75</td>
<td>32.97</td>
<td>5.33</td>
<td>1.41</td>
<td>60.91</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D: LIA Payout p.a.( $000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.79</td>
<td>8.47</td>
<td>1.61</td>
<td>0.32</td>
<td>7.83</td>
<td></td>
</tr>
</tbody>
</table>

Note: See Note to Table 1. Source: Authors’ calculations.
### Table 3: Welfare Gains and Ratio of 401(k) Devoted to Annuity at Age 66 Without and With Access to Longevity Income Annuity (LIA) Product: Optimal Annuitization Outcomes

<table>
<thead>
<tr>
<th>Case</th>
<th>Education</th>
<th>Alternative specifications</th>
<th>Optimal LIA Ratio (%)</th>
<th>Welfare Gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Female age 66</strong></td>
<td>Coll+</td>
<td><strong>Base Case</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>LIA unisex mortality</td>
<td>14.36</td>
<td>14,009</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LIA at age 80</td>
<td>26.72</td>
<td>15,802</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td></td>
<td>9.79</td>
<td>6,280</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td></td>
<td>5.27</td>
<td>2,204</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +34%</td>
<td>2.64</td>
<td>424</td>
</tr>
<tr>
<td><strong>Male age 66</strong></td>
<td>Coll+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td></td>
<td>14.26</td>
<td>35,837</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td></td>
<td>11.32</td>
<td>13,999</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +25%</td>
<td>8.94</td>
<td>5,696</td>
</tr>
</tbody>
</table>

**Note:** See Note to Table 1. LIA Ratio (%) refers to the fraction of the individual’s 401(k) plan assets used to purchase the LIA at age 65. Welfare Gain ($) refers to the retiree’s additional utility value from having access to the LIA versus no access at age 66. Source: Authors’ calculations.

### Table 4: Welfare Gains at Age 66 Without and With Access to Default Longevity Income Annuity (LIA) Product: Two Default Solutions

<table>
<thead>
<tr>
<th>Case</th>
<th>Education</th>
<th>Alternative specifications</th>
<th>Welfare Gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(No min assets) (Min $ 65K assets)</td>
<td></td>
</tr>
<tr>
<td><strong>Female age 66</strong></td>
<td>Coll+</td>
<td><strong>Base Case</strong></td>
<td>10% fixed fraction 10% fixed fraction + threshold default</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LIA unisex mortality</td>
<td>12,810</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LIA at age 80</td>
<td>11,008</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td></td>
<td>6,764</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td></td>
<td>5,467</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td>Mortality +34%</td>
<td>-1,160</td>
</tr>
<tr>
<td><strong>Male age 66</strong></td>
<td>Coll+</td>
<td></td>
<td>33,032</td>
</tr>
<tr>
<td></td>
<td>High school</td>
<td></td>
<td>13,245</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td></td>
<td>5,208</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +25%</td>
<td>1,840</td>
</tr>
</tbody>
</table>

**Notes:** In the case of the fixed fraction default approach 10% of retirees’ 401(k) accounts are converted into a LIA when they turn age 65. In this fixed fraction + threshold default approach, the 10% of assets are converted into longevity income annuities only when the worker’s 401(k) account equals or exceeds the threshold of $ 65,000. See Note to Tables 1 and 3. Source: Authors’ calculations.
### Table 5: Welfare Gains at Age 66 Without and With Access to Default Longevity Income Annuity (LIA) Product: Two Default Solutions with Unisex Pricing of LIA

<table>
<thead>
<tr>
<th>Case</th>
<th>Education</th>
<th>Alternative specifications</th>
<th>10% fixed fraction default (No min assets)</th>
<th>10% fixed fraction threshold default (Min $ 65K assets)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Female age 66</strong></td>
<td>Coll+</td>
<td></td>
<td>11,008</td>
<td>10,800</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td></td>
<td>7,557</td>
<td>7,796</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td></td>
<td>3,640</td>
<td>4,331</td>
</tr>
<tr>
<td></td>
<td>&lt; High school</td>
<td>Mortality +34%</td>
<td>-479</td>
<td>555</td>
</tr>
<tr>
<td><strong>Male age 66</strong></td>
<td>Coll+</td>
<td></td>
<td>28,451</td>
<td>28,445</td>
</tr>
<tr>
<td></td>
<td>High school</td>
<td></td>
<td>10,644</td>
<td>10,787</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td></td>
<td>4,007</td>
<td>4,481</td>
</tr>
<tr>
<td></td>
<td>&lt; High School</td>
<td>Mortality +25%</td>
<td>421</td>
<td>1,317</td>
</tr>
</tbody>
</table>

**Notes:** In the case of the fixed fraction default approach, 10% of retirees’ 401(k) accounts are converted into a LIA when they turn age 65. In the fixed fraction + threshold default approach, the 10% of assets are converted into longevity income annuities only when the worker’s 401(k) account equals or exceeds the threshold of $65,000. See Note to Tables 1 and 3. Source: Authors’ calculations.