Recreating Sustainable Retirement

Resilience, Solvency, and Tail Risk

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The Securitization of Longevity Risk and Its Implications for Retirement Security

Richard MacMinn, Patrick Brockett, Jennifer Wang, Yijia Lin, and Ruilin Tian

The simplest notion of individual longevity risk is that it is the possibility that one will outlive one's accumulated wealth. The risk of outliving one's accumulated wealth has many unpleasant consequences for individuals and societies, and because the risk is increasing it must be addressed. If an individual is covered by a defined contribution (DC) plan, the employee contributes to the plan until retirement. The employer may or may not match the employee's contribution. Accumulated wealth generated during the individual's working life yields the wealth that the individual will use to generate a retirement income stream. If the individual is covered by a defined benefit (DB) plan, the employer guarantees to provide the employee a designated amount of money upon retirement up until the employee's death. The employee may or may not contribute to the plan. The designated amount is based on the employee's earnings, length of employment, and age. With a DC plan, the employee is responsible for ensuring that enough money has been contributed to the account to avoid longevity risk. With a DB plan, the employer is liable for ensuring that the plan does not run out of money before the employee dies. In the first case the individual faces the longevity risk, while in the second case the pension provider faces the longevity risk.

Both plans have distinctive risks and in this chapter we examine the risks from the perspective of the individual and the institution. Longevity risk is important in part because of its size; international pension liabilities have been estimated at approximately \$21 trillion. Longevity risk is also important because as life expectancy increases, individuals must increase contributions to their DC plans to mitigate longevity risk and the size of the necessary additional contribution is uncertain, as it depends in part on how life expectancies change over time. Hence the questions of interest here include: What happens to the financial well-being of a retired cohort in the event of an unexpected change in life expectancy or financial stability? What happens if it does not manage these risks? What happens if it does manage these risks using currently available instruments? How might it manage the longevity risk with longevity instruments?

In what follows, we first explore the meaning of longevity risk and consider its magnitude. Next we consider how mortality and longevity risks have been transformed from pure risks to speculative risks. Subsequently we examine longevity risk and financial market risks from the perspective of those who bear them; we show a need for more longevity instruments in the retail market and some of the benefits of the existing longevity instruments in the wholesale market for longevity risk. A final section concludes.

Longevity Risk

Risk can be defined as the negative consequences of uncertainty. Both uncertainty (multiple possible outcomes) and negative consequences are necessary prerequisites for risk to exist (Baranoff et al. 2009). Since most people find longer life desirable, the term 'longevity risk' needs further explication to provide the context in which longevity presents a 'risk' rather than a 'benefit' to individuals and/or institutions. Longevity risk is the risk that an individual life span or the average population life span will exceed expectations. The negative consequences of an extended life span can include outliving one's friends, diminished mobility and cognitive flexibility/focus, and outliving one's financial resources after retirement without the remedial possibility of rejoining the workforce to produce necessary income at the later date wherein the probability (or the eventuality) that financial assets will soon be depleted becomes recognized. Longevity risk has an unsystematic or idiosyncratic component, as well as a systematic component. The idiosyncratic component is sometimes described as the risk of an individual outliving one's accumulated wealth; Milevsky (2006) describes this as 'retirement ruin.' The systematic component corresponds to the more general hazard that people in the aggregate will live longer than expected (Oppers et al. 2012), thus causing strain on pensions, employers, and society in general. The systematic component is often referred to as aggregate longevity risk in the literature (MacMinn et al. 2006; Blake et al. 2013). The aggregate longevity risk discussed here is the risk of living longer than one expects; it is also a systematic risk because life expectations are themselves random variables.

From a financial perspective, the unsystematic component of longevity risk may be handled by holding a sufficiently diversified and adequately funded asset portfolio to decumulate during retirement. If life expectancy were certain, the individual could purchase an annuity certain designed to provide the desired cash flow for the certain life expectancy. Alternatively, the individual could purchase a bond with the desired flow of coupon payments and leave the principal repayment as a bequest to beneficiaries but because life expectancy is uncertain, the individual must design a portfolio of assets to provide a desired cash flow for an undetermined period of time. This portfolio may consist of equity, bonds, and possibly a life annuity. The life annuity is an asset that provides a specified cash flow for the remaining years of an individual's life. If the unsystematic component of life expectancy were the only risk faced by an individual then a life annuity may be shown to dominate other assets (Yaari 1965). However, the risk of outliving one's accumulated wealth is not the only unsystematic risk. Increased life expectancy includes morbidity (and other) risks as well, implying that just a life annuity would not cover a long-term illness, dementia, etc., and therefore a diversified asset portfolio is still needed (Sinclair and Smetters 2004; Horneff et al. 2009; MacMinn and Weber 2010; Chai et al. 2011).

From a financial perspective, at the firm or society level, aggregate longevity risk (the systematic component) must be managed. The management choices include bearing the risk or transferring or trading the risk to some other entity willing to bear it. In deciding among these alternatives, a determination must be made on how to price this risk transfer appropriately. This aspect of longevity risk might be thought of as a trend risk or the risk of underestimating life expectancy (Blake et al. 2013). Alternatively, as noted, we may think of life expectancy itself as a random variable. Oeppen and Vaupel (2002) show the record life expectancy for females as projected by a number of authors and organizations and report the rather surprising result that the record female life expectancy has increased by three months per year for more than 150 years. Hence, Oeppen and Vaupel also show that each historical life expectancy prediction has been wrong. With reference to 2005 mortality rates, they note that U.K. mortality rates had declined over the previous 10–15 years by over 2 percent *per annum* for the age groups over 60. Referring to the 2 percent decline and using government actuarial department (GAD) figures, Turner made the following comment about mortality rates (2006: 562):

> If they continue at that rate, male life expectancy at sixty-five, currently estimated at nineteen, will reach about thirty by 2050. If the rate accelerates to 3 percent, life expectancy would soar to 37 years. Only if it decelerates to 1 percent would the GAD's 2002-based principal projection of 22 years in 2050 be correct. So the GAD 2002 projection—a major increase from previous projections—nevertheless still assumed a major deceleration of mortality rate improvement.

The errors in life expectancy estimates noted here highlight the systematic risk of longevity risk and the magnitude of the risk.

It is important to understand the vast financial size of the longevity risk problem. Turner (2006) estimated £2.5 ($4.3) trillion in liabilities subject to longevity risk in the U.K. Swiss Re has since estimated $20.7 trillion in pension liabilities subject to longevity risk internationally (Burne 2011). Oppers et al. (2012: 8) provide a different perspective by calculating the cost of maintaining the retirement living standard due to aging and longevity shocks as a percentage of gross domestic product (GDP) for advanced and emerging countries. Using the demographic trends predicted by the United Nations, they note:

> In the baseline population forecast and with a 60 percent replacement rate, the annual cost rises from 5.3 percent to 11.1 percent of GDP in advanced

> economies and from 2.3 percent to 5.9 percent of GDP in emerging economies . . . Taken over the full period, the cumulative cost of this increase because of aging in this scenario is about 100 percent of 2010 GDP for the advanced economies and about half that amount in emerging economies.

The authors also observe that a longevity shock of three years would add almost an additional 50 percent to these cumulative costs of aging by 2050. There is uncertainty surrounding all of these predictions, but the magnitudes are hard to ignore.

Ignoring longevity risk is indeed a significant problem. Oppers et al. (2012) use data from the U.S. Department of Labor (DOL) to estimate the longevity risk faced by DB plans. They report many plans used outdated mortality tables; the majority of the plans used the 1983 Group Annuity Mortality (GAM) until recently. Using out-of-date mortality tables exposes pension plans to longevity risk and risk of ruin. Dushi et al. (2010) compare pension liability values based on the plans' longevity assumptions versus the pension liability values forecast by the Lee–Carter mortality model and find that the outdated mortality tables could understate the pension liability for a typical male participant by approximately 12 percent.

Longevity risk can be borne or transferred, in whole or in part. The retail market for longevity risk allows consumers to transfer all or part of the risk with life annuities. In the U.K., this market amounts to £135 billion, but this is because consumers are required by law to at least annuitize before they turn 75 (Loeys et al. 2007).[1] The retail or life annuity market remains very small in the U.S. since there is no similar requirement that individuals annuitize when they retire. In fact, this general lack of a sizable life annuity market has been described as the annuity puzzle, since Yaari (1965) and Davidoff et al. (2005), among others, have shown that in the absence of a bequest motive, the life annuity instrument for retirement funding dominates other asset choices.

A wholesale market for longevity risk would allow pension funds and insurers the ability to transfer some of the longevity risk rather than bearing it. The U.K. wholesale market has been active and many of the transactions take the form of buyouts and buy-ins.[2] In a buyout, there is a transfer of pension assets and liabilities for a particular cohort; the cost of the buyout is the difference between the values of the assets and the liabilities transferred. The difference between the asset and liability values may be covered by a loan with a known cash flow that is well understood by investors. In the buy-in, there is a bulk purchase of annuities from an insurer to hedge the risk of the liabilities associated with one or more cohorts. This immunizes the pension fund from the liability risk for the cohorts covered.

The retail and wholesale markets noted here do provide a transfer mechanism for the market participants but the mechanisms are crude instruments. More than one risk is transferred and the risks are aggregated rather than disaggregated; this generates more concentration of risk and hence amplifies the eventual probability of insolvency for those concentrations. Given the size of the longevity risks, this becomes a new problem that is inconsistent with the history of financial markets. 'Indeed, the history of the development of risk instruments is a tale of the progressive separation of risks, enabling each to be borne in the least expensive way' (Kohn 1999: 2).

Securitization

Longevity risk has put corporations, governments, and individuals under a significant financial burden. One common way to manage this risk is securitization (i.e. isolating the cash flows that are linked to longevity risk and repackaging them into cash flows that are traded in capital markets). The earliest securitizations were 'block of business' securitizations used to capitalize expected future profits from a block of life business, such as to recover embedded values[3] or to exit from a geographical line of business. Cowley and Cummins (2005) introduced the early development of the securitization in life insurance. More recently, Blake et al. (2013) provided a more comprehensive and global overview of the emergence of the market in traded assets and liabilities linked to longevity and mortality and referred to this market as the New Life Market. They noted that the New Life Market would act as a catalyst to help facilitate the development of annuity markets both in the developed and the developing world and protect the long-term global viability of retirement income provision.

The idea of mortality securitization was initially proposed by Cox et al. (2000). The first mortality bond, known as Vita I, was issued by Swiss Re in 2003; it was designed to hedge mortality risk rather than to hedge longevity risk. Nevertheless, it provides an important successful example of a Life Market instrument. Vita I was a success, and led to additional bonds being issued to investors on less favorable terms.[4] Blake and Burrows (2001) were the first to advocate the use of mortality-linked securities to transfer longevity risk to capital markets. They suggested that the governments should help pension funds and insurance companies hedge their mortality risks by issuing survivor bonds. In 2004, the European Investment Bank (EIB) and BNP Paribas launched a longevity bond that was the first securitization instrument designed to transfer longevity risk; ultimately it was not issued due to insufficient demand. Design issues, such as the introduction of basis risk, pricing issues, institutional issues, and educational issues, were among the reasons why the EIB/BNP bond did not launch (Lin and Cox 2008).

The lack of success in issuing long-dated longevity bonds has led to a derivatives design effort. Various new securitization instruments and derivatives for longevity risk, such as mortality forwards, survivor swaps, survivor futures, and survivor options have received attention among academics and practitioners.[5] In 2007, J.P. Morgan introduced the first capital market derivative for a longevity hedge; it has become known as a '*q*-forward.' A *q*-forward can be used to hedge the value of the pension liability or the associated cash flows. More complex, life-related derivatives can be constructed by using the *q*-forward as a basic building block. A portfolio of q-forwards can be used to replicate and to hedge the longevity exposure of an annuity or a pension liability or to hedge the mortality exposure for a book of life business. Following the introduction of a q-forward transaction, a longevity swap was used to exchange actual pension payments for a series of pre-agreed fixed payments. This particular swap was legally constituted as an insurance contract and was not a capital market instrument. There have been 16 publicly announced transactions of longevity swaps executed between 2007 and 2012 in the U.K. (Blake et al. 2013).

A Financial Market Model

Suppose the financial market consists of the S&P 500 index, the Merrill Lynch corporate bond index, and a three-month T-bill. Following Cox et al. (2013), we describe the return dynamics of the S&P 500 index as $A_{1,t}$ and the Merrill Lynch corporate bond index as $A_{2,t}$ at time t as a combination of Brownian motion and a compound Poisson process. The stochastic process of the three-month T-bill return $A_{3,t}$ is simply described as Brownian motion. The three returns are as follows:

$$A_{1,t+\Delta} \mid \mathrm{F}_t = A_{1,t} \exp\left[(\alpha_1 - \frac{1}{2}\sigma_1^2 - \lambda_1 k_1)\Delta + \sigma_1 \Delta W_{1t} \right] \prod_{j>N_t^1}^{N_{t+A}^1} Y_{1j} \tag{8.1}$$

$$A_{2,t+\Delta} \mid \mathrm{F}_t = A_{2,t} \exp\left[(\alpha_2 - \frac{1}{2}\sigma_2^2 - \lambda_2 k_2)\Delta + \sigma_2 \Delta W_{2t} \right] \prod_{j>N_t^2}^{N_{t+A}^2} Y_{2j} \tag{8.2}$$

$$A_{3,t+\Delta} \mid \mathrm{F}_t = A_{3,t} \exp\left[(\alpha_3 - \frac{1}{2}\sigma_3^2)\Delta + \sigma_3 \Delta W_{3t} \right] \tag{8.3}$$

where the constants $\alpha_1, \alpha_2, \alpha_3$ and $\sigma_1, \sigma_2, \sigma_3$ are the drift and volatility measures of the S&P 500 return, the Merrill Lynch corporate bond return, and the three-month T-bill rate given no jumps. The parameter $k_1 \equiv E(Y_1 - 1)$ is the expected percentage change in the S&P 500 return and k_2 is similarly defined for the Merrill Lynch corporate bond return if a Poisson event occurs. The parameters λ_1 and λ_2 are the mean numbers of arrivals per unit time of the Poisson processes N_t^1 and N_t^2 respectively. The jump size Y_{1j} or Y_{2j} is independent and identically distributed as a lognormal random variable with the size parameter m_1 and the volatility parameter S_1 or m_2 and S_2. Y_{1j} and Y_{2j} are independent for all i and j. The correlation between the standard Brownian motions of the S&P 500 index and the Merrill Lynch corporate bond index, W_{1t} and W_{2t}, is captured by the correlation coefficient ρ_{12} (i.e. $Cov(W_{1t}, W_{2t}) = \rho_{12}\sigma_1\sigma_2 t$).

We further assume the three-month T-bill is uncorrelated with either the S&P 500 index or the Merrill Lynch corporate bond index. Based on the annual data of the S&P 500 and the Merrill Lynch corporate bond provided by the DataStream

Table 8.1 Maximum likelihood parameter estimates of three pension assets using annual data

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
α_1	0.0866	α_2	0.0691	α_3	0.0515
σ_1	0.0864	σ_2	0.0547	σ_3	0.0329
λ_1	0.2742	λ_2	0.0505	ρ_{12}	0.6016
m_1	−0.3048	m_2	−0.1468		
s_1	0.0000	s_2	0.0000		

Notes: α_1, α_2, α_3 and σ_1, σ_2, σ_3 are the drift and volatility measures of the returns of the S&P 500 index, Merrill Lynch corporate bond index, and three-month T-bill. λ_1 and λ_2 are the mean numbers of arrivals per unit time of the Poisson processes for the returns of the S&P 500 and Merrill Lynch corporate bond indices. m_1 (m_2) and s_1 (s_2) are the size parameter and volatility parameter of the lognormal jump size for the return of S&P 500 (Merrill Lynch corporate bond) index. ρ_{12} is the correlation coefficient between the geometric Brownian motions of the returns of S&P 500 and Merrill Lynch corporate bond indices.

Source: Authors' calculations; see text.

and the three-month T-bill rates from FRED at Federal Reserve Bank of St. Louis from 1989 to 2010, we estimate models (8-1), (8-2), and (8-3) to obtain the model parameters. The estimated parameters are based on annual data and are reported in Table 8.1.

In what follows we will use these estimates to forecast returns for investor portfolios and for DB plans.

DC Plans

Given the financial market model developed in the previous section, suppose the individual investor or, equivalently, the retiree selects a portfolio $(\omega_1, \omega_2, \omega_3)$ in the S&P 500 index, the Merrill Lynch corporate bond index, and the three-month T-bill, respectively. Given our interest in longevity risk we investigate the length of a sustainable retirement period under the following two assumptions: (1) The individual invests in a TIAA-CREF-type lifecycle fund;[6] (2) The individual invests in a portfolio based on his own preferences.

Assumption (1): Investment in a TIAA-CREF-type Lifecycle Fund

The TIAA-CREF Lifecycle Funds consist of a series of target retirement date funds in five-year increments (2010, 2015, 2020, etc.), where an investor selects the fund that most closely matches his or her retirement year (e.g. a Lifecycle 2040

Table 8.2 Asset allocation at different ages (percentage)

Age	65	66	67	68	69	70	71	72	73	74	75 and older
Equity	50	49	48	47	46	45	44	43	42	41	40
Bonds	38	38	38	38	39	39	39	39	39	40	40
Risk Free	12	13	14	15	15	16	17	18	19	19	20

Note: All the numbers for equity, bonds, and risk-free assets are in percentages.
Source: TIAA-CREF (2013).

Fund is for an investor planning to retire in or around 2040). The funds are professionally managed and automatically adjust over time. For a retiree who invests in a TIAA-CREF-type Retirement Fund, the portfolio allocation at different ages is illustrated in Table 8.2.[7]

Assumption (2): Investment in a Self-selected Portfolio.

To consider all possible combinations of ω_1 and ω_2, we note that short selling is not allowed. In this case the investment in equity and bond indices satisfies $0 \le \omega_1 + \omega_2 \le 1$.

Investor Portfolio Analysis

We investigate the sustainable length of a retiree's retirement savings when the financial market experiences the following two scenarios:

Base Case

Suppose the financial market maintains the same trend and volatility that it has demonstrated over the past 20 years.[8] As such, we use the parameters in Table 8.1 calibrated with historical data to forecast the returns of the three pension assets. Under the Base case, the stock market will experience about 10/(1/0.2742) = 2.74 crashes every ten years. The crashes in the bond market take place less frequently. Every ten years, an investment in corporate bonds is expected to face 10/(1/0.505) = 0.51 crashes.[9]

BaseX2 Case

Suppose financial crashes happen twice as frequently as what the market experienced in the past 20 years. As such, we double the parameters λ_1, m_1, and s_1 for the stock index and λ_2, m_2, and s_2 for the bond index. Under the BaseX2 case, the stock and bond markets will experience 5.48 and 1.01 crashes every ten years.

We simulate the returns of three pension assets from $t = 0$ when the individual retires. Based on the United States male population mortality data from 1901 to 2007,[10] we assume the maximal age he can live is 103. For each yield path, we simulate 58 years after $t = 0$. Suppose the initial retirement fund at time 0 is $M_0 = M$ and the retiree withdraws W_d per period starting from $t = 1$. The value of the retirement fund M_t at time t depends on the amount invested in asset i at time $t-1$, $A_{i,t-1}$ and its return in period t and $r_{i,t}$. Hence,

$$M_t = \sum_{i=1}^{3} A_{i,t-1}(1 + r_{i,t}), \quad t = 1,2,3,... \tag{8.4}$$

and the following equation holds for the retiree:

$$\sum_{i=1}^{3} A_{i,t} = M_t - W_d, \quad t = 1,2,3,... \tag{8.5}$$

The sustainable length of the retirement fund, $\tilde{S}$, is calculated as:

$$\tilde{S} = \max\left\{t \in N^+ \mid M_t \geq W_d\right\} \tag{8.6}$$

We run 1,000 simulations with the market parameters to generate 1,000 yield paths for each pension asset. For each yield path, we calculate sustainable length S_i based on equations (8.4), (8.5), and (8.6). For the random variable $\tilde{S}$, we investigate three measures (the mean, $VaR_{1\%}$, and $CVaR_{1\%}$) as shown in models (8.7), (8.8), and (8.9) respectively.

$$E(\tilde{S}) = E\left(\sum_{i=1}^{1000} S_i\right) \tag{8.7}$$

$$VaR_{1\%}(\tilde{S}) = s = \min\left\{s \in R \mid \mathrm{P}\left\{\tilde{S} \geq s\right\} \leq 99\%\right\} \tag{8.8}$$

$$CVaR_{1\%}(\tilde{S}) = E\left\{\tilde{S} \mid \tilde{S} \leq s\right\} \tag{8.9}$$

where $\tilde{S}$ stands for the sustainable length of the retiree's retirement fund, $VaR_{1\%}(\tilde{S})$ gives the smallest sustainable period such that the probability of observing a sustainable period greater than it is 99 percent, and $CVaR_{1\%}(\tilde{S})$ gives the expected sustainable period conditional on the sustainable period being shorter than $VaR_{1\%}(\tilde{S})$. The impact of portfolio allocation on the mean, $VaR_{1\%}$, and $CVaR_{1\%}$ of the sustainable length is sensitive to the initial retirement savings M and the annual withdrawal W_d, which can be explained following two lines of thought that lead to opposite conclusions. First, due to the risk and return tradeoff, we should be able to observe a negative relationship between the mean and $CVaR_{1\%}$ (or $VaR_{1\%}$) because the latter is a measure of risk. Second, the mean and the tail expectation of a random variable could move in the same direction, since both come from the same distribution.

Results for the TIAA-CREF-type Lifecycle Fund

Given an initial retirement fund M = \$1,000,000, we assume the retiree invests in a TIAA-CREF-type retirement fund. That is, the portfolio allocation changes over time as specified in Table 8.2. The sustainable length of the fund for the Base case and the BaseX2 case under different annual withdrawal strategies is illustrated in Table 8.3.

We further investigate how the individual's funding status would be affected if the financial market deteriorates due to more frequent crashes (BaseX2 case). Setting the Base case as the benchmark, the influence of market deterioration on the individual retirement fund is expressed as the difference of the sustainable length between the Base and BaseX2 cases. The differences in mean, $VaR_{1\%}$, and $CVaR_{1\%}$ are illustrated in Figure 8.1. Figure 8.1 shows that if the financial markets

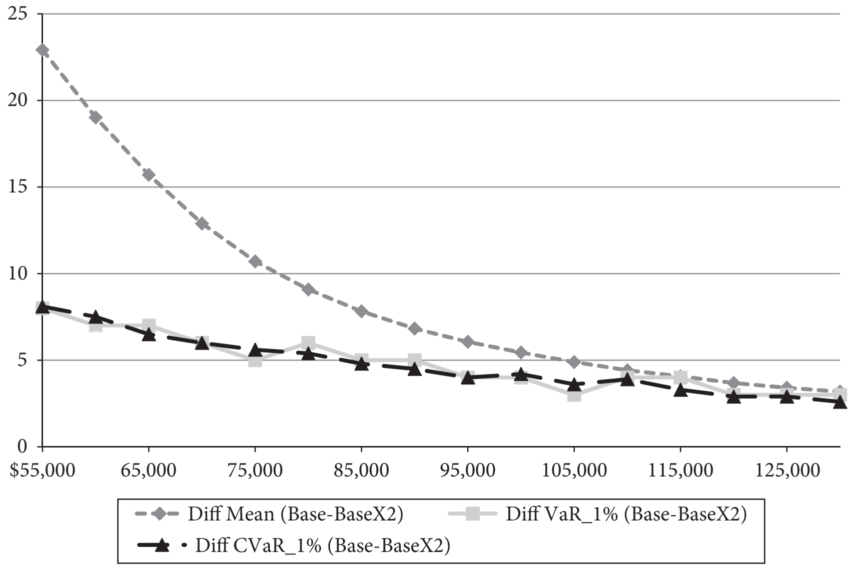


Figure 8.1. Comparison between Base and BaseX2 cases for an investor holding a TIAA-CREF-type retirement fund.

Note: The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years. The BaseX2 case supposes financial crashes happen twice as frequently as the market has experienced in the past 20 years. The vertical axis shows the sustainable periods in years and the horizontal axis is the annual withdrawal.

Source: Authors' calculations; see text.

TABLE 8.3 Sustainable lengths when investing in a TIAA-CREF-type retirement fund

Base Case				BaseX2 Case			
Withdraw ($)	Mean ($)	$VaR_{1\%}$ ($)	$CVaR_{1\%}$ ($)	Withdraw ($)	Mean ($)	$VaR_{1\%}$ ($)	$CVaR_{1\%}$ ($)
55,000	35.305	14	13.4971	55,000	12.382	6	5.3951
60,000	30.558	13	12.5974	60,000	11.541	5.99	5.09
65,000	26.501	12	11.4963	65,000	10.795	5	4.9999
70,000	23.054	11	10.5974	70,000	10.163	5	4.5975
75,000	20.281	10	9.8996	75,000	9.574	5	4.2936
80,000	18.154	10	9.395	80,000	9.068	4	4
85,000	16.411	9	8.799	85,000	8.588	4	4
90,000	15.016	9	8.4963	90,000	8.19	4	4
95,000	13.862	8	7.8996	95,000	7.801	4	3.8996
100,000	12.877	8	7.5974	100,000	7.433	4	3.3951
105,000	12.024	7	6.8996	105,000	7.131	4	3.2936
110,000	11.267	7	6.8996	110,000	6.846	3	3
115,000	10.612	7	6.2935	115,000	6.542	3	3
120,000	10.008	6	5.8996	120,000	6.323	3	3
125,000	9.483	6	5.8996	125,000	6.07	3	3
130,000	9.023	6	5.5975	130,000	5.837	3	3

Notes: The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years. The BaseX2 case supposes financial crashes happen twice as frequently as what the market experienced in the past 20 years.

Source: Authors' calculations; see text.

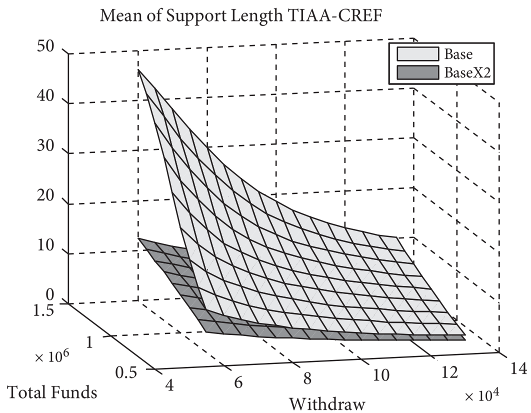
become more volatile, in the sense that crashes become more frequent, then the individual can expect to lose more than 20 periods from the sustainable retirement fund. Figure 8.1 and Table 8.3 also show that, under the same circumstances, a retiree can lose eight sustainable periods from the $VaR_{1\%}$ at the lowest annual withdrawal ($55,000). Finally, the difference in CVaR represents the loss in sustainable periods in the tail of the distribution sustainable periods and Figure 8.1 shows that for the lowest withdrawal rate the investor can expect to lose 8.1 of the sustainable retirement periods; the expected tail loss eventually diminishes with the withdrawal rate since the number of sustainable retirement periods also diminishes.

When both the initial fund M and annual withdrawal W_d are allowed to change, we demonstrate the impact of M and W_d on the sustainable number of periods in Figure 8.2. This figure shows that the BaseX2 case deteriorates from the Base case. As one expects, the figure shows that the sustainable number of retirement years increases with M and decreases with W_d. Given M = $1,000,000 and W_d = $55,000, the expected number of sustainable retirement periods is 35 in the Base case, while it is about 13 periods if W_d is increased to $100,000.

If the investor realizes returns in the tail of the portfolio distribution, then the $CVaR_{1\%}1$ yields 13 and 8 periods for these two withdrawal rates respectively in the Base case. Again given M = $1,000,000, the expected number of sustainable retirement periods is greater than the life expectancy of a 65-year-old U.S. male (i.e. 19.4 years) if he withdraws no more than $75,000 per year; even here, however, a withdrawal rate of $55,000 will not sustain that 65-year-old to his life expectancy. Since there is a 30 percent chance that the 65-year-old will live to 90, the M = $1,000,000 and W_d = $55,000 may be adequate unless he experiences returns in the tail of the portfolio distribution; then the sustainable retirement period is clearly inadequate. In the event that crashes are more frequent, Table 8.3 shows that given M = $1,000,000 and W_d = $55,000, the expected and conditional tail-expected values for sustainable periods become 12.38 and 5.4 respectively. Hence, the expectations fall far short of the life expectancy if the financial market deteriorates, as in the BaseX2 case. One of the additional difficulties for the investor facing financial risk and longevity risk is that his perceived life expectancy may fall short of his actual life expectancy (i.e. the individual may get the trend wrong).

Results for Other Portfolios

Next we suppose the investor selects a portfolio for retirement based on his own preferences.[11] Given an initial retirement asset M at $t = 0$ of $1,000,000, we show how the sustainable periods are affected by the annual withdrawal W_d and portfolio allocation under the Base and BaseX2 scenarios. Figures 8.3 and 8.4 show the mean and $CVaR_{1\%}$ of the sustainable lengths respectively. The three surfaces from top to bottom stand for sustainable periods with withdrawal rates of $75,000, $100,000, and $125,000 respectively. As Figure 8.3 shows, in the Base case, the initial investment of M = $1,000,000 and W_d = $75,000 allows the investor to



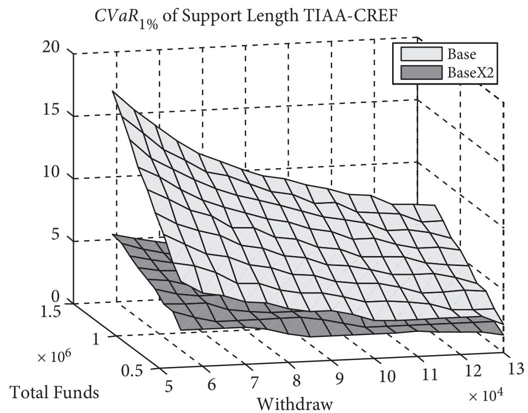


Figure 8.2. Mean and $CVaR_{1\%}$ of sustainable lengths given different initial savings and annual withdrawals for a TIAA-CREF-type lifecycle portfolio.

Note: The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years. The BaseX2 case supposed financial crashes happen twice as frequently as the market has experienced in the past 20 years.

Source: Authors' calculations; see text.

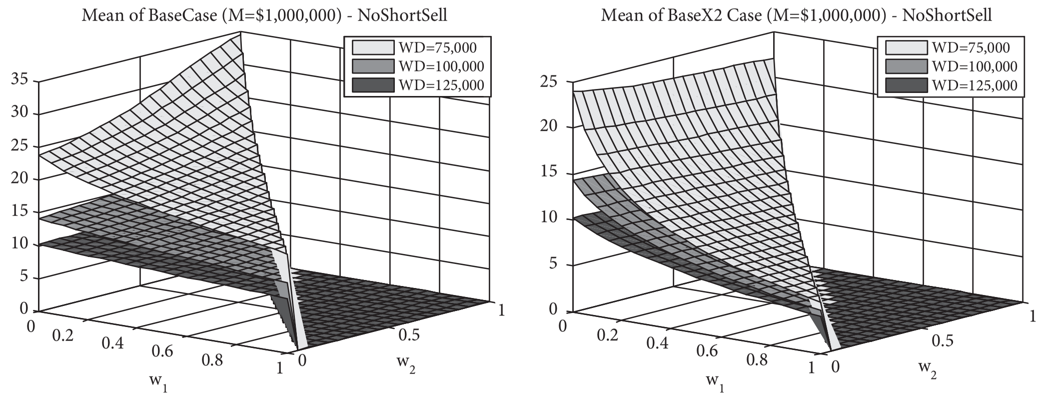


Figure 8.3. Mean of sustainable length when investing in customized portfolios.

Note: The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years. The Base X2 case supposed financial crashes happen twice as frequently as the market has experienced in the past 20 years. w_1 *and* w_2 stand for the proportions of the retiree's fund invested in equity and long-term fixed-income securities, respectively.

Source: Authors' calculations; see text.

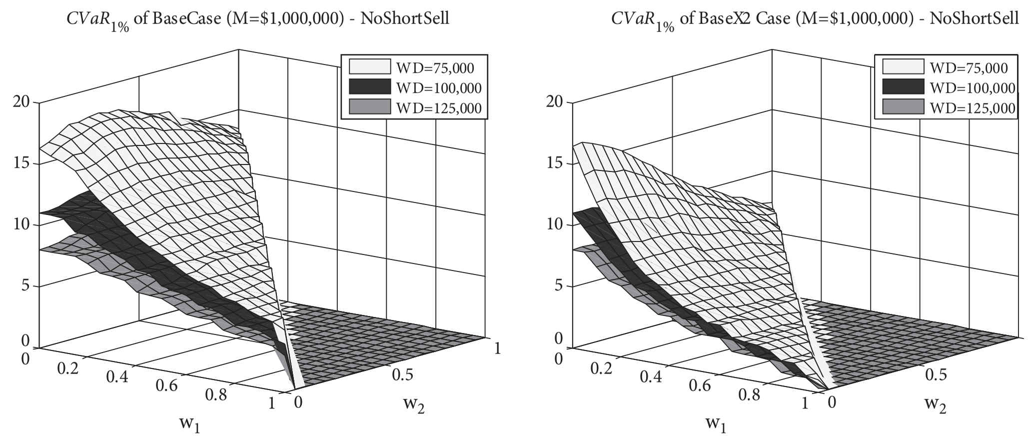


Figure 8.4. $CVaR_{1\%}$ of sustainable length when investing in customized portfolios.

Note: The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years. The Base X2 case supposed financial crashes happen twice as frequently as the market has experienced in the past 20 years. w_1 *and* w_2 stand for the proportions of the retiree's fund invested in equity and long-term fixed-income securities, respectively.

Source: Authors' calculations; see text.

generate an expected sustainable retirement of almost 35 periods given the portfolio $(\omega_1, \omega_2, \omega_3)$ = (0, 1, 0) (i.e. the investor plunges in the bond index fund). Figure 8.3 also shows that in the BaseX2 case, the investor can generate an expected sustainable retirement of almost 24 years by investing in the portfolio $(\omega_1, \omega_2, \omega_3) =$ (0, 0, 1) (i.e. the investor plunges in T-bills). Again in the case of an initial investment of M = \$1,000,000 and W_d = \$75,000, Figure 8.4 shows that to maximize the number of sustainable retirement years in the tail measured by $CVaR_{1\%}$, the investor should invest 30 percent of his fund in the bond index and the remaining 70 percent in T-bills (i.e., $(\omega_1, \omega_2, \omega_3)$ = (0, 0.3, 0.7)) in the Base case; this portfolio yields almost 18 sustainable retirement periods. Figure 8.4 also shows that in the BaseX2 case, the investor should choose the portfolio with 10 percent invested in the bond index and 90 percent invested in T-bills (i.e. $(\omega_1, \omega_2, \omega_3)$ = (0, 0.1, 0.9))[12] to maximize the number of sustainable retirement periods in the tail; this portfolio yields 11 sustainable retirement periods. In other words, to reduce tail risk, the investor should choose a more conservative portfolio and invest more in risk-free assets such as T-bills.

DB Plans

DB plans put longevity risk on the pension provider, not the individual. The DB provider is a trustee who should act in the interests of the retirement cohort; we consider one retirement cohort. Since the DB plan is exposed to financial and longevity risks, one objective is to minimize the total unfunded liability (TUL) of the plan subject to any appropriate constraints. The TUL up to the terminal age of the retirees is defined as the present value of the sequence of unfunded liabilities. Hence TUL is

$$TUL = \sum_{t=1}^{\infty} \frac{UL_t}{(1+\rho)^t}$$

where the random variable UL_t is the underfunding at time t. We suppose that T is the retirement date of the cohort. Then for $t \leq T$, the plan's unfunded liability UL_t equals

$$UL_t = PBO_t - (PA_t + C), \quad t = 1, 2, \ldots, T. \tag{8.10}$$

In (8.10) PBO_t is the pension benefit obligation and PA_t is the date t pension asset value. When $t > T$, UL_t equals

$$UL_t = PBO_t - PA_t + B \cdot {}_{t-T}\hat{p}_{x,T} \qquad t = T+1, T+2, \ldots \tag{8.11}$$

where B is the survival benefit and ${}_{t-T}\hat{p}_{x,T}$ is the conditional expected probability that a plan member age x at time T survives $t-T$ years when $t > T$.

Following Cox et al. (2013), who investigate capital market and longevity risks, we solve the following constrained minimization problem, or equivalently the pension optimization problem:

$$\begin{aligned} &\text{Minimize Var}\left[\sum_{t=1}^{\infty}\frac{UL_t}{(1+\rho)^t}\right] \\ &\text{subject to } E\{TUL\} = 0 \\ &\qquad CVaR_\alpha(TUL) = \tau \\ &\qquad 0 \le \omega_i \le 1,\ i = 1,2,\ldots,\mathrm{n} \\ &\qquad \sum_{i=1}^{n}\omega_i = 1 \\ &\qquad C \ge 0 \end{aligned} \tag{8.12}$$

In (8.12), we require the expected TUL to equal zero. To control the underfunding risk, we impose an α-level conditional value-at-risk (CVaR) constraint on the total unfunded liability (i.e. $E(TUL \mid TUL \ge VaR_{95\%}) = \tau$).[13] Short selling is not allowed for the plan, so $\omega_i \ge 0$.

DB Base Case

To obtain the optimal solutions for the Base case for a DB plan given the pension optimization problem in (8.12) with the Lee and Carter (1992) mortality model and the pension asset models (8.1), (8.2), and (8.3), suppose the DB plan has members who all join the plan at age $x_0 = 45$ ($t = 0$) and retire at age $x = 65$ ($T = 20$). The annual survival benefit payment after retirement is $B = \$10$ million and the pension fund at $t = 0$ is \$5 million. Following Cox et al. (2013), we set the pension valuation rate at $\rho = 0.08$ and the life annuity discount rate at $r = 0.05$. In addition, the plan will amortize the unfunded liability over $m = 7$ years as in Panteli (2010) and following Maurer et al. (2009), we set the penalty factors on the supplementary contributions and withdrawals at $\psi_1 = \psi_2 = 0.2$. Our objective is to find the optimal pension asset allocation and contribution strategies for the plan throughout the life of the cohort.

We set year 2007 as our base year $t = 0$ and run a Monte Carlo simulation with 1,000 iterations to generate forecasts for the three financial asset returns and pension liabilities PBO_t for $t = 1,2,\ldots$. The downside risk parameter is set at 60 and given $\tau = 60$, the optimal solution for (8.12) is shown in Table 8.4.

To achieve the lowest underfunding variance $\mathcal{J}$ and the target $CVaR_{95\%}(TUL)$ of 60, the plan should invest 27 percent of its funds in the S&P 500 index, 43 percent in the Merrill Lynch corporate bond index, and the remaining 30 percent of its funds in three-month T-bills. In addition, given $E(TUL) = 0$, the optimal annual

Table 8.4 Optimal solution for the Base case with model (5.13)

C	ω_1	ω_2	ω_3	J
3.34	0.27	0.43	0.30	624.41

Notes: C stands for the normal contribution. J is the value of the objective function in Model (5-13), which measures the variance of total unfunded liability. (ω_1 , ω_2 , ω_3) represents the investment strategy where ω_1 , ω_2 , and ω_3 are the proportions invested in the S&P 500 index, Merrill Lynch corporate bond index, and 3-month T-bill, respectively. The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years.

Source: Authors' calculations; see text.

contribution is $C = 3.34$. The total pension cost represents the present value of all normal contributions, C, supplementary contributions, SC_t and withdrawals, W_t . A higher TPC lowers the plan's underfunding risk but imposes a higher cost on the plan sponsor. To achieve the level of $CVaR_{95\%}(TUL) = 60$, the expected total pension cost is $ETPC = 36.08$.

Longevity Risk Effect

To examine the adverse effect of an unexpected mortality improvement on the plan, we change the value of the Base case g in the model to other possible values; in the Lee–Carter model, mortality is a function of a common risk factor and the risk factor is described as a random walk with drift g. This drift g may be thought of as the systematic risk component of mortality. A more negative value of g implies a more substantial mortality improvement. Given $CVaR_{95\%}(TUL) = 60$, the adverse effect of the longevity risk is captured by the higher E(TPC), since the plan must adjust E(TPC) upward to reflect higher longevity risk.

Table 8.5 shows the optimization results given different assumptions on g after solving the optimization problem (8.12). As g decreases from the Base case −0.20 to −0.40, E(TPC) increases notably from 36.08 to 39.06 (i.e. an 8.3 percent rise); this is due to the higher normal contribution $C = 3.62$ with $g = -0.40$, compared to $C = 3.34$ with $g = -0.20$. The increased longevity risk increases the normal contribution and puts more weight in the tails of the underfunding distribution. Hence an increase in longevity risk increases the variance J of the underfunding distribution. That variance increases from 624.41 to 709.96 as g decreases from −0.2 to −0.4. In addition, as g decreases, the plan invests more in the safe asset. Equivalently, the plan manager must elect a higher portfolio weight for the safe asset, so as to satisfy his downside risk constraint (that is, the $CVaR_{95\%}(TUL) = 60$).

Table 8.5 Optimal normal contribution and asset allocation for the Base case given $CVaR_{95\%}(TUL) = 60$ and $E(TUL) = 0$ and different mortality improvement parameters g in the Lee and Carter Model (1992)

g	C	ω_1	ω_2	ω_3	$E(TPC)$	$\mathcal{J}$
−0.20	3.34	0.27	0.43	0.30	36.08	624.41
−0.30	3.50	0.28	0.39	0.33	37.77	670.84
−0.40	3.62	0.28	0.36	0.36	39.06	709.96

Notes: C stands for the normal contribution. $\mathcal{J}$ is the value of the objective function in Model (13), which measures the variance of total unfunded liability. (ω_1, ω_2, ω_3) represents the investment strategy where ω_1, ω_2, and ω_3 are the proportions invested in the S&P 500 index, Merrill Lynch corporate bond index, and 3-month T-bill, respectively. g is the mortality improvement parameter in the Lee and Carter Model (1992). $E(TPC)$ represents the expected total pension cost of the plan. The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years.

Source: Authors' calculations; see text.

Capital Market Risk Effects

To examine the capital market risk effect on the pension plan, we double the values of the volatility, jump size, and jump arrival rate parameters for the S&P 500 index and the Merrill Lynch corporate bond index in Table 8.1, simulate, and resolve the optimization model (8.12). The results with the doubled parameter values of the S&P 500 index and the Merrill Lynch corporate bond index are shown in Table 8.6; they provide important insights for the pension plan with respect to possible market crashes. Given g equal to the Base case level of −0.20 and a more volatile capital market, the $E(TPC)$ increases by 16.6 percent from 36.08 in Table 8.5 to 42.08 in Table 8.6. In addition, to meet the downside risk constraint, the annual normal contribution C increases by 17.8 percent to 3.93 and the proportion invested in the low-risk three-month T-bill rises by 54 percent to $\omega_3 = 0.47$, compared with the Base case levels of $C = 3.34$ and $\omega_3 = 0.30$ in Table 8.5.

It is worth noting that if the adverse longevity and capital market events both occur, it will push up the expected total pension cost $E(TPC)$ dramatically by 24.9 percent, from 36.08 in Table 8.5 to 45.05 in Table 8.6. These changes could cause significant financial consequences to the pension sponsor. Both longevity risk and capital market risk affect the financial stability of a pension sponsor.

Next we investigate how pension hedging strategies can mitigate the adverse effects arising from these two sources of risk.

Pension Hedging Strategies

Here we investigate two pension longevity risk-hedging strategies: a ground-up hedging strategy, and an excess-risk hedging strategy. The ground-up hedging strategy not only reduces longevity risk but also manages capital market risk, as it

TABLE 8.6 Optimal normal contribution and asset allocation for the BaseX2 case given $CVaR_{95\%}(TUL) = 60$ and $E(TUL) = 0$ and different mortality improvement parameters g in the Lee–Carter Model (1992)

g	C	ω_1	ω_2	ω_3	$E(TPC)$	$\mathcal{J}$
−0.20	3.93	0.08	0.45	0.47	42.08	619.92
−0.30	4.12	0.08	0.43	0.48	43.97	660.39
−0.40	4.22	0.08	0.42	0.49	45.05	697.99

Note: C stands for the normal contribution. $\mathcal{J}$ is the value of the objective function in Model (5-13), which measures the variance of total unfunded liability. (ω_1, ω_2, ω_3) represents the investment strategy where ω_1, ω_2, and ω_3 are the proportions invested in the S&P 500 index, Merrill Lynch corporate bond index, and 3-month T-bill, respectively. g is the mortality improvement parameter in the Lee and Carter Model (1992). $E(TPC)$ represents the expected total pension cost of the plan. The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years.

Source: Authors' calculations; see text.

transfers both pension asset and liability risks to pension risk takers. The ground-up hedging strategy, given a full hedge, is equivalent to a pension buyout, and the excess-risk hedging, also given a full hedge, is equivalent to a mortality option; see Cox et al. (2013) for a discussion of both.

The Ground-up or Buyout Hedging Strategy

Suppose the plan implements a ground-up hedging strategy and transfers a proportion h^G of pension assets and liabilities to a hedge provider by paying a price equal to

$$HP^G = \frac{h^G\left(1+\delta^G\right)B\bar{a}\left(x(T)\right)}{\left(1+\rho\right)^T}$$

where $B\bar{a}\left(x(T)\right)$ is the expected present value of pension payments at retirement T and δ^G is the unit hedge cost. Given that the plan pays a hedge price HP^G, the available fund for pension asset investment at $t = 0$ is $PA_0^G = M^G = M - HP^G$, which is lower than that of the no-hedge case with $PA_0 = M$. In our example, $M = 5$. With the hedge ratio h^G, the pension liability retained by the plan becomes

$$PBO_t^G = \begin{cases} \dfrac{\left(1-h^G\right)Ba\left(x(T)\right)}{\left(1+\rho\right)^{T-t}} & t = 1,2,\ldots,T \\ \left(1-h^G\right)Ba\left(y(\mathrm{t})\right) & t = T+1, T+2,\ldots \end{cases}$$

In this expression, $a\big(x(T)\big)$ is the life annuity factor for age x at retirement T and $a\big(y(t)\big)$ is the life annuity factor for age y after retirement T with $t = T+1, T+2, \ldots$.

Suppose the plan adopts the optimal asset allocation and normal contribution strategies shown in Table 8.5. Table 8.7 shows how the ground-up hedging strategy mitigates the funding downside risk caused by the capital market risk and longevity risk for the Base case with different hedge ratios.

With the ground-up hedging strategy, when $h^G > 0$, all $CVaR_{95\%}(TUL)$s in Table 8.7 are lower than $CVaR_{95\%}(TUL) = 60$ without hedging. Table 8.7 also shows that as h^G increases, $CVaR_{95\%}(TUL)$ and E(TPC) decrease, indicating a lower pension risk to the plan. For example, when $\delta^G = 0$, as h^G increases from 0.1 to 0.15, $CVaR_{95\%}(TUL)$ decreases from 46.91 to 40.38 and E(TPC) decreases from 34.81 to 34.18. The hedge cost δ^G, however, reduces the risk reduction effect of the ground-up hedging. For example, when $\delta^G = 0$ and $h^G = 0.15$, $CVaR_{95\%}(TUL)$ is only 40.38 but it increases to 42.16 when $\delta^G = 0.1$ and $h^G = 0.15$. As a robustness check, we also examine the ground-up hedging strategy with different combinations of g and the pension asset parameters. All of them echo the pattern we observe in Table 8.7. We conclude that the ground-up hedging strategy can effectively reduce the capital market and longevity risks imbedded in a pension plan.

TABLE 8.7 Ground-up edging strategy for Base case with $g = -0.2$

	$\delta^G = 0$		$\delta^G = 0.05$		$\delta^G = 0.1$	
	$h^G = 0.1$	$h^G = 0.15$	$h^G = 0.1$	$h^G = 0.15$	$h^G = 0.1$	$h^G = 0.15$
C	3.34	3.34	3.34	3.34	3.34	3.34
ω_1	0.27	0.27	0.27	0.27	0.27	0.27
ω_2	0.43	0.43	0.43	0.43	0.43	0.43
ω_3	0.31	0.31	0.31	0.31	0.31	0.31
E(TUL)	−7.93	−11.90	−7.31	−10.96	−6.68	−10.02
E(TPC)	34.81	34.18	35.03	34.52	35.26	34.86
$CVaR_{95\%}(TUL)$	46.91	40.38	47.50	41.27	48.09	42.16
J	625.75	628.73	631.62	637.40	637.58	646.26
HP^G	2.36	3.54	2.48	3.72	2.60	3.90

Note: C stands for the normal contribution. (ω_1, ω_2, ω_1) represents the investment strategy where ω_2, ω_2, and ω_3 are the proportions invested in the S&P 500 index, Merrill Lynch corporate bond index, and 3-month T-bill, respectively. g is the mortality improvement parameter in the Lee and Carter Model (1992). TUL and TPC represent the total unfunded liability and total pension cost of the plan. J is the value of the objective function in Model (5-13), which measures the variance of total unfunded liability. HP^G, h^G, and δ^G are the hedge price, hedge ratio, and unit hedge cost under the ground-up hedging strategy. The Base case assumes the financial market maintains the same trend and volatility that it has demonstrated throughout the past 20 years.

Source: Authors' calculations; see text.

The Excess-risk Hedging or Insurance Option Strategy

The second pension hedging strategy, the excess-risk hedging strategy, focuses on transferring the high-end longevity risk. With the excess-risk hedging strategy, the plan needs to determine a strike level on the s-year survival probability ${}_s\tilde{p}_{x,T}$ for age x at retirement T in each year t, $t = T+1, T+2, \ldots$ above which to transfer a proportion h^E of the longevity risk. The conditional expected s-year survival rate, ${}_s\hat{p}_{x,T}$, is defined as ${}_s\hat{p}_{x,T} = E[{}_s\tilde{p}_{x,T} \mid \tilde{p}_{x,T}, \tilde{p}_{x+1,T+1}, \ldots, \tilde{p}_{x+s-1,T+s-1}]$, where $\tilde{p}_{x+s-1,T+s-1}$ is the one-year survival rate for age $x+s-1$ in year $T+s-1$.

Suppose at $t=0$, the plan purchases a series of European call options with strike levels set at the expected survival rates, ${}_s\bar{p}_{x,T} = E[{}_s\hat{p}_{x,T}]$, $s = 1,2,\ldots$. The option payoffs in years $T+1, T+2, \ldots$ are determined by $\max\{0, B\,{}_s\hat{p}_{x,T} - B\,{}_s\bar{p}_{x,T}\}$, $s = 1,2,\ldots$. Accordingly, to hedge a proportion h^E, the plan needs to pay a hedge price of

$$HP^E = \frac{h^E\left(1+\delta^E\right)E\left[\sum_{s=1}^{\infty} v^s \max\left\{B\,{}_s\hat{p}_{x,T} - B\,{}_s\bar{p}_{x,T}, 0\right\}\right]}{\left(1+\rho\right)^T}$$

where δ^E is the hedge cost per unit of longevity risk ceded. With the hedge ratio h^E, the plan's liability becomes

$$PBO_t^E = \begin{cases} \dfrac{Ba\left(x(T)\right) - Bh^E\sum_{s=1}^{\infty} v^s \max\left\{0, {}_s\hat{p}_{x,T} - {}_s\bar{p}_{x,T}\right\}}{\left(1+\rho\right)^T} & t = 1,2,\ldots,T \\ Ba\left(y(\mathrm{t})\right) - Bh^E\sum_{s=t-T+1}^{\infty} v^{s-(t-T)} \max\left\{0, {}_s\hat{p}_{x,T} - {}_s\bar{p}_{x,T}\right\} & t = T+1, T+2, \ldots \end{cases}$$

With the hedge price HP^E, the fund available for investment at time 0 is reduced to $M^E = M - HP^E$. Again, in our example, we assume $M = 5$ and the pension plan implements the optimal asset allocation and normal contribution strategies shown in Table 8.5. With different combinations of g and the pension asset parameters, we find the same pattern as Table 8.7 with the ground-up hedging strategy (results available on request from the authors). That is, with a positive hedge ratio h^E, $CVaR_{95\%}(TUL)$ and E(TPC) are lower than those without hedge. However, the magnitude of risk reduction achieved by the excess-risk hedging strategy is much lower than that of the ground-up hedging strategy. For example, when $g = -0.20$, the pension asset parameters in Table 8.1 and $h^G = 0.1$, $CVaR_{95\%}(TUL) = 46.91$ with the ground-up hedging strategy. However, at the same levels of g and the pension asset parameters, the excess-risk strategy only reduces $CVaR_{95\%}(TUL)$ to 56.44 even with a full hedge of longevity risk above the expected survival rates (i.e., $h^E = 1$). This is explained by the fact that the excess-risk strategy only transfers the high-end longevity risk but not the pension asset risk, while the ground-up strategy reduces both pension asset and liability risks. In many cases, the capital market risk on pension assets seems to impose a more significant effect on the pension plan than the longevity risk.

Conclusion

Concern regarding capital market risk often eclipses that due to longevity risk in pension management. When it comes to retirement issues, however, these two risks are integrally linked. The number of sustainable years for a retirement portfolio is determined, in part, by market crashes, changes in market volatility, and changes in life expectancy. There are instruments to handle capital market volatility, which include futures and forward contracts to hedge interest rate, currency, and price risks; there are also derivatives to hedge credit risks, weather risks, and more. There are insurance instruments to handle the volatility of life; these instruments include life insurance and life annuities. These insurance instruments have not, however, been designed to deal with the systematic component of the life risks. If life expectancy unexpectedly increases (e.g. a cure for cardiovascular disease or cancer is found), then life insurance becomes more profitable for the insurer but life annuities become less profitable, or may even threaten insurer solvency and adversely impact retirement plans of individuals and pension funds.

To address these issues, we created scenarios to assess risk for both the individual and the institution. In the case of the individual, scenario analysis showed that if the individual invested in a lifecycle fund such as that offered by TIAA-CREF and the financial markets were driven by historical parameters, then a $1 million investment at retirement combined with a withdrawal rate of $75,000 per year would yield approximately 20 sustainable years. This is one year more than the life expectancy of a 65-year-old male in 2013. Most financial planners would consider this as an inadequate retirement horizon, and many would advocate planning for a much longer horizon.[14] The same analysis shows that one could only say that the fund would last for ten years with a 99 percent probability. Similarly, if market parameters were doubled so that crashes occurred more often and the market was more volatile (i.e. BaseX2 case), one could expect the fund to last less than ten years and the fund would last for four years with 99 percent probability. This leaves the investor with considerable uncertainty. Yet the TIAA-CREF-type lifecycle fund held 40 percent in equity, 40 percent in bonds, and 20 percent in T-bills. The analysis also showed that the investor could select an alternative portfolio to increase the number of sustainable years. If the investor held all in the bond fund, he could expect the portfolio to continue paying the same $75,000 per year for almost 35 years. Additionally, if the investor's returns were in the worst 1 percent of the portfolio payoffs then he could still expect almost 18 sustainable retirement years, but only if the portfolio was changed to a 30 percent investment in bonds and a 70 percent investment in T-bills. In the BaseX2 case (i.e. a more volatile market), the same investor could expect the portfolio fund to last almost 24 years if he plunged in the T-bill fund. In this BaseX2 case, if the investor's returns were in the worst 1 percent of the portfolio payoffs then he could expect 11 sustainable retirement years, but only if the portfolio was altered to a 10 percent investment in bonds and a 90 percent investment in T-bills. These numbers do not account for the possible changes in life expectancy that will doubtless make even the best numbers here seem even less sufficient. The DC plans leave the investor with considerable longevity risk, and without the foresight of increasing the size of the investment fund they can only reduce the annual withdrawal or change the portfolio to attempt to keep the retirement fund sustainable for more years. These results emphasize the need for financial instruments that provide a more effective means of transferring some of the longevity risk to those better able to bear it.

Pension providers bear the longevity risk for DB plans. The pension provider has a fiduciary responsibility to act in the interest of the plan members and therefore our scenario objective was to minimize pension underfunding subject to constraints on the expected underfunding, short selling, and the size of the tail of the underfunding distribution. When longevity risk was increased, the solution to the constrained minimization problem showed an 8.3 percent increase in the expected total pension cost. When increased capital market risk was also added to longevity risk, the solution to the constrained minimization problem showed a 24.9 percent increase in the expected total pension cost.

To mitigate risk, two longevity risk-hedging schemes were considered. The first was similar to a partial buyout of the pension plan and the analysis showed that this hedge could lower the pension failure risk. When longevity risk was increased, the reduction in pension risk between the hedged and unhedged scenarios became more pronounced; when capital market risk was also increased, the reduction in pension failure risk between the hedged and unhedged scenarios was even more pronounced. The second hedge was a longevity option. Here there was no exchange of assets; rather, there was only an exchange of liabilities in the tail. As was the case with the partial buyout strategy, the longevity option strategy also demonstrated a reduction in pension failure risk that increased with the size of the hedge.

In sum, improvements can result from managing longevity risk in the context of both defined contribution and defined benefit schemes. Today's DC risk management schemes are currently far too limited (e.g. life annuities and reverse mortgages, among others), and additional financial instruments can help fill the gap. The DB risk management possibilities are limited also, but they have received more attention in academia and in capital markets. Other hedging schemes must also be addressed.

Notes

1. See BBC News (2010).
2. For more discussion of buyouts and buy-ins, see Blake et al. (2013).
3. Embedded value is the present value of future profits plus adjusted net asset value.
4. For example, Vita II–Swiss Re 2005 ($362 million), Vita III–Swiss Re 2007 ($705 million), Vita IV–Swiss Re 2009/10 ($175 million), Vita V–Swiss Re 2012 ($275 million), Tartan–Scottish Re 2006 ($155 million), and OSIRIS–AXA 2006 ($442 million).
5. For more on this, see Feinstein (1993), Blake et al. (2006), Wang et al. (2007), and Coughlan (2014).
6. TIAA-CREF stands for a Teachers Insurance and Annuity Association–College Retirement Equities Fund. A lifecycle fund refers to a fund designed to provide long-term appreciation and capital preservation based on the age and retirement date of the investors in the fund.
7. Data were retrieved from TIAA-CREF (2013).
8. In our simulations based on the parameters estimated from 1989–2010 data, the expected risk premium of S&P 500 over three-month T-bill is around −0.03 each year in the Base case and −0.20 in the BaseX2 case. The negative risk premium of S&P 500 is consistent with the observation that the return on equities has been 7.6 percentage points a year lower than that on government bonds in the U.S. since the end of 1999 (*The Economist* 2012).
9. If there is a Poisson event or equivalent crash in the equity market then the expected loss in the S&P index is approximately 26 percent, while if there is a crash in the bond market then the expected loss in the bond index is approximately 14 percent.
10. The mortality data for 1901–1999 are taken from the Human Life Table Database (MPIDR 2013), and for 2000–2007 from the Human Mortality Database (HMD 2013); these were provided by the University of California at Berkeley and the Max Planck Institute for Demographic Research.
11. The analysis is static and so the portfolio is assumed to remain the same through retirement.
12. The portfolio (0, 0.05, 0.95) also maximizes the sustainable years in the tail of the portfolio distribution.
13. The equality constraint is used rather than an inequality constraint so that the variance is not pushed to zero using T-bills (that would also inflate the total pension cost). The equality constraint is also important because it generates base cases which we later use for comparison with two different longevity risk hedges; there we use the same portfolio of assets generated in the Base case and add a hedging instrument. For this comparison to work, we use the equality constraint for *CVaR* and no constraint on the total pension cost.
14. See Krueger (2011). Also, former Society of Actuaries President Anna Rappaport is quoted in Powell (2012) as saying 'The planning horizon should be long, and if mortality data is used to pick it, it should not be life expectancy, but rather the age that there is a 90 percent or 95 percent chance of survival.'

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