Recreating Sustainable Retirement

Resilience, Solvency, and Tail Risk

EDITED BY

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Chapter 7

Model Risk, Mortality Heterogeneity, and Implications for Solvency and Tail Risk

Michael Sherris and Qiming Zhou

Mortality improvements have been systematic in that they have impacted individuals of all ages, although to varying extents by age and across time for many countries. Mortality improvement rates have also shown varying trends (Njenga and Sherris 2011). Pension funds and insurance companies issuing life annuities have been exposed to this systematic risk, and this has the potential to impact solvency, especially in the tail of the distribution of survivors. Although some of this risk has been transferred to reinsurers using reinsurance and longevity swaps, much of this risk is accumulating with insurers, pension funds, and reinsurers, and it has not been diversified into the broader financial markets (Blake et al. 2011).

Systematic longevity risk is usually modeled with a doubly stochastic survival model, where the mortality rate follows a stochastic process and all individuals of the same age and gender are assumed to experience the same realized mortality rate. Given the mortality rate, individual survival is subject only to idiosyncratic risk, which can be diversified in large pools of lives. Even if there is only idiosyncratic risk, at older ages in the tails of the survival distribution, the number of lives surviving becomes small and the variability in benefit payments and liability values increases. This is exacerbated by systematic risk from uncertain but common rates of improvement across individuals.

Many models of systematic mortality risk have been proposed. These vary from models such as the Lee–Carter model (Lee and Carter 1992) and variations, to models that model random changes in a parametric survival curve (Cairns et al. 2006), to those that model the dynamics of mortality rates in a financial framework similar to that used for interest rate models (Biffis 2005). These models do not include allowance for heterogeneity. Individuals of the same age are assumed to experience the same aggregate mortality rate.

Increasingly, attention is being devoted to the impact of mortality heterogeneity and its effect on insurers and pension funds (Lin and Liu 2007; Liu and Lin 2012 ; Su and Sherris 2012). Along with systematic mortality risk, this mortality heterogeneity has implications for the solvency and tail risk of annuity and pension providers. Even if there were no systematic, or aggregate, mortality risk, heterogeneity generates variability in future experience and volatility in financial results. Heterogeneity requires underwriting of risks to avoid adverse selection. Without full information

about the risks that insurers underwrite, the financial consequences of adverse selection has its greatest impact for annuities in the tail of the survival distributions long after the annuities have been issued.

Solvency and tail risk for life annuities and pensions have two dimensions. First, there is an effect on insurer profitability from adverse experience as well as an impact on variability at the older ages. Trends in mortality that arise from uncertain mortality improvements and from the deaths of less healthy lives in a heterogeneous pool have their greatest influence at the older ages. Second, the volatility of financial results arises from both systematic mortality changes, with higher volatility experienced at older ages, and from heterogeneity, also producing higher volatility at older ages (Su and Sherris 2012; Meyricke and Sherris 2013).

There are many different approaches to modeling mortality heterogeneity. Recent advances have seen the calibration and application of more advanced models in the form of Markov aging models (Lin and Liu 2007; Liu and Lin 2012; Su and Sherris 2012) that are extensions of the Le Bras model (1976). The other, more commonly used, approach is to apply frailty models to capture unobserved heterogeneity (Vaupel et al. 1979).

In this chapter, we develop and apply a stochastic Markov aging model of heterogeneity that also includes systematic mortality risk, calibrated to population aggregate mortality and health data. We compare results with a well-known frailty model and the Le Bras–Markov multiple state model to assess model risk, neither of which includes systematic mortality risk. These models are used to quantify solvency and tail risk for a portfolio of life annuities using risk measures standard deviation and value-at-risk for fund values at the older ages. Results illustrate the effects of heterogeneity and model risk on the assessment of longevity risk for these portfolios, as well as the impact of selection and pool size.

Mortality Heterogeneity Models

The main approaches to modeling mortality heterogeneity that we consider are frailty models and Markov multiple state models. Frailty models treat heterogeneity as unobservable. An often-used frailty model is that of Vaupel et al. (1979), where an individual is assumed to have frailty z at age x with force of mortality: $\mu(x,z) = zae^{bx} + c$. The frailty factor z is Gamma distributed $z \sim Gamma(1, \sigma^2)$ so that the average frailty at age x is

$$\bar{z}(x) = \left(1 + \sigma^2 \frac{a}{b}\left(e^{bx} - 1\right)\right)^{-1}$$

and the average force of mortality is given by $\bar{\mu}(x) = \bar{z}(x)ae^{bx} + c$.

The Markov multiple state mortality model was developed by Le Bras (1976), who used a continuous time Markov chain with an infinite number of states and a

discrete state space to model senescence. The model starts at state 1 and progresses to state 2, 3, etc. In any state, the rate of jump to the next higher state and the rate of death are assumed proportional to the state number. All individuals start in state 0 at time 0. In state *i*, the transition rate to state $i + 1$ is $\lambda_0 + i\lambda$, and the transition to death (an absorbing state) is $\mu_0 + i\mu$. For the Le Bras model, the probability of being in state *i* at time *t* is (Yashin et al. 2000):

$$P_i(t) = \frac{e^{-(\lambda_0+\mu_0)t}}{i!}\left(\frac{\lambda - \lambda e^{-(\lambda+\mu)t}}{\lambda+\mu}\right)^i \prod_{k=1}^{i}\left(\frac{\lambda_0}{\lambda} + (k-1)\right)$$

The probability of survival to time *t*, given the individual was in state *n* at time 0, is given by

$$S_n(t) = e^{-(\lambda_0+\mu_0+n(\lambda+\mu))t}\left(\frac{\lambda+\mu}{\mu+\lambda e^{-(\lambda+\mu)t}}\right)^{\frac{\lambda_0+n\lambda}{\lambda}}$$

Yashin et al. (1994) show the representation of the average force of mortality in the fixed frailty model to be equivalent to the Le Bras model.[1] Markov aging models allow for heterogeneity because of the differing mortality rates in the different states.

There have been several applications of Markov chains to failure time distributions in mortality, also known as phase-type distributions. Lin and Liu (2007) devised a deterministic survival rate model based on a Markov aging process. Each state in the model represents a 'physiological age,' as opposed to calendar age. The model assumes that there is a maximum physiological age, *n*, and that $n = 200$ is appropriate as an approximation to the potentially infinite aging process in the Le Bras model. Subsequently, Su and Sherris (2012) developed the Lin and Liu model (2007) to assess population heterogeneity for life annuity portfolios and relate states and mortality rates to aggregate population mortality.

These two Markov aging models have parameters that capture the changes in observed period life tables. Liu and Lin (2012) make the model stochastic by adding a time change component. The small number of states and the transition matrix facilitate the incorporation of health information. The time change allows a probabilistic statement of mortality uncertainty. The initial distribution is estimated from health condition data, and closed forms for the expected value and variance of the survival probability exist if the stochastic time change process has a closed form moment generating function.

These Markov aging models are the basis of the model used in what follows. We extend the Su and Sherris (2012) approach to include health states calibrated to health conditions data as well as aggregate population mortality data. We also subordinate this underlying model to a Gamma time change, so that survival distributions are stochastic. The underlying model allows an assessment of model risk by comparison

of results for solvency and tail risk with the other models of heterogeneity. The subordinated model shows the significance of heterogeneity if mortality is stochastic.

The Markov aging model used has a time-inhomogeneous five-state transition matrix fitted to ages 30–110. Transition occurs as a Markov process from one transient state to its next state, or to the absorbing state, and the model takes into account both health status and mortality data. Aggregate survival rates are determined by a deterministic underlying multiple states survival model $S_0(\cdot)$ and a time change process γ_t. The underlying model assumes the individual mortality process moves through a series of deteriorating health statuses. Health and mortality is made stochastic by a random time change. The aggregate survival rate at time t is $S_t = S_0(\gamma_t)$. Time until death in this system has a phase-type representation (π, T), where π is the initial distribution on the transient states, and T is the states' transition rates matrix. The probability of survival up to time x is $S_0 = \pi \exp(Tx)e$ where e is a column of ones. Under the assumption that deterioration in health is more likely than improvement, transition is assumed to be acyclic. Since all acyclic phase-type distributions have a Coxian representation, T can be written as:

$$\begin{pmatrix} -(\lambda_{1,t}+q_{1,t}) & \lambda_{1,t} & 0 & 0 & 0 \\ 0 & -(\lambda_{2,t}+q_{2,t}) & \lambda_{2,t} & 0 & 0 \\ 0 & 0 & -(\lambda_{3,t}+q_{3,t}) & \lambda_{3,t} & 0 \\ 0 & 0 & 0 & -(\lambda_{4,t}+q_{4,t}) & \lambda_{4,t} \\ 0 & 0 & 0 & 0 & -q_5 \end{pmatrix}$$

where

$$q_{1,t} = q_{2,t} = q_{3,t} = q_{4,t} = a \times e^{bt}.$$

$$q_{5,t} = a \times e^{bt} + c$$

$$\lambda_{i,t} = m_i \times (t-1) + n_i \quad for\, i = 1 \ldots 4$$

$$a, b, c, m, n \geq 0$$

Here, $\lambda_{i,t}$ is the rate of transition from state i to state $i+1$ at time t, and $q_{i,t}$ is the rate of transition from state i to the absorbing (death) state at time t. The time change is modeled as a Gamma process which is non-decreasing, additive, and has a closed form moment generating function. It is defined as starting at $\gamma_0 = 0$ with independent increments $(\gamma_{t+s} - \gamma_t)$, which are Gamma distributed with mean s and variance vs.

The Markov aging model is used in two ways. Its deterministic component (i.e. the underlying Markov process) is used for comparisons with other deterministic heterogeneity models. The subordinated model is used to assess the impact of systematic mortality risk.

Data

Modeling mortality heterogeneity requires a basis to divide the population into groups of individuals anticipated to experience similar rates of mortality, distinct from other groups. Calibration of these models requires information about the health status distribution and survival probability. This can be done using socioeconomic status, health conditions, or health risk factors. Socioeconomic status and income level are related to mortality, yet the correlation is not definitive and mortality is driven by more specific factors than socioeconomic status. Health risk factors based on individual panel data can be used to relate failure time to health characteristics of individuals. Characteristics include various factors such as diastolic and systolic blood pressure, body mass index, cholesterol, blood sugar, vital capacity, and cigarettes per day. This approach has significant data availability imitations at a population level.

Health risk factors such as obesity or smoking habits are less effective in capturing heterogeneity than existing health conditions such as heart disease or lung cancer. In addition, health condition data is more readily available than health risk factor information, which requires both the risk factor and its duration. The ideal form of data is that which records a cohort's experience through time. However, health data are generally only available for the population alive in a particular year, so period mortality data must be used to match period health data.

For calibration of the Markov aging model, the data used for estimating severity of the health conditions and health status distribution were derived from a variety of sources. The National Health Survey (NHS) data (ABS 2009) are used to capture prevalence of long-term conditions, at ten-year intervals from age 15 to 75, from years 2007–2008. We also use estimated average dementia prevalence by Ritchie et al. (1992) in five-year age intervals from 60 to 85. The Australian Cancer Incidence and Mortality Books (ACIMB) (AIHW 2012) are used for cancer incidence and mortality for five-year age intervals up to 85, to the year 2008. Mortality by cause data (other than cancer) was taken from the following sources: the WHO mortality database (WHO 2010) for Australia gives the number of deaths from a health condition, for five-year age intervals until 95, to the year 2006; the Australian Bureau of Statistics Causes of Death database (ABSCD) (ABS 2013) gives number of deaths from each condition, aggregate of all ages, to the year 2010. Infectious diseases or accidents were not taken into account, which means that the calibrated model assumes all individuals to have the same exposure to these baseline risks.

In order to determine population health status distributions, health conditions are ranked according to their severity and divided into five groups (or health states); the distribution of the population for these five health states was estimated from the prevalence of health conditions. Health conditions are ranked by the probability of death from cause-of-death data given the prevalence of a condition. Since deaths by cause from WHO are only available up to 2006, and prevalence is only available for 2007–2008, the 2006 WHO data are scaled by the ratio of 2008 to 2006 numbers of deaths in ABSCD.

Some assumptions are made in estimating the proportion of the population in each health state. It is assumed that the prevalence of a condition for individuals for a ten-year age range could be used to represent the expected prevalence at the midpoint age, since health data are available at ten-year intervals, but the model requires distributions across ages. It is also assumed that long-term conditions are independent and that for a person affected by more than one condition, the highest death rate among all of the conditions is assumed to be the death rate. The proportion of individuals with a specific condition as their most severe condition is assumed equal to the proportion of individuals not affected by any worse condition multiplied by the proportion of the total population affected by the specific condition.

Aggregate mortality data are taken from the Human Mortality Database (HMD 2013). The 2008 Australian period life table (male and female combined) is used for coherence with health data.

Calibration of Mortality Heterogeneity Models

Figure 7.1 shows the survival curve for the fitted Le Bras model and the Australian 2008 life table used for calibration. The model provides a better fit to the survival curve when fitted for ages above 20. The parameter values estimated for the Le Bras model 20+ are given in Table 7.1. The model is equivalent to the frailty model.

The Markov aging model is fitted using observed health and survival distributions as expected values. The sum of squared differences with the model's estimation of $E(S(t))$ is minimized. A lower limit of 0.001 is imposed for ν to prevent a near zero denominator in the numerical estimation procedure. Other parameters are assumed to have a lower limit of 0. Parameter estimates for the Markov aging model are given in Table 7.2.

Figure 7.2 shows the fitted survival curve. Figure 7.3 shows the fitted versus observed data by the health states for the model. The model provides a good fit to the survival distribution and health states data used for calibration.

Solvency and Tail Risk

In order to assess solvency and tail risk arising from heterogeneity, a portfolio of life annuities is projected using simulation. Annuity contracts are assumed to be written at age 65 under differing assumptions about the health status of the lives purchasing the annuity. The annuities pay an annual payment of $1 for as long as the individual lives. Expenses and other costs are not included. The distribution of health status is generated from each model. For comparison purposes health status ranges are aggregated into comparable groups for the purpose of calculating premiums and simulating annual balances.

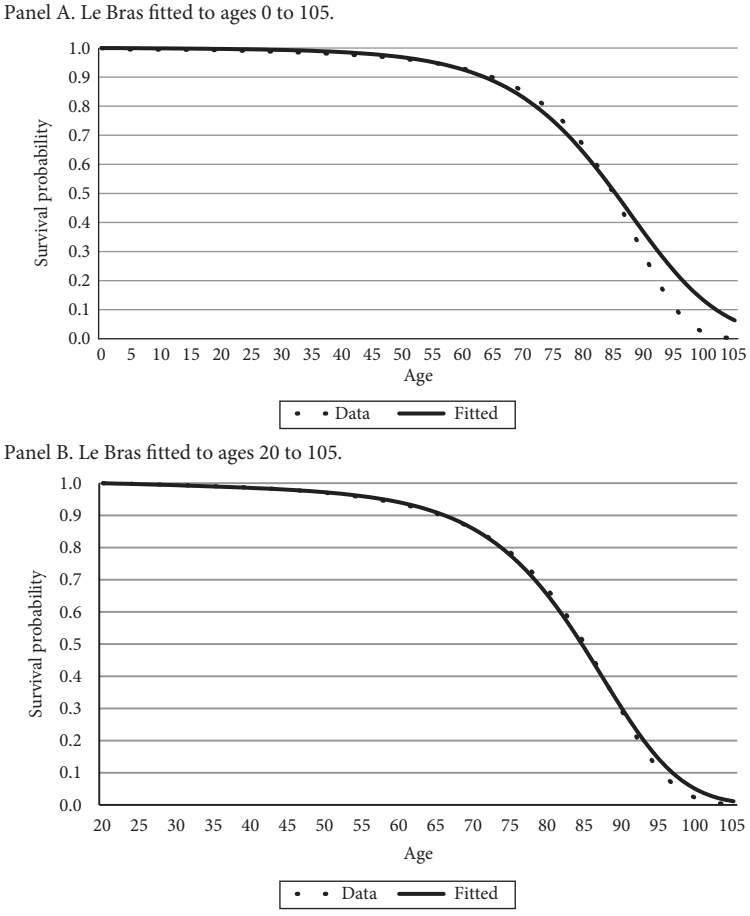


Figure 7.1. Survival curve fit of the Le Bras model.

Notes: Figures show the fit of the Le Bras model to the 2008 Australian life table survival curve (male and female combined). The model provides a better fit to survival data starting from age 20 than that starting from birth.

Panel A. Le Bras fitted to ages 0–105.
Panel B. Le Bras fitted to ages 20–105.

Source: Authors' calculations.

Premiums are calculated to be equal to the actuarial expected present value of all payments. Survival rates conditional on health states are used to allow for selection. Population average survival rates are used for the cases where no anti-selection is assumed for mixed health status groups. A fixed interest rate of 3 percent *per annum* is assumed along with an assumption of random investment returns.

TABLE 7.1 Parameter estimates for Le Bras Model fitted to ages above 20

λ_0	0.489972
μ_0	0.000608
λ	0.117869
μ	0.00001

Notes: The table shows the parameters estimates for the Le Bras model based on the Yashin et al. parameterization (1994). Parameter definitions are given in the text.

Source: Authors' calculations.

TABLE 7.2 Parameter estimates for subordinated Markov model fitted to ages 30–110.

a	0.000022
b	0.143882
c	0.907697
m_1	0.001753
n_1	0.004911
m_2	0.000919
n_2	0.020675
m_3	0.00038
n_3	0.046633
m_4	0
n_4	0.032396
v	0.146892

Notes: The table shows the parameters estimates for the subordinated Markov aging model. Parameter definitions are given in the text.

Source: Authors' calculations.

Random returns are simulated using a model (including calibration) adopted directly from Nirmalendran et al. (2012). Assets were assumed allocated according to the Australian Prudential Regulation Authority statistics (APRA 2010) of 5.5 percent in cash, 86.8 percent in bonds, and 7.7 percent in stocks (rebalanced every year). Cash rates and stock prices are modeled with geometric Brownian motion. Short rates generated by the Vasicek model are used for single-period bond returns. For the random returns case, premiums are calculated with discount factor based on bonds yields. However, unlike Nirmalendran et al. (2012), the market price of investment risk is not included.

The distributions of healthy states for the Markov aging model are given in Table 7.3. These percentages are calibrated to the health data. The table shows the shift

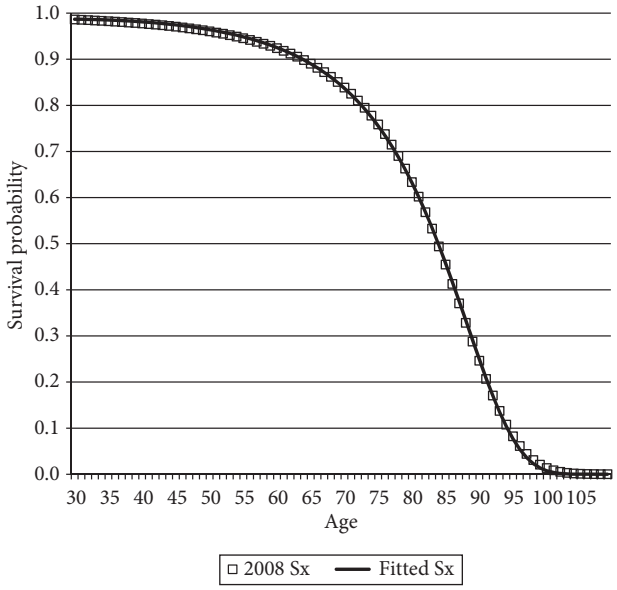


Figure 7.2. Survival curve fit of the Markov aging model of heterogeneity based on both health and survival data.

Note: Figure shows the fit of the Markov aging model used in the chapter compared to the 2008 Australian life table survival curve. The model fit is shown for ages 40 and above.

Source: Authors' calculations.

from the healthier states to the less healthy states and eventually to the death states with increase in age. The percentage in the healthiest state diminishes rapidly from age 50 to 70, with a reduction from 47.5 percent to 14.5 percent. By age 70 the distribution across health states has shifted to the less healthy states with higher mortality.

Figure 7.4 shows the distribution of heterogeneity at age 65 given by the three models by showing the distribution of expected future lifetimes for the different models. The Vaupel frailty model and the Le Bras Markov model forecast a higher proportion with higher life expectancies than the Markov aging model. The Markov aging model reflects a calibration to health status data as well as population mortality. By not reflecting health status, the expected future lifetime is overstated in the other models.

Impact of Heterogeneity and Adverse Self-selection

The impact of heterogeneity is illustrated for the three models in Figure 7.5 with a comparison of a 'best health' case and a 'mixed' case using the standard deviation of the fund values in the older ages for a pool size of 1,000 individuals. The best

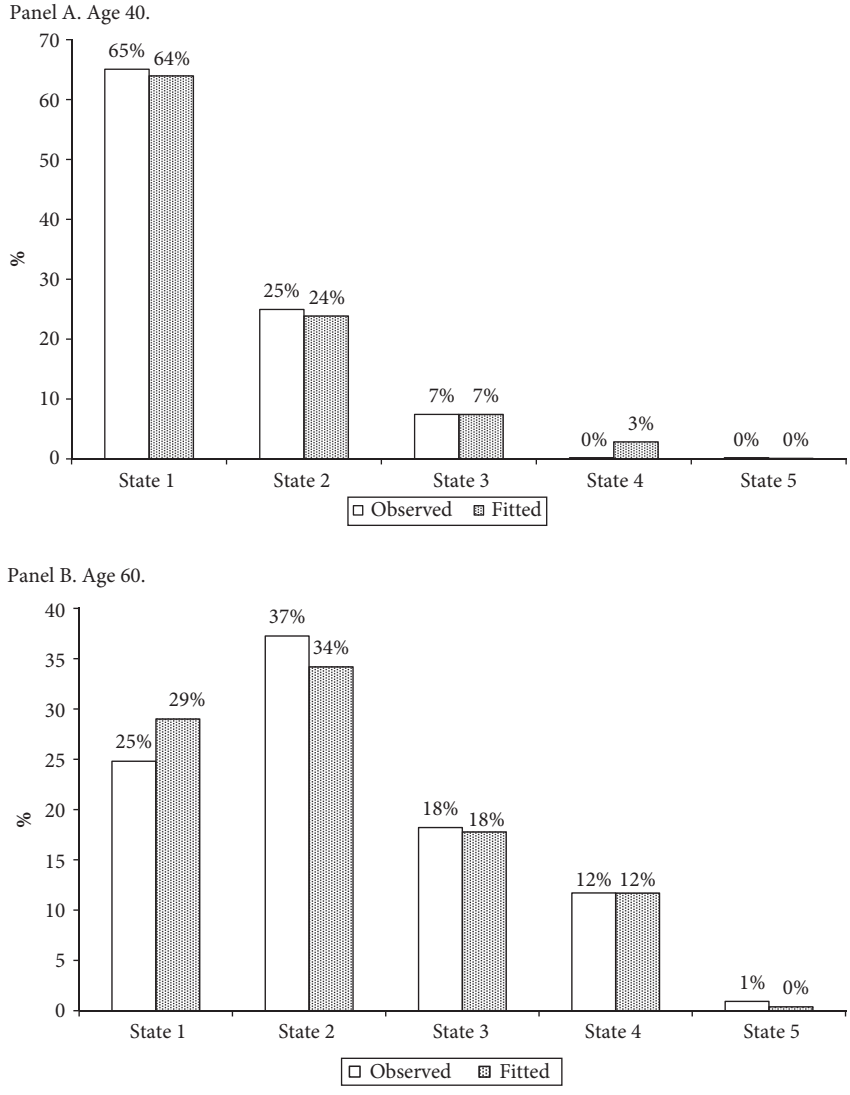


Figure 7.3. Fitted versus observed data for Markov aging model.

Note: Figure shows distribution of health states for the Markov aging model used in the paper compared to the actual data use dot fit the model. The model fit is shown for ages 40 and 60.

Panel A. Age 40.
Panel B. Age 60.

Source: Authors' calculations.

TABLE 7.3 Markov aging model: Percentage distribution of health states for ages 40–70

	State: 1(%)	2(%)	3(%)	4(%)	5(%)	Deceased (%)
Age:						
30	72.7	20.5	5.0	0.4	0.0	1.3
40	65.1	25.0	7.4	0.2	0.2	2.1
50	47.5	31.3	12.5	4.7	0.3	3.7
60	24.8	37.3	18.2	11.7	0.9	7.1
70	14.5	29.6	20.4	18.3	2.2	14.9

Notes: The table shows the distribution of health states for varying ages based on the Markov aging model. Health state 1 is the best health state with the lowest mortality rate, and 5 is the worst health state with the highest mortality rate.

Source: Authors' calculations.

Figure 7.4. Heterogeneity based on expected future lifetimes at age 65.

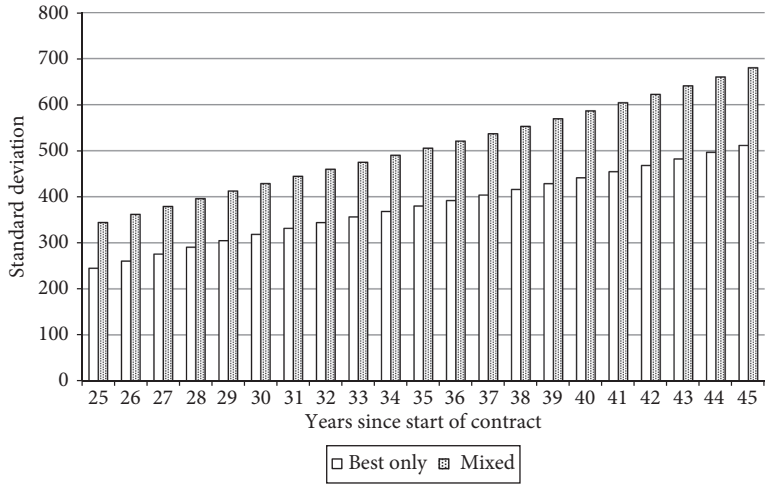
Note: The figure shows the distribution of future expected lifetime according to the three modes used in the text to quantify heterogeneity of mortality. The Markov model has a noticeably different distribution from the other models, reflecting its calibration to both health and survival data.

Source: Authors' calculations.

health case assumes that only individuals in the best health class of the Markov aging model purchase annuities. The mixed cases assume a portfolio of annuitants with similar health proportions to that of the population purchases annuities with an average premium for the group, and there is no selection based on health.

The standard deviation of the annuity pool amount increases with older ages for all models. Even though frailty models imply reduced relative heterogeneity in

Panel A. Markov model standard deviation.



Panel B. Le Bras model standard deviation.

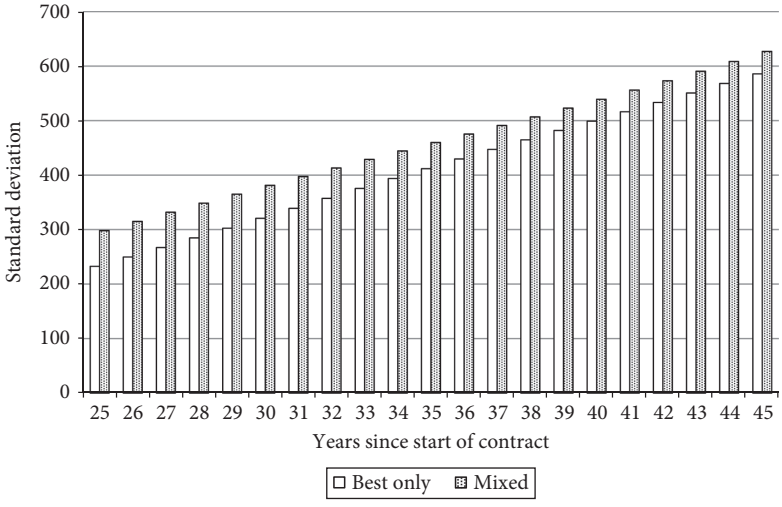


Figure 7.5. Standard deviation risk measure for the annuity pool amount at older ages for the different models of heterogeneity for a pool size of 1,000.

Note: The figures show standard deviation of the annuity fund for annuities commencing at age 65 at the older ages for a pool size of 1,000 individuals. The standard deviations are shown for the three different models and for the assumption that only the best health individuals purchase annuities (best only), and also assuming a mixture of health states representative of the population purchase annuities (mixed).

Panel A. Markov model standard deviation.
Panel B. Le Bras model standard deviation.
Panel C. Vaupel model standard deviation.

Source: Authors' calculations.

Figure 7.5. (*Continued*)

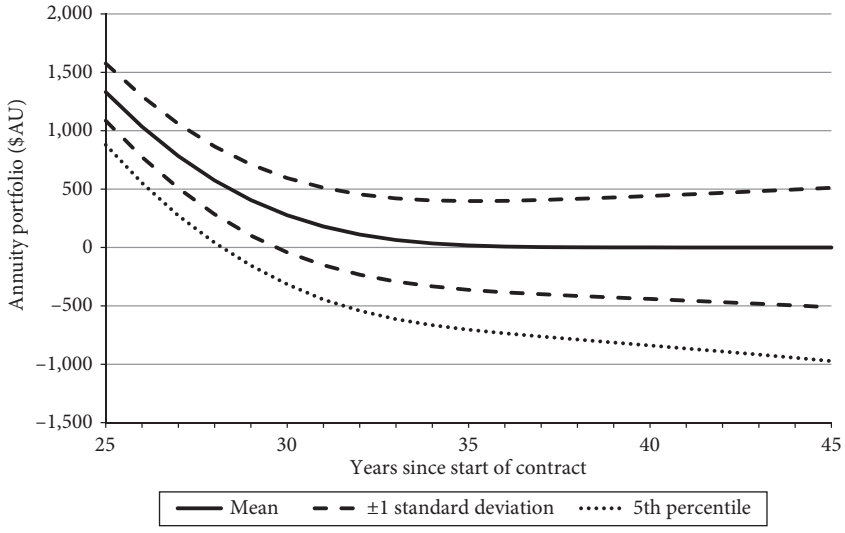
mortality at older ages, there is an increase in variability of pool fund amounts. The Le Bras and Vaupel models produce similar results, with the Vaupel model producing higher standard deviations.

The most interesting aspect shown here is the Markov aging model, whose measure of heterogeneity is specifically calibrated to population health data. The heterogeneity for cases when only people in the best health states purchase annuities is significantly lower than for the mixed-population pool. These differences do not arise in the other two models, where heterogeneity in health is derived from aggregate survival rates only.

Figure 7.6 shows the Markov aging model results for the best health state compared with the mixed health case in order to illustrate the differences in the expected value of the fund as well as the variability. The best health case expected value starts higher but both fund values converge to zero, since the premiums are fair. In the mixed population case the distribution of fund sizes is much wider, with significantly higher probabilities of adverse fund sizes.[2] This illustrates how the strategy of writing annuities for a select group of individuals reduces the volatility arising from heterogeneity and is a lower risk strategy for an annuity provider.

In practice individuals can self-select against the annuity provider. This is referred to as adverse selection. To consider this we assume that the premium charged is based on the mixed population distribution of health states but individuals purchase annuities based on their health state. Thus lives in better health than the mixed group find the annuity rate attractive and purchased annuities. As shown in Figure 7.7, the effect of this anti-selection is that the average fund size

Panel A. Best health state only annuity fund balance.



Panel B. Mixed population annuity fund balance.

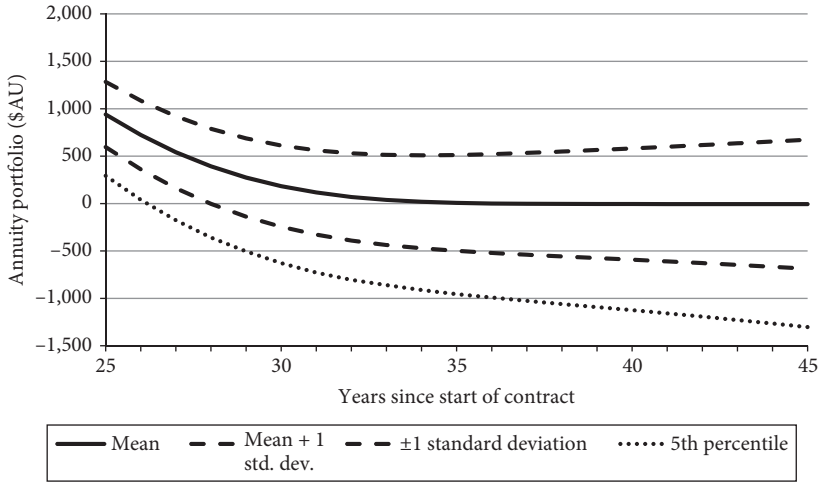


Figure 7.6. Balance of annuity fund for the best health state and the population mix showing uncertainty and downside risk.

Note: The figures show the annuity fund for annuities commencing at age 65 at the older ages for a pool size of 50 individuals. Panel A shows an annuity portfolio with only the best health state and Panel B shows annuities assuming a mixture of health states representative of the population purchase annuities (mixed).

Panel A. Best health state only annuity fund balance.
Panel B. Mixed population annuity fund balance.

Source: Authors' calculations.

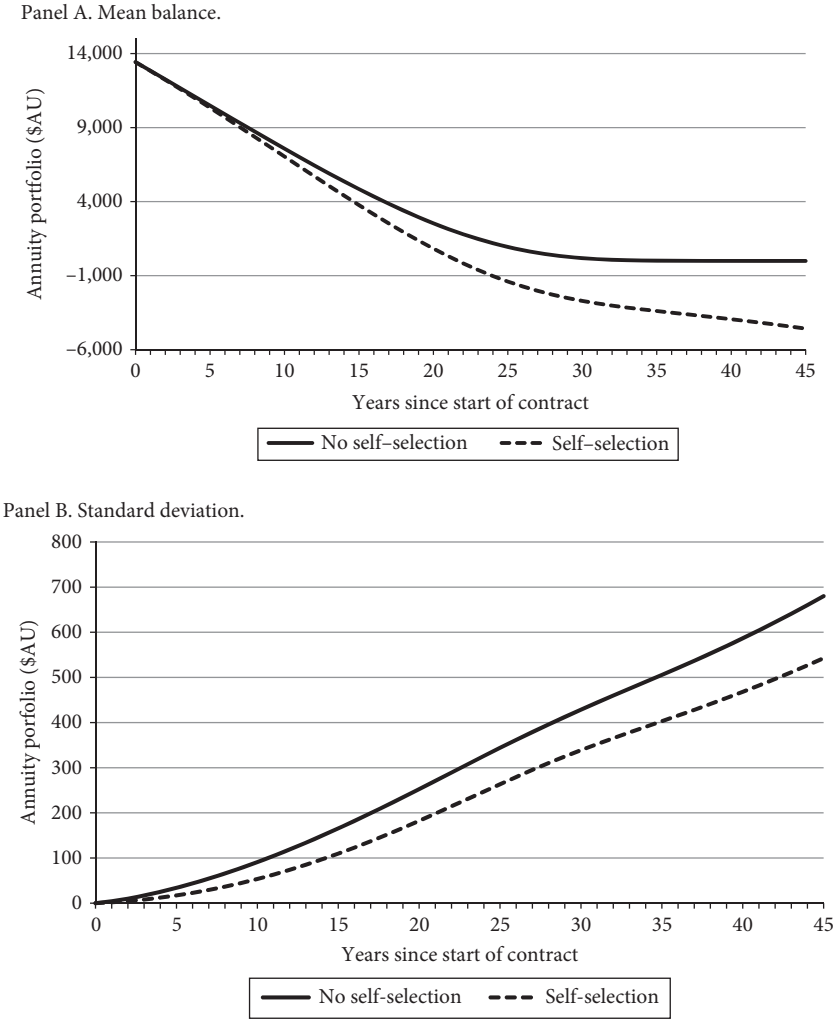


Figure 7.7. Mean and standard deviation of balance of annuity fund showing the impact of adverse selection.

Note: The figures show the annuity fund for annuities commencing at age 65 for a pool size of 50 individuals assuming that a population annuity rate is charged. The top figure shows the mean balance and the bottom figure the standard deviation. Two cases are shown: one where there is no self (adverse) selection and the other where only the healthy lives purchase annuities.

Panel A. Mean balance.
Panel B. Standard deviation.

Source: Authors' calculations.

TABLE 7.4 Annuity premiums and tail risk measures assuming a fixed investment return for different models of heterogeneity

Mortality model	Heterogeneity	Annuity premium	Risk measures at age 110 Stdev	95% VaR
Markov	best health only	15.22	511.76	821.67
	state 2	14.94	519.94	855.21
	state 3	13.97	566.82	922.96
	state 4	11.45	687.93	1112.05
	state 5	0.64	118.03	199.83
	mixed	13.42	682.91	1122.83
	mixed w/self-selection	13.42	540.05	5452.06
Le Bras	best health only	17.49	588.21	947.61
	mixed	14.15	634.72	1052.37
	mixed w/self-selection	14.15	608.96	6816.63
Vaupel	best health only	18.39	639.50	1029.39
	mixed	14.72	676.94	1130.14
	mixed w/self-selection	14.72	656.93	7369.68

Notes: The table shows the premium for a life annuity of 1 *per annum* and tail risk measures for a pool of 1,000 individuals aged 65 assuming different pool compositions for health statuses for a fixed investment return of 3% *per annum*. Results are shown for the different deterministic models of heterogeneity. See text.

Source: Authors' calculations.

drops significantly, as expected, and the chance of major losses increases. Adverse selection does produce lower standard deviations of pool fund balances, but this is primarily because the mean level of the fund falls more rapidly and the self-selected group is less heterogeneous than the mixed group.

Table 7.4 shows the premiums and risk measures at the ultimate age of 110 for the cases of best health, mixed health, and adverse selection for a pool size of 1,000. Annuity premiums vary significantly according to health state in the Markov aging model. They vary from 15.22 for the best health state to 0.64 in the worst health state. The Le Bras and Vaupel models produce higher premiums, reflecting the higher life expectancy in these models. The three models agree on the impact of self-selection, although they differ on the reduction in volatility when the best health group is priced separately. These results illustrate the extent of model risk in allowing for heterogeneity when assessing a pricing strategy and solvency of an annuity pool. Large variations in premiums occur as well as in tail measures of risk.

Impact of Random Investment Returns

Table 7.5 shows the annuity premiums and risk measures for pool sizes of 1,000 assuming random investment returns. Premiums are lower since the average interest rate in the stochastic model is higher than the deterministic 3 percent used in Table 7.4. The best health annuity premium for the Markov aging model is now 12.68,

TABLE 7.5 Annuity premiums and tail risk measures assuming random investment returns for different models of heterogeneity

Mortality model	Heterogeneity	Annuity premium	Risk measures at age 110 Stdev	95% VaR
Markov	best health only	12.68	4,570.53	7,372.69
	state 2	12.49	4,454.89	7,150.40
	state 3	11.80	4,250.96	6,755.93
	state 4	9.85	3,494.09	5,688.37
	state 5	0.63	386.70	638.02
	mixed	11.34	4,096.90	6,528.22
	mixed w/self-selection	11.34	3,912.58	17,878.91
Le Bras	best health only	13.93	5,480.72	9,047.16
	mixed	11.84	4,328.60	6,910.74
	mixed w/self-selection	11.84	4,218.89	19,635.98
Vaupel	best health only	14.35	5,725.39	9,188.87
	mixed	12.14	4,500.81	7,286.31
	mixed w/self-selection	12.14	4,428.48	20,553.27

Notes: The table shows the premium for a life annuity of 1 *per annum* and tail risk measures for a pool of 1,000 individuals aged 65 assuming different pool compositions of health statuses for a random investment return. Results are shown for the different deterministic models of heterogeneity. See text.
Source: Authors' calculations.

compared to 13.93 for the Le Bras model and 14.35 for the Vaupel model. Risk is substantially increased with the addition of investment return risk. The Le Bras and Vaupel models show similar risk measures for the different cases of selection and these are higher than the Markov aging model. For the Markov aging model the better health states contribute significantly to the overall fund risk measures.

Impact of Stochastic (Systematic) Mortality

The subordinated Markov aging model incorporates stochastic mortality through a Gamma time change. This changes the survival probabilities for all individuals in a random manner. The result is a distribution of survival probabilities for each health state. The degree of uncertainty in future survival probabilities is determined by the variance ν of the Gamma time change.

Table 7.6 shows the impact of the Gamma time change parameter on the fund standard deviation at age 110. The annuity fund tail risk is not very sensitive to this assumption. Higher values of the parameter result in reduced standard deviations for a mixed health state fund as compared with the best health only case.

Impact of Pool Size

Table 7.7 compares the standard deviation at age 110 for pool sizes 100 to 100,000 given by the Markov aging model with and without the stochastic time change.

Table 7.6 Standard deviation of annuity fund for different assumptions of stochastic mortality risk

	Stdev at age 110	
ν	Best health	Mixed
0.01	956.16	1,027.44
0.05	1,859.89	1,817.99
0.1	2,603.98	2,509.03
0.5	5,660.73	5,474.22

Notes: The table shows the standard deviation for a life annuity fund of a pool of 1,000 individuals age 65 assuming different pool compositions for health statuses for annual payments of $1 and a fixed investment return of 3% *per annum*. See text.

Source: Authors' calculations.

Table 7.7 Standard deviation at age 110 for different pool sizes using Markov model without and with stochastic mortality risk

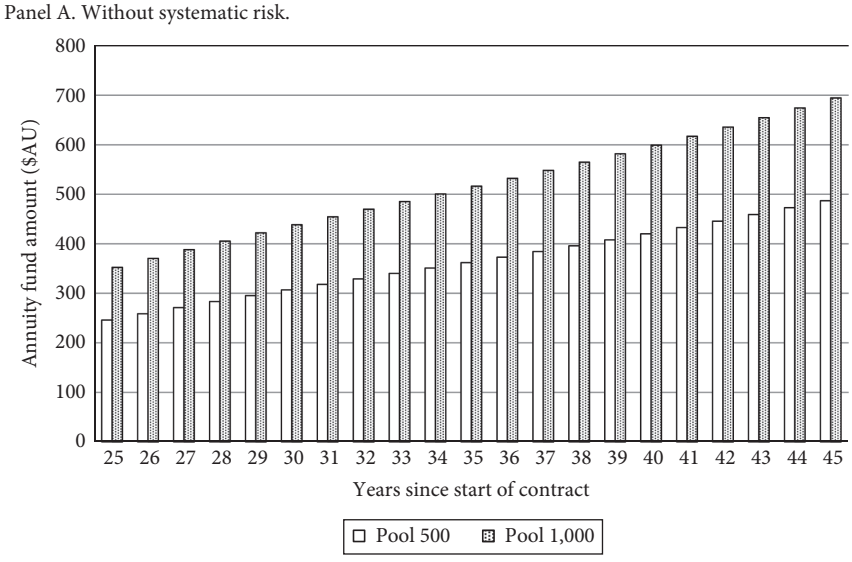
Pool size	Deterministic Markov	Subordinated Markov
100	215.54	354.96
1,000	684.12	2,954.66
10,000	2,147.19	28,913.66
100,000	6,832.39	287,749.89

Notes: The table shows the standard deviation of the fund at age 110 for life annuity of 1 *per annum* for best health individuals age 65, assuming a fixed investment return of 3% *per annum*. The stochastic model assumes variance of Gamma time change $\nu = 0.095$. See text.

Source: Authors' calculations.

With deterministic mortality rates, standard deviation increases approximately in proportion to the square root of pool size, showing a diversification of idiosyncratic mortality risk. Thus as the pool size grows by 10 times from 1,000 to 10,000, the tail risk, as given by the fund standard deviation at age 110, increases by approximately 3.16 times (square root of 10). In contrast, with the inclusion of systematic risk, the effect of diversification of mortality risk increases by 9.8 times (almost 10). Systematic mortality risk dominates as the pool size increases.

Figure 7.8 shows how the impact of systematic mortality risk increases through the older ages. The standard deviation of the pool amount for ages above 90 for the deterministic and subordinated Markov aging models, for pool sizes 500 and 1,000, increases significantly. The effect of larger pool sizes at the older ages is clearly seen.



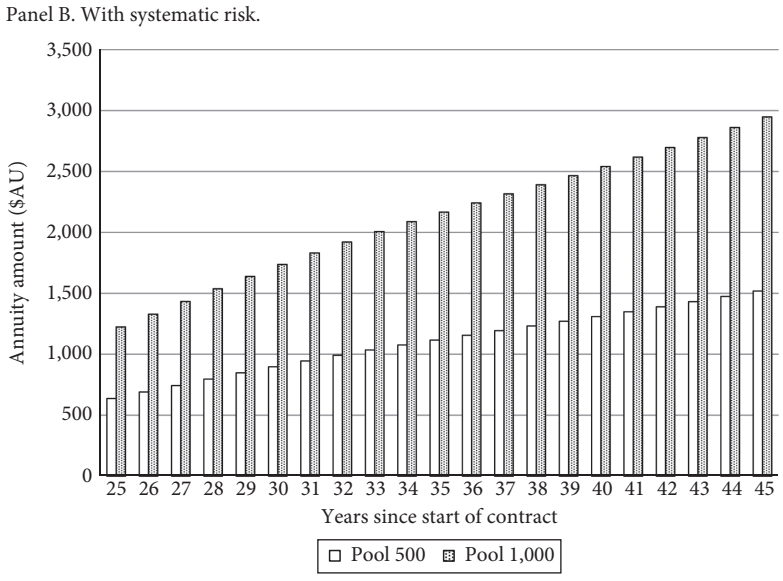


Figure 7.8. Standard deviation of annuity pool amount at older ages for the Markov aging model.

Note: The figures show the standard deviation of the annuity fund for annuities of $1 *per annum* for best health individuals aged 65, assuming a fixed investment return of 3 percent *per annum*.

Panel A. Without systematic risk.
Panel B. With systematic risk.

Source: Authors' calculations.

Conclusion

This chapter has deployed a recently developed Markov aging model for mortality heterogeneity, along with more commonly used frailty models, to show the impact of heterogeneity and systematic mortality risk on annuity fund values at the older ages, the tail of the mortality distribution. Model risk for longevity arises from a misspecification of the underlying process being modeled. Systematic mortality risk models have been developed and applied. Markov aging models for heterogeneity have also been developed. Standard models of heterogeneity do not capture observed health differentials or the effect of systematic mortality risk. Using a model that captures only one of these aspects of mortality risk has limitations because of model risk.

We illustrate the impact of this model risk in the determination of annuity premiums and fund risk measures. Heterogeneity results in a wide variation in annuity premiums depending on health status. Selection of lives in better health states by insurers when writing life annuities is a less risky strategy than writing annuities on all health states in the population, even if premium rates vary by health state and there is no adverse selection. Adverse selection negatively impacts both profitability and fund risk.

Increasing pool sizes increase tail risk almost linearly with the size of the pool for the cases where the Markov aging mortality model includes systematic risk. This effect is not captured by standard models of heterogeneity where mortality pooling results in only a square root of pool size increase in fund risk.

Notes

1. The two are equivalent when: $a = [(\lambda_0)/(\lambda)] \times \mu$, $b = \lambda + \mu$, $c = \mu_0 - [(\lambda_0)/(\lambda)] \times \mu$, $\sigma^2 = [(\lambda)/(\lambda_0)]$.
2. The other two models show a smaller magnitude; see Table 7.3.

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