ARC CENTRE OF EXCELLENCE IN POPULATION AGEING RESEARCH

Model Risk, Mortality Heterogeneity and Implications for Solvency and Tail Risk

2013 Pension Research Council Symposium
Recreating Sustainable Retirement: Resilience, Solvency, and Tail Risk
April 25-26
Wharton School of the University of Pennsylvania

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Overview

- Annuity portfolios - mortality/longevity; volatility of financial results arises from
  – systematic mortality changes, with higher volatility experienced at older ages, and
  – heterogeneity also producing higher volatility at older ages (Su and Sherris 2012; Meyricke and Sherris 2013).
- Tail risk and solvency in annuity/pension funds impacted by adverse selection and uncertainty (expected profitability and volatility)

What we do

- Develop and apply a stochastic Markov ageing model of heterogeneity calibrated to population aggregate mortality and health data that also includes systematic mortality risk.
- Compare results with a well-known frailty model and the Le Bras Markov multiple state model to assess model risk, neither of which includes systematic mortality risk.
- Quantify solvency and tail risk for a portfolio of life annuities using risk measures - standard deviation and value-at-risk for fund values at the older ages.
- Demonstrate the impact of heterogeneity and model risk on the assessment of longevity risk for these portfolios, as well as the impact of selection and pool size.

Models Used

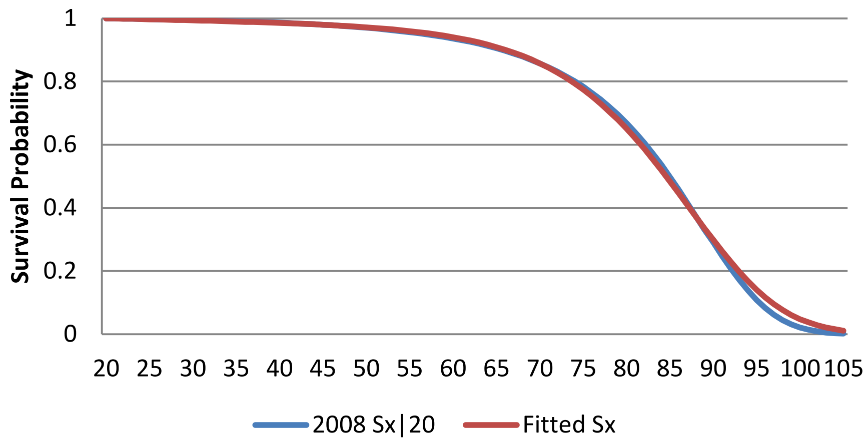
- Frailty (Vaupel model) – Gamma frailty factor
- Multiple states (Le Bras) – equivalent parameterization to frailty model (Yashin, Vaupel and Iachine, 1994)
- Non-homogenous Markov ageing model calibrated to health data with systematic risk – an extension of Liu and Lin (2012) and Su and Sherris (2012)

Data

- National Health Survey (NHS) data for prevalence of long term conditions, Self-assessed health and estimated average dementia prevalence.
- Australian Cancer Incidence and Mortality Books (ACIMB) and WHO mortality database for Australia for number of deaths from a health condition.
- Australian Bureau of Statistics Causes of Death (ABSCD) for number of deaths from each condition, the aggregate of all ages.
- Australian life tables (from Human Mortality Database) up to age 110, and Australian cohort mortality rates for mortality rate data.

Model Fits - Survival Curves

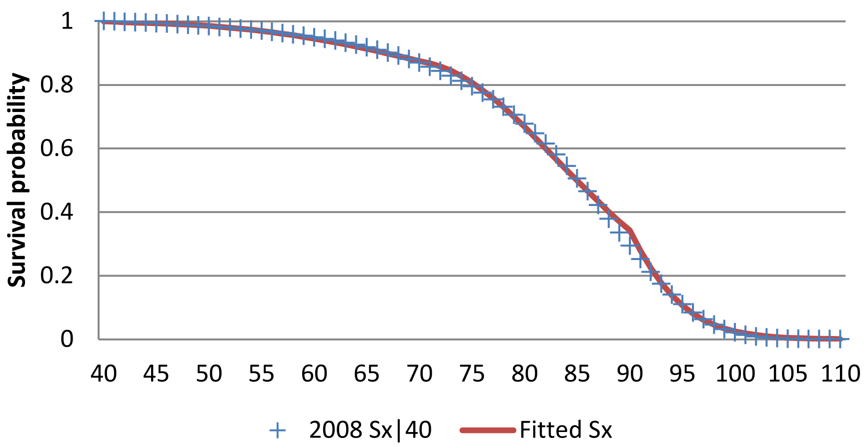
Le Bras fitted to ages 20 to 105



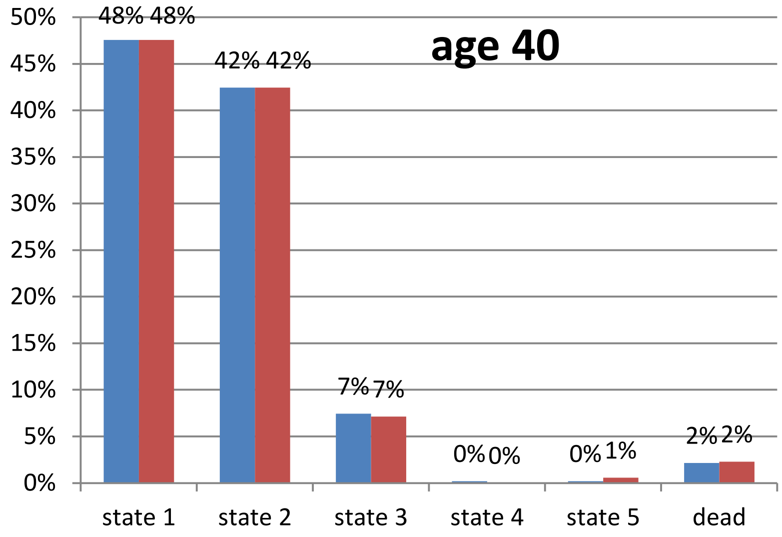
Frailty and Le Bras model have similar calibration

Markov ageing model with 3 transition matrices

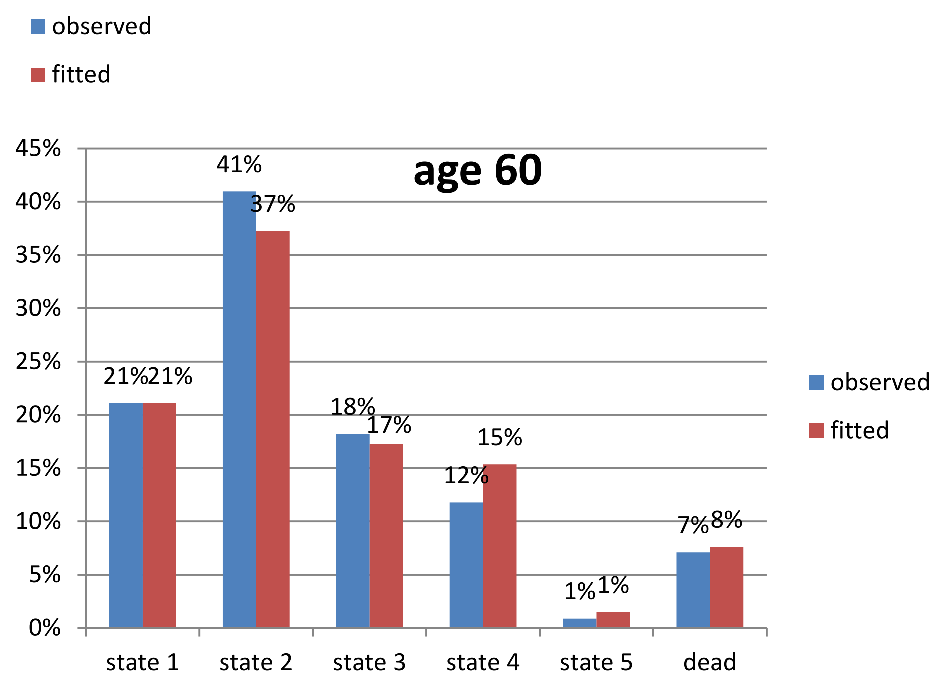
Markov ageing model based on both health and survival data



Markov ageing model – Health States

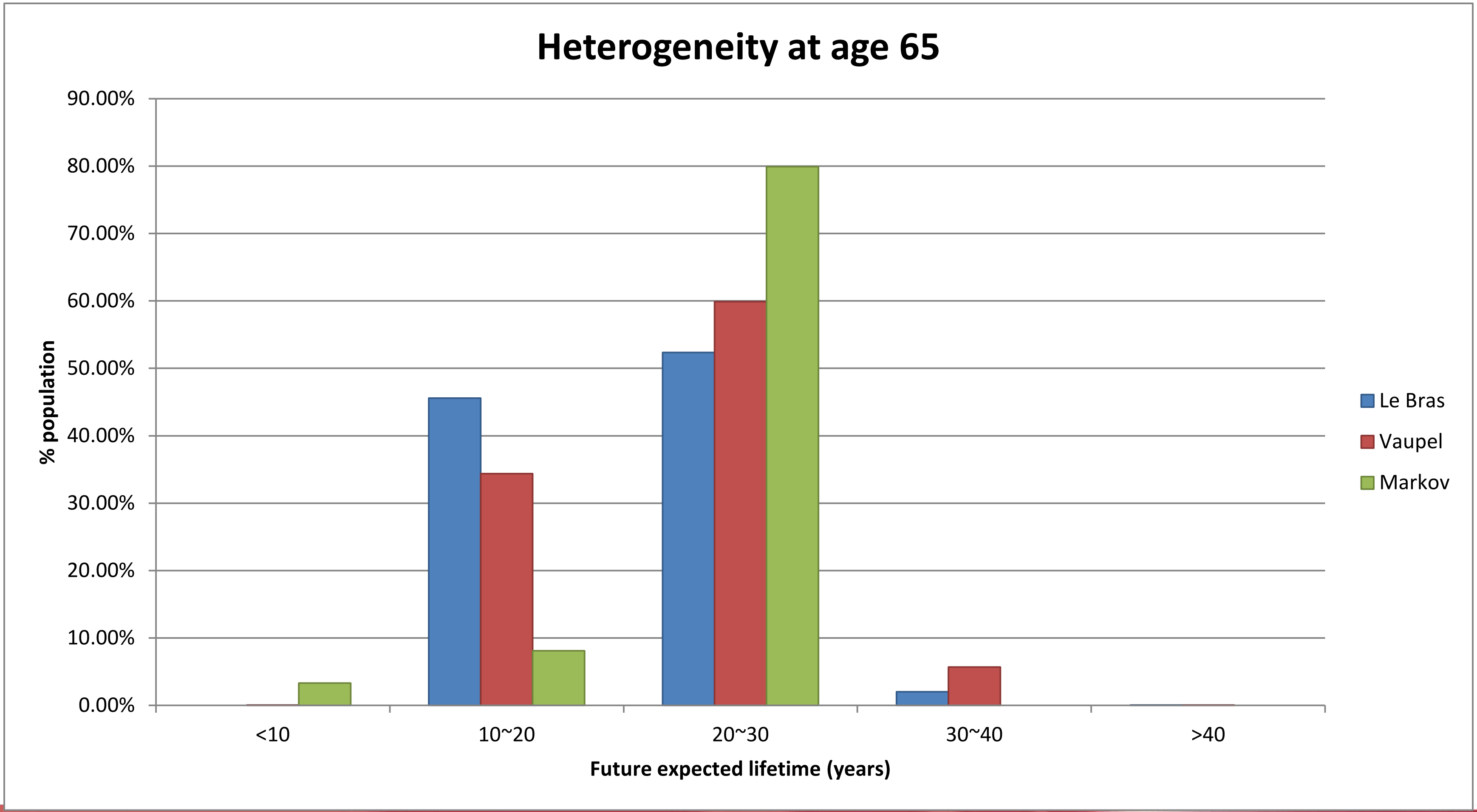


Fitted to ages above 40



Fitted to survival curve at older ages

Heterogeneity - Model comparison



Simulation of Annuity Fund

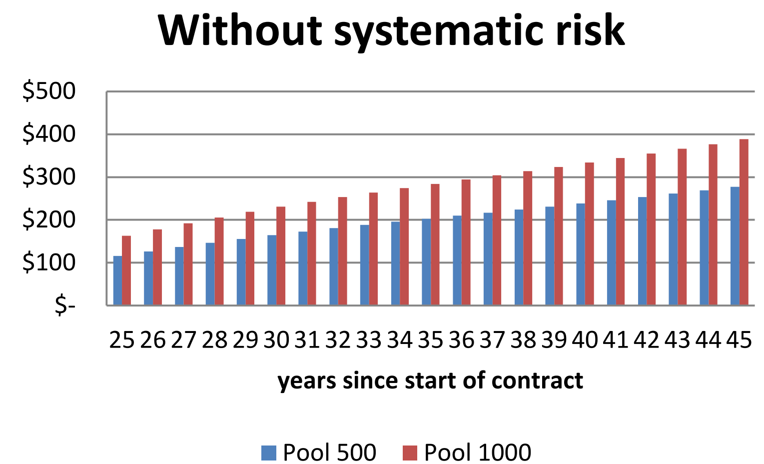
- Annuity contracts assumed written at age 65 under differing assumptions about the health status of the lives purchasing the annuity. Ranges of health status were aggregated into groups for the purpose of calculating premiums and simulating annual balances
- All annuities are for $1 p.a. No expenses or other costs assumed.
- Premiums are equal to the actuarial expected present value of all payments.
- Survival rates conditional on health states are used to allow for selection and population average survival rates are used for the case of no anti-selection.
- A fixed interest rate of 3% p.a. was assumed as well as an assumption of random investment returns.
- Random returns were simulated using a model (including calibration) from Nirmalendran et al (2012). Assets allocated according to APRA's 2010 statistics of 5.5% in cash, 86.8% in bonds, and 7.7% in stocks (rebalanced every year).
- Cash rates and stock prices modeled with geometric Brownian motion.
- For the random returns case, premiums were calculated with discount factor based on bonds yields.

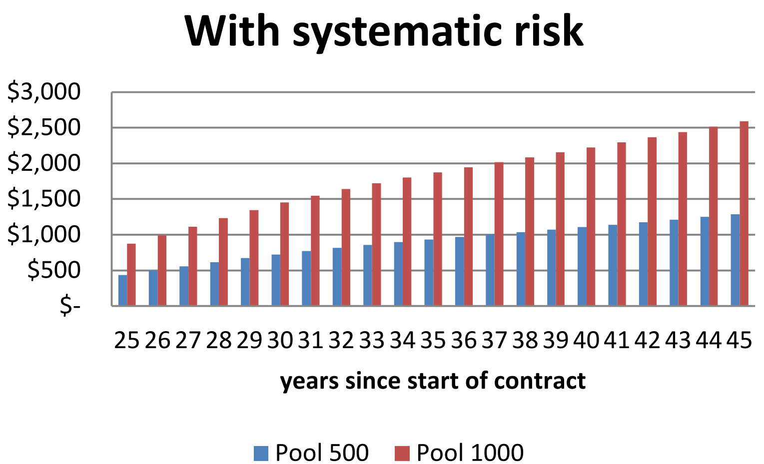
Effect of Pool Size – Systematic Risk

Pool size	Deterministic Markov	Subordinated Markov
100	122.66	286.21
1000	388.23	2588.74
10000	1216.31	25649.07
100000	3914.59	254307.38

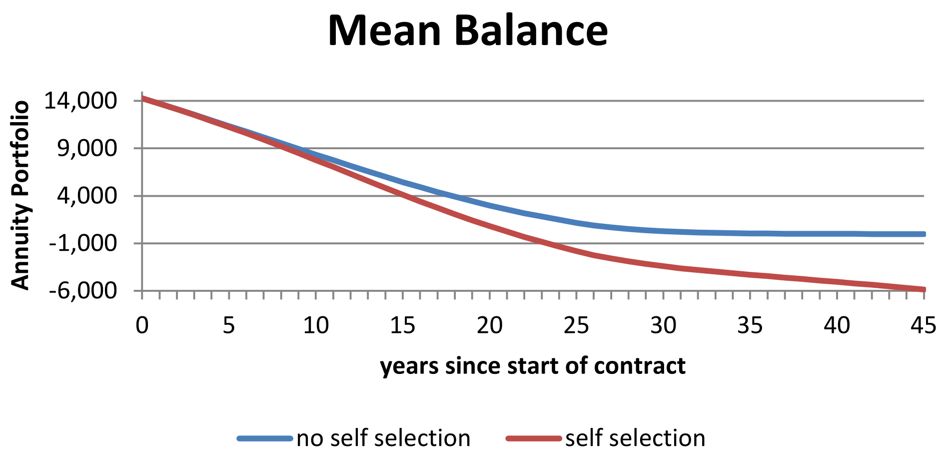
Standard deviation of the fund at age 110 for life annuity of 1 p.a. for best health individuals aged 65

Fixed investment return of 3% p.a. Stochastic model has variance of Gamma time change *v=0.095*.





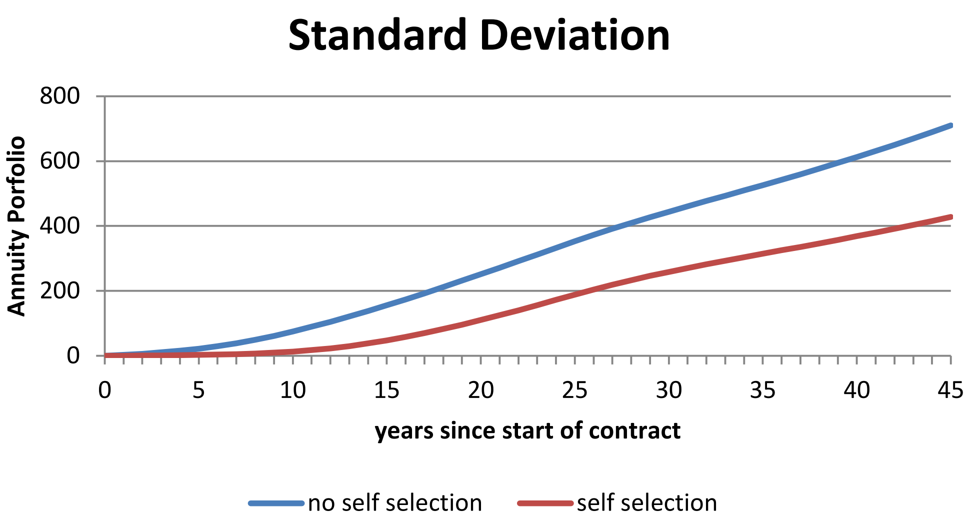
Effect of Adverse Selection



Pool size 1000
Markov ageing model

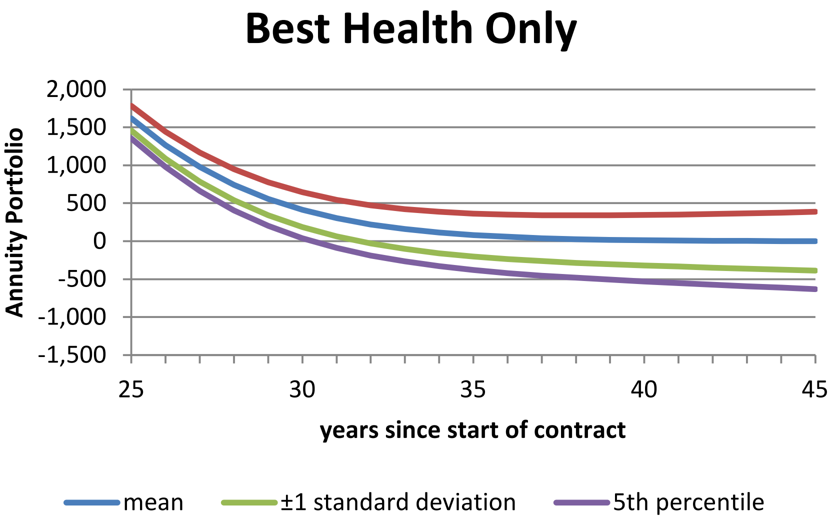
Reduced profitability (selection is adverse)

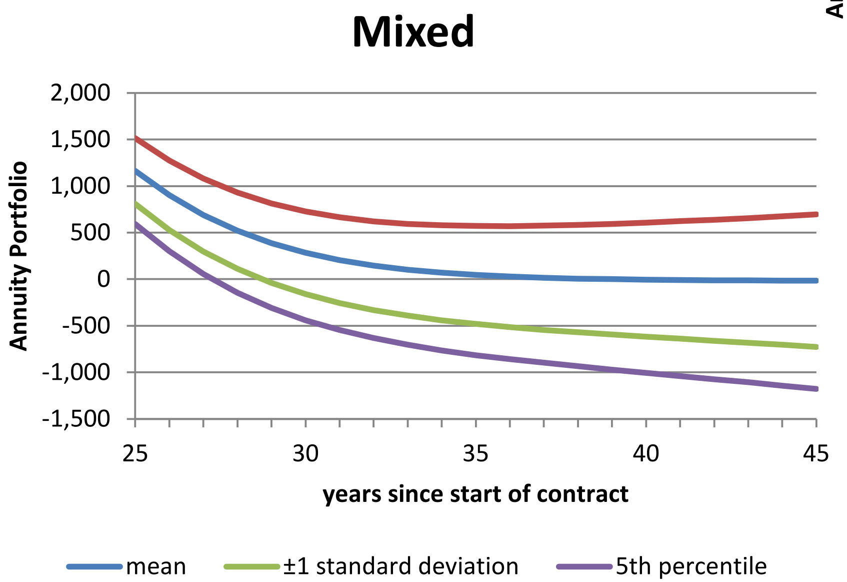
Reduced volatility



Effect of Portfolio Mix

Writing a portfolio representative of the population – less profitable and more risky

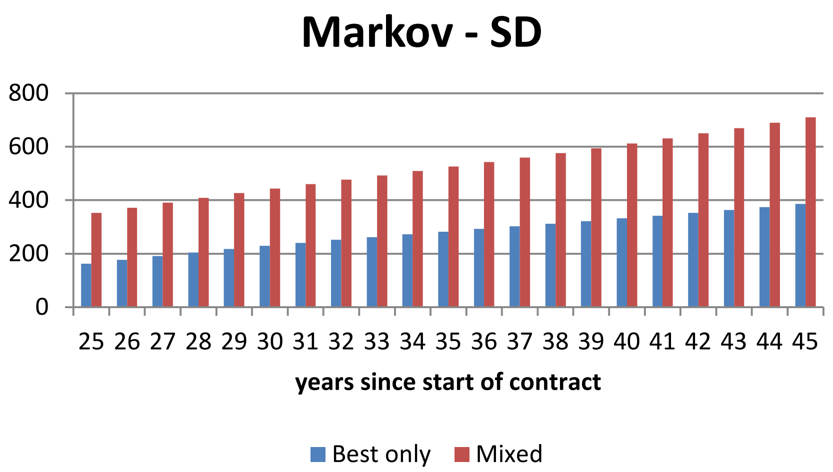


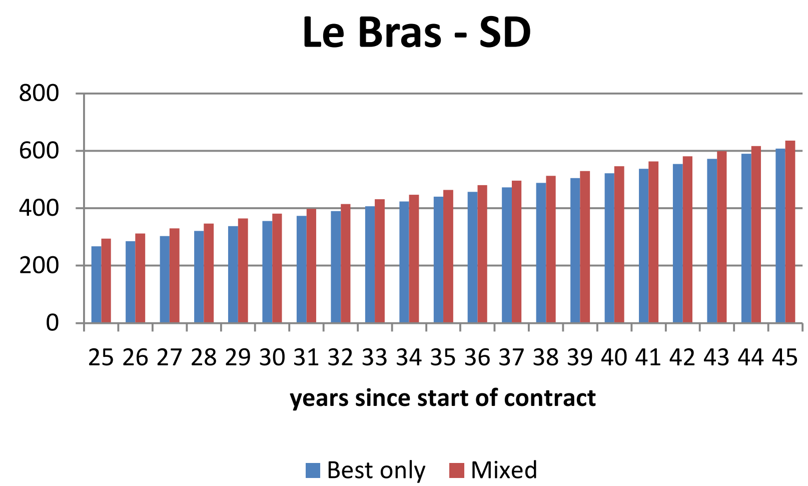


Markov ageing model

Le Bras model – lower mean and less volatility

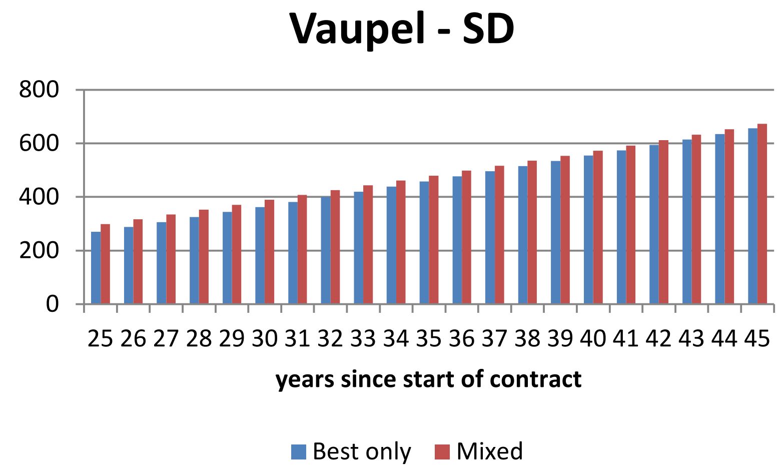
Model Risk





Frailty and Le Bras model similar volatility for mixed and select lives

Systematic risk higher for mixed portfolio



Conclusions

- Tail and solvency risk for longevity for annuity portfolio.
- Introduce a new model with both systematic risk and heterogeneity (calibrated to population health status data).
- Systematic risk is not the full story.
- Heterogeneity is important – adverse selection, impact of population mix in pool.
- Longevity risk magnified by investment risk in the tail.

Additional Slides

Systematic Longevity Risk

- Stochastic age-period life table and variations (e.g. Lee and Carter 1992)
- Random changes in a parametric survival curve (e.g. Cairns, Blake, and Dowd 2006)
- Dynamics of mortality rates in a financial framework as for interest rate models (e.g. Biffis 2005).

Heterogeneity

- Commonly used approach - frailty models to capture unobserved heterogeneity (Vaupel, Manton, and Stallard 1979, Su and Sherris 2012).
- Extensions of the Le Bras model (Le Bras 1976)
- Markov ageing models (Su and Sherris 2012; Lin and Liu 2007; Liu and Lin 2012)

Heterogeneity

- Markov ageing model of physiological age calibrated to aggregate population mortality data (Lin and Liu 2007)
- Markov ageing model of physiological age and comparison with frailty models (Su and Sherris 2012)
- Subordinated Markov ageing model, small number of health states, calibrated to aggregate population mortality data (Liu and Lin 2012)

Method of Calibration

- Population health status distributions estimated from prevalence of health conditions.
- Health conditions ranked according to their severity and divided into 5 groups (or health states).
- All individuals assumed to have same exposure to baseline risks infectious diseases or accidents.
- Health conditions ranked by the probability of death given the existence of a condition (calculated from number of deaths by cause and condition prevalence).
- Expected prevalence of a condition for individuals at the midpoint age for a 10 year age range.
- Long term conditions are independent and for a person affected by more than one condition, the highest death rate among all of the conditions was assumed to be the death rate.
- Proportion of individuals with condition X as their most severe condition was assumed equal to the proportion of individuals not affected by conditions worse than X multiplied by the percentage prevalence of X in the population. The proportion of individuals not allocated to any condition was assumed to have the best health status.

Markov ageing model

Survival probability (non-homogenous)

$$S_0(x) = \begin{cases} \pi \exp(T_1 x)\, e & for\ 0 \le x < s_1 \\ \pi \exp(T_1 s_1) \exp(T_2(x - s_1))\, e & for\ s_1 \le x < s_2 \\ \pi \exp(T_1 s_1) \exp(T_2(s_2 - s_1)) \exp(T_3(x - s_2))\, e & for\ s_2 \le x < s_3 \\ etc & \end{cases}$$

Subordinated process

$$S(t) = S_0(\gamma_t)$$

$\gamma_0 = 0$, independent increments $(\gamma_{t+s} - \gamma_t)$, Gamma distributed with mean s and variance νs

Three transition matrices fitted to age intervals 40-70, 70-90 and 90-110.

Markov ageing model

Matrix 1:

$$\begin{pmatrix} -0.040674 & 0.040674 & 0 & 0 & 0 \\ 0 & -0.038392 & 0.038390 & 0 & 0 \\ 0 & 0 & -0.077902 & 0.077895 & 0 \\ 0 & 0 & 0 & -0.041452 & 0.036872 \\ 0 & 0 & 0 & 0 & -0.324648 \end{pmatrix}$$

Matrix 2:

$$\begin{pmatrix} -0.538303 & 0.538173 & 0 & 0 & 0 \\ 0 & -0.286794 & 0.286664 & 0 & 0 \\ 0 & 0 & -0.197219 & 0.197089 & 0 \\ 0 & 0 & 0 & -0.142874 & 0.142744 \\ 0 & 0 & 0 & 0 & -0.163605 \end{pmatrix}$$

Matrix 3:

$$\begin{pmatrix} -0.942212 & 0.942212 & 0 & 0 & 0 \\ 0 & -0.922036 & 0.922036 & 0 & 0 \\ 0 & 0 & -0.594132 & 0.594132 & 0 \\ 0 & 0 & 0 & -0.383907 & 0.383907 \\ 0 & 0 & 0 & 0 & -0.386949 \end{pmatrix}$$

Markov ageing model - Distribution of health states

Age\State	1	2	3	4	5	deceased
40	47.60%	42.50%	7.50%	0.20%	0.20%	2.20%
50	4.09%	37.90%	12.60%	4.70%	0.30%	3.70%
60	21.10%	41.00%	18.20%	11.80%	0.90%	7.10%
70	13.00%	31.10%	16.90%	21.80%	2.20%	14.90%

Distribution of health states for varying ages - Markov ageing model.
Health state 1 is best health state with lowest mortality rate and 5 is the worst health state with highest mortality rate.

Annuity premiums and tail risk measures

Fixed investment return of 3% p.a.

Mortality model	Heterogeneity	Annuity premium	Risk measures at age 110		
			Mean	Stdev	95% VaR
Markov	best health only	16.32	-0.07	386.09	631.73
	mixed	14.29	-15.86	710.31	1176.89
	mixed w self selection	14.29	-5872.49	428.07	6566.69
Le Bras	best health only	15.84	4.24	607.33	986.31
	mixed	14.16	11.56	635.70	1022.46
	mixed w self selection	14.16	-3105.13	613.12	4109.81
Vaupel	best health only	16.29	-0.88	658.73	1072.07
	mixed	14.72	-1.61	673.32	1109.78
	mixed w self selection	14.72	-2610.51	666.36	3694.48

Effect of adverse selection

Premium for a life annuity of 1 p.a. and tail risk measures for a pool of 1000 individuals aged 65.

Annuity premiums and tail risk measures

Random investment return

Mortality model	Heterogeneity	Annuity premium	Risk measures at age 110		
			Mean	Stdev	95% VaR
Markov	best health only	13.48	-199.80	4912.11	7843.93
	state 2	12.54	-198.90	4387.30	7117.17
	state 3	10.04	-111.25	3192.87	5144.76
	state 4	6.74	-54.63	1917.96	3131.44
	state 5	5.00	-35.88	1478.46	2441.54
	mixed	11.99	-132.34	4420.42	7051.55
	mixed w self selection	11.99	-14675.61	4112.85	21204.18
Le Bras	best health only	12.95	-109.05	4901.30	7811.46
	mixed	11.84	-59.61	4283.44	6883.19
	mixed w self selection	11.84	-7006.90	4244.59	13922.83
Vaupel	best health only	13.14	-141.61	5040.23	8067.82
	mixed	12.13	-112.90	4476.47	7234.56
	mixed w self selection	12.13	-5777.86	4397.70	12874.70

Investment risk magnifies longevity risk

Premium for a life annuity of 1 p.a. and tail risk measures for a pool of 1000 individuals aged 65
Results are shown for the different deterministic models of heterogeneity.