

# **Time is Money: Life Cycle Rational Inertia and Delegation of Investment Management**

Hugh H. Kim, Raimond Maurer, and Olivia S. Mitchell

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Wharton School, University of Pennsylvania  
3620 Locust Walk, 3000 SH-DH  
Philadelphia, PA 19104-6302  
Tel: 215.898.7620 Fax: 215.573.3418  
Email: [prc@wharton.upenn.edu](mailto:prc@wharton.upenn.edu)  
<http://www.pensionresearchcouncil.org>

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## Abstract

We investigate the theoretical impact of including two empirically-grounded insights in a dynamic life cycle portfolio choice model. The first is to recognize that, when managing their own financial wealth, investors incur opportunity costs in terms of current and future human capital accumulation, particularly if human capital is acquired via learning by doing. The second is that we incorporate age-varying efficiency patterns in financial decisionmaking. Both enhancements produce inactivity in portfolio adjustment patterns consistent with empirical evidence. We also analyze individuals' optimal choice between self-managing their wealth versus delegating the task to a financial advisor. Delegation proves most valuable to the young and the old. Our calibrated model quantifies welfare gains from including investment time and money costs, as well as delegation, in a life cycle setting.

### **Hugh Hoikwang Kim**

SKK Graduate School of Business, Finance Area  
Sungkyunkwan (SKK) University  
25-2 Sungkyunkwan-ro, Seoul, 110-745, Korea  
h.kim@skku.edu

### **Raimond Maurer**

Finance Department  
Goethe University of Frankfurt  
Grueneburgplatz 1 (Uni-PF. H 23)  
60323 Frankfurt am Main, Germany  
maurer@finance.uni-frankfurt.de

### **Olivia S. Mitchell**

The Wharton School, University of Pennsylvania and NBER  
3620 Locust Walk, 3000 SH-DH  
Philadelphia, PA 19104  
mitchelo@wharton.upenn.edu

# Time is Money: Life Cycle Rational Inertia and Delegation of Investment Management

## 1. Introduction

Most people devote sparse attention to their financial portfolios and do not actively manage their own finances.<sup>1</sup> Individuals' tendency to maintain their portfolio allocations for long periods of time, which we term *investor inertia*, has been interpreted by some as evidence of irrationality or financial illiteracy.<sup>2</sup> Here, by contrast, we incorporate time costs associated with investment management and show that such inertia can be consistent with optimal behavior. Additionally, we explain why some investors rationally delegate the responsibility to make their investment decisions to a financial advisor.

To this end, we develop a life cycle model with rational agents that can replicate empirically-observed household portfolio inertia patterns.<sup>3</sup> In a dynamic consumption and portfolio framework with endogenous labor supply, we account for time costs devoted to portfolio management; this time becomes important when the investor must accumulate job-specific human capital via learning by doing. Our structure for financial decisionmaking costs posits an age-related time efficiency pattern for financial decisionmaking, in keeping with observed empirical evidence. We evaluate the role of financial advisors who, for a fee, help investors manage their financial portfolios. This possibility enables individuals to continue to invest in their job-related human capital.

A long literature on household finance has focused on optimal dynamic portfolio allocation patterns by a rational forward-looking consumer who decides, on his own, how to

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<sup>1</sup> See for instance Abel, Eberly and Panageas (2013); Biliias, Georgarakos, and Haliasos (2009); Dellavigna and Pollet (2008); the *Economist* (2011); and Tang, Mitchell, Mottola and Utkus (2010).

<sup>2</sup> C.f., Biliias, Georgarakos, and Haliasos (2009); Lusardi and Mitchell (2014); and Mitchell, Mottola, Utkus, and Yamaguchi (2006).

<sup>3</sup> See Brunnermeier and Nagel (2008).

allocate his wealth between stocks and bonds.<sup>4</sup> More recently, this approach has been extended to include flexible labor supply.<sup>5</sup> The key contribution of the present paper, which builds on these prior studies, is that we develop and solve a life cycle model of consumption, labor supply, and portfolio management strategy, integrating the impact of time cost on investors' portfolio choices in the context of endogenous human capital accumulation in a life cycle setting. This allows us to diagnose reasons for portfolio inertia and predict the age-related demand for delegation to financial advisors.

We find that when investors cannot delegate, young investors exhibit inertia while middle-aged investors are more active. Since the young have little human capital but face the longest time horizon, their opportunity costs from financial investment are higher than those of middle-aged investors having more job-specific human capital. Retirees who must forgo leisure and are less efficient in decisionmaking are less likely to engage in self-management, particularly at older ages. In other words, different portfolio management approaches are selected optimally over the life cycle. We also find that the average portfolio allocation to equities is rather stable by age, at around 40-60%, consistent with empirical evidence.<sup>6</sup>

In a world without access to a financial advisor, self-management is implemented mostly by middle-aged workers and early retirees; almost no young investors elect self-management. By contrast, the opportunity to delegate money management proves to be quite valuable for specific subgroups: about 10% of young investors, 15% of middle-aged investors, and 50% of older investors optimally turn over their management responsibilities to financial advisors. The

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<sup>4</sup> C.f. Cocco, Gomes, and Maenhout (2005); Gomes and Michaelidis (2003); and Horneff, Maurer, Mitchell, and Stamos (2009).

<sup>5</sup> Among these are Bodie, Merton, and Samuelson (1992); Chai, Horneff, Maurer, and Mitchell (2011); and Gomes, Kotlikoff, and Viceira (2008).

<sup>6</sup> See for example Ameriks and Zeldes (2004) and U.S. Census Bureau (2012)

calibrated model shows that having a delegation option from the beginning of the lifetime boosts welfare by 2.5%, in terms of certainty equivalent consumption streams.

In what follows, Section 2 describes the general specification of the investor's portfolio problem when time is the primary cost of financial management. Section 3 explores when investors find it optimal to choose portfolio inertia, first when no delegation is feasible, and then we introduce the possibility of hiring a financial advisor. Section 4 presents a calibration and numerical solution of the model. Section 5 outlines results and provides measures of the welfare impact of delegation. We conclude with a discussion of implications of our findings for the financial advisory industry, retirement plan sponsors, and policymakers.

## **2. Dynamic Portfolio Choice with No Delegation**

### **2.1 Financial Decisionmaking Efficiency and Time Budgets over the Life Cycle**

This section specifies the investor's problem when allocating his portfolio, assuming that asset self-management requires that he devote time to the process. Our model incorporates a dynamic consumer determining his equity share and labor supply, both of which influence his current and future labor income, and his financial wealth. The investor is endowed with an available time normalized to 1 each period, and he can allocate this time to work ( $l_t$ ) or leisure ( $L_t$ ). Time ( $t = 0, 1, \dots, T$ ) is measured in years, assuming that at  $t = 0$  the investor starts his work life at age 20. The investor also faces mortality risk over the course of his (uncertain) lifetime, and his maximum age is set to 100 ( $T = 80$ ).

When the individual is not a financial expert, investing his saving requires him to devote both time and mental resources to the task (Abel et al. 2013). This can be costly: for instance, managing financial assets requires locating and opening brokerage accounts, analyzing financial

products' risk/return characteristics, and evaluating product fee structures. After deciding how to allocate his wealth between risk-free and risky assets, he must then devote time to implement these choices. For example, a buyer of mutual funds must read and compare many prospectuses and execute trading orders, and he may need to form a portfolio of various mutual funds to achieve his desired investment allocation. This imposes on him an opportunity cost, since his labor earnings depend on job-specific skills (i.e., human capital) accumulated mainly through work experience.<sup>7</sup>

We capture the explicit opportunity cost of adjusting the wealth portfolio by the fraction of the investor's time ( $\phi_t$ ) devoted to financial decisionmaking. Someone who is not well-informed regarding financial management will need to allocate more time to acquire and process information related to portfolio management. Thus the investor faces the following time budget constraint:

$$l_t + L_t + \phi_t \mathbf{1}_{\{a_t=1\}} = 1, \quad (1)$$

where  $a_t = 1$  is a variable taking the value of 1 if he self-manages, and 0 otherwise. We also posit that the time cost of making an efficient financial decision  $\phi_t$  can vary with age, so that middle-aged consumers are more efficient in managing their wealth, compared to younger and older individuals. This is modeled as a U-shaped age-related function for the time cost of financial decisionmaking over the life cycle, as in Agarwal et al. (2009) who show that task performance skills (the sum of analytic and experiential skills) peak in middle age.<sup>8</sup> Moreover,

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<sup>7</sup> Some people may enjoy self-management or believe they can outperform the market and professional investors. Yet few can do so in practice, and their performance is often worse than average (Lusardi and Mitchell 2007; Mitchell et al. 2009).

<sup>8</sup> Technically, this inefficiency cost comes from the complexity that the typical investor faces when implementing his choices in a dynamic programming problem; see Johnson et al. (2001).

the investor incurs time costs every time he self-manages his financial portfolio, since he must re-solve his life cycle model and implement new choices each period.

## 2.2 The Human Capital Accumulation Process

We posit that job-specific human capital is accumulated through learning by doing, as in Arrow (1962) and Becker (1964). Here we denote as  $H_t$  and  $l_t$ , respectively, the time devoted to developing job-specific human capital and work time each period. The law of motion for job-specific human capital is:

$$H_{t+1} = [(1 - \delta_t)H_t + F_t(H_t, l_t)] \times \lambda_t, \quad (2)$$

where  $F_t(H_t, l_t)$  is an experience formulation function and  $\delta_t$  is a depreciation rate<sup>9</sup> for job-specific human capital. An idiosyncratic temporary shock ( $\lambda_t$ ) also affects the accumulation level of human capital in the next period.

This formulation makes clear that work in the current period ( $l_t$ ) not only generates current labor income, but it also raises the stock of future human capital thus generating higher future labor income.<sup>10</sup> Previous research on endogenous labor supply in a dynamic portfolio choice model incorporates wage income as an important source of risk (Bodie et al. 1992; Gomes et al. 2008; Chai et al. 2011), but there the decision to work is assumed to affect only current income. Consequently, those prior studies implicitly assume that work time substitutes for current leisure time, and the price of leisure is simply the current wage. By contrast, here we model the investor who considers how taking time away from work today influences his human capital accumulation, his future labor earnings, and his age-related efficiency pattern of financial decisionmaking.

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<sup>9</sup> This can also be interpreted as a rate of skill obsolescence; that is, some knowledge becomes outdated by the advent of new technology.

<sup>10</sup> This could also be interpreted as a reputation effect in the labor market.

We specify the experience acquisition function following Ben-Porath (1967), as follows:

$$F_t(H_t, l_t) = a(H_t \cdot l_t)^\theta \quad (3)$$

where  $a$  is a parameter that represents the individual efficiency or the learning ability for accumulating human capital.<sup>11</sup> The elasticity of human capital accumulation  $\theta$  is assumed to have decreasing returns to scale ( $\theta \in (0,1)$ ).

### 2.3 Labor Income and Asset Returns

Labor income ( $E_t$ ) is determined by the individual's job-specific human capital level ( $H_t$ ) and wage shock ( $Y_t$ ):

$$E_t = l_t H_t Y_t . \quad (4)$$

Here  $l_t$  represents (normalized) working hours. The pattern for human capital accumulation  $H_t$  is similar to the age-specific deterministic wage trend found in the life cycle literature (Cocco et al. 2005; Gomes et al. 2008). In the present model, however,  $H_t$  is endogenously accumulated over time by the individual's labor supply. The wage shock ( $y_t \equiv \log Y_t$ ) follows an AR(1) process and is influenced by an idiosyncratic shock  $y_t = \eta + \rho y_{t-1} + \epsilon_t^y$  where  $\epsilon_t^y \sim iid N(0, \sigma_y)$ . After the (exogenous) retirement age of 65 ( $t = 45$ ), the individual enjoys full time leisure and receives a Social Security benefit equal to a fraction of his final labor earnings, similar to the US. retirement system.

Two asset classes are available for the consumer's investment portfolio: risky stocks and riskless bonds. The real stock return ( $R_t$ ) is assumed to be serially independent and identically log normally distributed with parameters  $\mu_S$  and  $\sigma_S$ , i.e.  $\log R_t \sim N(\mu_S, \sigma_S)$ .<sup>12</sup> The stock log return and wage shock are correlated with a coefficient of  $\sigma_{\epsilon_S}$ . The riskless bond has a return  $\bar{R}$

<sup>11</sup> Our notion of human capital is informed by job-specific skills accumulated by working, as in Becker (1964).

<sup>12</sup> Tang et al. (2010) report that people receive lower returns when they manage their own portfolios, compared to having professionals manage them. For simplicity, we assume that equity returns are the same for all portfolio management methods (inertia, self-management, and delegation).



in all periods. We denote  $R_{t+1}$  as the stock return from  $t$  to  $t + 1$ , so that the fraction of the individual's wealth invested in stocks is determined in period  $t$ , and returns are realized in  $t + 1$ .

## 2.4 Portfolio Choice and Wealth Dynamics

At time  $t$ , the individual selects the equity portion ( $\pi_t$ ) for his portfolio, and the portfolio then generates an uncertain return of:  $R_{t+1}^p = (1 - \pi_t)\bar{R} + \pi_t R_{t+1}$ . We note that  $R_{t+1}^p$  is a random variable at time  $t$  when the portfolio weight is selected and is realized at time  $t + 1$ . Denoting  $C_t$  as consumption, the dynamic budget constraint can be formulated as:<sup>13</sup>

$$W_{t+1} = R_{t+1}^p(W_t + E_t - C_t) \quad (5)$$

Total cash-on-hand in period  $t$  consists of financial wealth ( $W_t$ ) and labor income ( $E_t \equiv l_t H_t Y_t$ ). After consuming  $C_t$  in period  $t$ , the consumer invests his remaining assets and earns returns of  $R_t^p$ .

## 2.5 Preferences and Time Horizon

As in Gomes et al. (2008), we suppose the investor has a standard time-separable, modified Cobb-Douglas power utility function over current consumption ( $C_t$ ) and time devoted to leisure ( $L_t$ ) in each period, given by  $U_t(C_t, L_t) = \frac{1}{1-\gamma} (C_t L_t^\alpha)^{1-\gamma}$ . Here  $\alpha > 0$  captures an investor's preference for leisure relative to consumption. The parameter  $\gamma$  measures risk aversion.

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<sup>13</sup> We could introduce a direct transaction cost ( $TC(\pi_t, \pi_{t+1})$ ) for portfolio adjustment in which case the wealth dynamics would be  $W_{t+1} = R_{t+1}^p(W_t + E_t - TC(\pi_t, \pi_{t+1})\mathbf{1}_{\{adjustment\}} - C_t)$ . Nevertheless, we do not focus here on direct monetary costs; see Bonaparte and Cooper (2009) for a discussion.

### 3 Dynamic Portfolio Choice Problem with Inertia, Self-Management, and Delegation

#### 3.1 Portfolio Inertia

Technically speaking, portfolio inertia in period  $t$  is defined as retaining the previous period's portfolio in the next period while incurring no time cost. When an investor continues to follow his previous period's portfolio allocation ( $\pi_t$ ) next period ( $\pi_{t+1}$ ), he saves time ( $\varphi_t$ ) that he would otherwise devote to collecting and analyzing new information to rebuild an optimal financial portfolio and implement the change. For this reason, when the investor engages in portfolio inertia, his next period portfolio share is identical to that he set previously, so his time constraint is not impacted by the cost of financial decisionmaking. Accordingly, someone electing portfolio inertia has the following equity share and time constraint (where  $l_t$  and  $L_t$  denote labor supply and leisure, respectively):

$$\pi_{t+1} = \pi_t$$

$$l_t + L_t = 1$$

It is worth emphasizing that holding the same portfolio over time does not necessarily imply that the investor is engaging in portfolio inertia. That is, he could decide to incur the time cost to self-manage and still end up selecting his previous portfolio as the optimum for next period. In this event, his portfolio choice is not naïve, and he will sacrifice a portion ( $\varphi_t$ ) of his available time to end up in this position.

Portfolio inertia permits a previous period's portfolio choice to affect the current period's decision regarding which portfolio management method to use. Thus the previous portfolio ( $\pi_t$ ) serves as a state variable in our model. Other state variables include wealth ( $W_t$ ), accumulated human capital level ( $H_t$ ), and the wage shock ( $y_t$ ). In total, then, we have four choice

variables: the portfolio management method (i.e., portfolio inertia or self-management), labor supply ( $l_t$ ), the next period's equity share ( $\pi_t$ ), and consumption ( $C_t$ ).

We define  $V_t^a(W_t, H_t, \pi_t, y_t)$  as the discounted lifetime utility of an investor when he chooses to self-manage his portfolio. Similarly,  $V_t^i(W_t, H_t, \pi_t, y_t)$  denotes the discounted lifetime utility of an investor when he chooses portfolio inertia. Then the value function at time  $t$  may be specified as:

$$V_t(W_t, H_t, \pi_t, y_t) \equiv \max\{V_t^a(W_t, H_t, \pi_t, y_t), V_t^i(W_t, H_t, \pi_t, y_t)\}. \quad (6)$$

Let  $\beta < 1$  be the investor's time preference and  $p_t$  the probability that he survives to the next period. Then the value function for self-management method is as follows:

$$\begin{aligned} V_t^a(W_t, H_t, \pi_t, y_t) &= \max_{\{C_t, \pi_t, l_t\}} U_t(C_t, L_t) + p_t \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\ &\quad s. t. C_t \leq W_t + E_t \\ W_{t+1} &= R_{t+1}^p(W_t + E_t - C_t) \\ H_{t+1} &= [(1 - \delta_t)H_t + F_t(H_t, l_t)] \times \lambda_t \\ l_t + L_t + \phi_t &= 1. \end{aligned} \quad (7)$$

The value function for the individual electing portfolio inertia is:

$$\begin{aligned} V_t^i(W_t, H_t, \pi_t, y_t) &= \max_{\{C_t, l_t\}} U_t(C_t, L_t) + p_t \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\ &\quad s. t. C_t \leq W_t + E_t \\ W_{t+1} &= R_{t+1}^p(W_t + E_t - C_t) \\ H_{t+1} &= [(1 - \delta_t)H_t + F_t(H_t, l_t)] \times \lambda_t \\ l_t + L_t &= 1. \end{aligned} \quad (8)$$

When an investor employs portfolio inertia, he does not maximize the value function with respect to equity share but instead takes the previous equity share as his next period's portfolio.

That is, if  $V_t^a \geq V_t^i$ , the investor opts for self-management ( $a_{t=1}$ ); else he opts for portfolio inertia.

The key differences between the two value functions have to do with the time constraint and the next period's portfolio choice. The appeal of portfolio inertia is that the time saved can then be used to work and accumulate more human capital, or to enjoy more leisure. During retirement the investor does not work, so if he decides to self-manage his portfolio he sacrifices only his leisure time. Nevertheless, increasing inefficiency in financial decisionmaking makes inertia appealing at older ages.<sup>14</sup>

### 3.2 The Role of Financial Advisors

Next we extend our model to examine how introducing financial advisors can add value to life cycle decisionmakers.<sup>15</sup> Reasons for delegating portfolio management can include time costs, efficiency gains due to lower transaction costs, and beliefs regarding professional managers' skills. In what follows, we focus mainly on the investor's loss of human capital associated with having to manage his own portfolio.

When an investor elects to delegate the portfolio management to an advisor, he must pay a management fee out of his total cash-on-hand ( $W_t + E_t$ ). The advantage of hiring the financial advisor is the saved time which can then be used to work and accumulate more job-specific knowledge, or to enjoy leisure. If, instead, he self-manages his portfolio, he need not pay the adviser fee but he does incur the time cost associated with his age-based efficiency pattern of financial decisionmaking. In the financial advisory service industry, the fee generally consists of a fixed minimum dollar amount ( $\varphi_{min}$ ) and a variable component ( $\varphi_{ptg}$ ), where the latter is

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<sup>14</sup> Sufficient conditions for the selection of portfolio inertia are provided in Appendix A.

<sup>15</sup> In the U.S., Registered Investment Advisors (RIAs) must file with the Securities and Exchange Commission so they can advise on financial investments including stocks, bond, mutual funds, etc. They also manage portfolios of securities for households and employers, helping implement clients' optimal portfolio choices. See Mitchell and Smetters (2013).

expressed as percentage of assets under management. Formally, this structure may be expressed as follows:

$$\varphi_t = \max(\varphi_{min}, (W_t + E_t) * \varphi_{ptg}) \quad (9)$$

The financial advisor not only chooses a portfolio for the consumer but also proposes the optimal levels of consumption and labor supply. The value function for the delegated portfolio management method is:

$$\begin{aligned} V_t^d(W_t, H_t, \pi_t, y_t) &= \max_{\{C_t, \pi_t, l_t\}} U_t(C_t, L_t) + p_t \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\ & \text{s. t. } C_t \leq W_t + E_t - \varphi_t \\ W_{t+1} &= R_{t+1}^p (W_t + E_t - \varphi_t - C_t) \\ R_{t+1}^p &= (1 - \pi_t) \bar{R} + \pi_t R_{t+1} \\ H_{t+1} &= [(1 - \delta_t) H_t + F_t(H_t, l_t)] \times \lambda_t \\ l_t + L_t &= 1 \end{aligned} \quad (10)$$

Note that the investor does pay the management fee  $\varphi_t$  out of his cash on hand, but he does not incur the time cost  $\phi_t$ .

One important issue when delegating portfolio management is the possibility of a conflict of interest between the investor seeking to maximize his utility over consumption and leisure, and the financial advisor who seeks to maximize the client's wealth (and thereby his own fees).<sup>16</sup> Such moral hazard is mitigated in a dynamic setting since the financial advisor must take into

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<sup>16</sup> The conflict of interest between clients and financial advisors has been analyzed theoretically by Sharpe (1985), Stoughton et al. (2011), and Mullainathan et al. (2012), among others. Even though the investor cannot observe the financial advisor's portfolio choice at the beginning of time  $t$ , he can easily obtain information about the past return process and his total wealth at the end of time  $t = 1, \dots, T$ . In a competitive market, each advisor will be monitored by his competitors which implies that reputation costs will reduce his chances of being hired by some other investor. Ou-Yang (2003) examines a continuous-time dynamic optimization problem in a delegated portfolio management problem, where he finds that a financial advisor exactly follows the investor's optimal portfolio policy if a symmetric (i.e., reward and punishment) remuneration scheme is offered. For this reason, the investor need not consider the incentive compatibility problem.

account his reputation and potential future revenue, which naturally depend on client outcomes. Accordingly, for the investor to implement his first-best choice, he must be able to verify the advisor's choices, which in our model is possible since we assume the return process is observed *ex-post*. Accordingly, we only need to solve the investor's dynamic programming problem; his optimal choices will be implemented by his financial advisor so the problem can be summarized as:

$$\begin{aligned}
& V_t(W_t, H_t, \pi_t, y_t) \\
&= \max_{\{a_t, l_t, \pi_{t+1}, c_t\}} U_t(C_t, L_t) + \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\
&\quad s. t. C_t \leq W_t + E_t \\
& W_{t+1} = R_{t+1}^p(W_t + E_t - \mathbf{1}_{\{a_t=2\}}\varphi_t - C_t) \\
& R_t^p = (1 - \pi_t)\bar{R} + \pi_t R_t \\
& H_{t+1} = [(1 - \delta_t)H_t + F_t(H_t, l_t)] \times \lambda_t \\
& l_t + L_t + \varphi_t \mathbf{1}_{\{a_t=1\}} = 1 \\
& \pi_{t+1} = \pi_t \text{ if } a_t = 0
\end{aligned} \tag{11}$$

where  $a_t = 0$  denotes portfolio inertia,  $a_t = 1$  self-management, and  $a_t = 2$  hiring a financial advisor. In addition,  $V_t \equiv \{V_t^i, V_t^a, V_t^d\}$  where  $V_t^i$  is the value function for the portfolio inertia case,  $V_t^a$  is the value function for self-management, and  $V_t^d$  is the value function for delegating portfolio management.

## 4 Model Calibration and Solution

### 4.1 Existence of a Solution and Numerical Procedure

There is no simple Euler equation linking the marginal benefit of today's portfolio adjustment with future marginal benefits, inasmuch as the investor is unsure about which portfolio management method he will select in the future (Adda and Cooper 2003). Although the existence of solutions is guaranteed (see Appendix B), deriving these is analytically intractable. For this reason we solve the model numerically via backward induction, polynomial approximation of the value function, and Monte-Carlo integration.

### 4.2 Parameter Calibration

To characterize the model's output, we calibrate the model using a reasonable set of base case parameters. We set the discounting factor  $\beta$  to 0.98, the coefficient of risk aversion at  $\gamma = 3$ , and the leisure preference to  $\alpha = 1.3$ , as is conventional. The one-period survival rates  $p_t$  which enter into the utility function are calculated by the cohort mortality table from the 2009 US Social Security Administration Trustees Report for females born in 1990 (Bell and Miller 2012).

To calibrate human capital accumulation process we assume that human capital  $H_t$  depreciates at rate of  $\delta_t = 0.07\% \times age$  per year, the elasticity parameter in the experience accumulation function is set to  $\theta = 0.2954$ , and the accumulation rate to  $a = 0.2192$ . The idiosyncratic shocks of human capital development follow an independent identically lognormal distribution  $\ln(\lambda_t) \sim N(-0.5 \cdot 0.2917^2, 0.2917)$ . These parameters produce a similar age-dependent wage rate profile as used in Gomes et al. (2008).<sup>17</sup> For the permanent wage shock process  $y_t$ ,

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<sup>17</sup> Technically, we numerically minimize the distance of various moments (i.e., mean, standard deviation, median, skewness, kurtosis, max, min and the age of maximum wage) of the simulated human capital accumulation process

the drift parameter  $\eta$  is set to zero, the AR(1) autocorrelation coefficient to 0.85, and the wage shock standard deviation is 0.2917 prior to retirement, and afterwards it is 0.28 (as in Love 2010). After retirement, when labor supply is zero, shocks may be interpreted as income or consumption surprises typical of those experienced by older households (e.g. unexpected out-of-pocket medical expenses or long-term care expenses).

Retirement benefits replace about 50% of the individual's last labor income  $E_t = 0.2H_{45}Y_{45}$  ( $t = 45, 46, \dots, T$ ). This formulation generates higher (lower) replacement rates for worker with lower (higher) average career earnings, consistent with the progressive benefit rules of the U.S. Social Security system (Chai et al. 2011). The riskless asset return  $\bar{R}$  is set to 1.02 and the risk premium for stocks is 4% with a standard deviation of 20,5% (Cocco et al. 2005). In our baseline calibration, the fixed minimum management fee  $\varphi_{min}$  to delegate portfolio management to an advisor is set to zero.<sup>18</sup> The variable portfolio management fee  $\varphi_{ptg}$  is set to 1.3%, which is the median expense ratio for U.S equity mutual funds (ICI 2013).<sup>19</sup>

The efficiency function for financial decisionmaking is assumed to be convex (Agarwal et al. 2009). Investors around age 50 are assumed to be most financially savvy with  $\phi_{22} = 0.03$ ; that is, they must sacrifice only 3% of their normalized time to manage their own portfolios. Young investors are assumed to be the least efficient ( $\phi_1 = 0.09$ ). The functional form for efficiency is assumed to be as follows:  $\phi_t = \frac{0.09-0.03}{30^4} (age - 30)^4 + 0.03$ , where the 4th power generates a flatter efficiency pattern for the middle-aged. Although we have no direct empirical evidence for this functional form and the selected parameters, we believe that this is a reasonable calibration which allows us to explore how the efficiency patterns affect portfolio management

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given in equations (2) and (3) relative to the (deterministic) age-dependent wage profile used in Gomes et al. (2008). In this procedure we assume a fixed level ( $l_t = 0.5$ ) of labor supply until retirement.

<sup>18</sup> Even though financial advisors can play somewhat different roles from that of a mutual fund portfolio manager, their fee levels are similar.

<sup>19</sup> To be precise we set, to avoid numerical problems,  $\varphi_{ptg}$  equal to 0.0001% of the first year's maximum wage.



schemes. In sensitivity analysis below, we also report results using different inefficiency patterns by age. Baseline parameters are summarized in Table 1.

*Table 1 here*

## **5. Simulated Life Cycle Profiles**

We describe investor behavior using the optimal controls of the baseline parameterization of our life cycle model to generate 2,000 simulated lifetimes reflecting realizations of stock returns and labor income shocks. We summarize results for portfolio management methods (inertia, self-management, delegation), and the allocation to risky stocks over the life cycle.

Average life cycle profiles of choice variables when delegation is not an option are presented in Figure 1. Panel A shows that the model generates consumption and wealth profiles over the life cycle consistent with other studies, namely hump-shaped with age (e.g. Gomes et al. 2008). Panel B refers to labor earnings and pension benefits; these payments rises with age due to human capital accumulation and then drop sharply at retirement. Panel C reveals that investors' average equity share pattern is relatively flat with age, ranging between 40 and 60%, consistent with empirical evidence (U.S. Census Bureau 2012). Panel D traces the level of labor supply which is relatively flat over time, and the stock of human capital which rises with age until retirement.

*Figure 1 here*

In Figure 2 we trace the pattern of portfolio management methods (inertia versus self-management) over the life cycle (Panel A), and the equity share conditional for the subgroup of those electing each method (Panel B). Inertia proves to be the dominant method for young investors up to about age 30; thereafter self-management rises steadily to retirement and many

retirees switch to self-management. The explanation is that the young have little wealth but much to lose if they devote time to manage their meager assets; at older ages, individuals have more wealth and a lower opportunity cost of time. Moreover, portfolio inertia plays an important role in generating the flat equity share pattern. That is, the young investors begin with a low equity share and select portfolio inertia. The middle-aged are more active, since this group is the most efficient in terms of financial decisionmaking and they have already accumulated substantial human capital. For this reason, sacrificing a small amount of time has only a modest impact on their future wage rates. Still, however, almost 85% of the middle-aged group does not change portfolio allocations; such a high level of inactivity is consistent with several empirical studies (Mitchell et al. 2006; Vissing-Jorgensen 2003). Among the older group, their lower decisionmaking efficiency somewhat depresses their interest in self-management, though they are still more active than young investors. And interestingly, retirees are much more likely to switch to self-management due to having more leisure time; in fact, the fraction of self-managing investors jumps from 15% at age 64 to about 30% at age 65. At very old ages, investors are quite inefficient in financial decisionmaking, needing ever-larger fractions of their time when they self-manage their assets. This discourages self-management.

*Figure 2 here*

Next, Panel B of Figure 2 shows the average equity share for the two different subgroups. Here we see, that compared to inertia investors, those who chose self-management allocate a much higher fraction of their financial assets to stocks: on average, self-managing investors hold 90% of their financial assets in stocks during their work lives. Interestingly, active investors sharply curtail their equity exposure at retirement from 90% to 70%. A detailed look into individual simulated life cycle profiles reveals that active investors typically have more wealth than do the inertia investors. Thus wealthier investors allocate more time, i.e. become active, to

improve the risk and return profile of their financial assets. It is well known (c.f., Bodie et al. 1992) that in a world with flexible labor supply, investors should hold much of their wealth in risky stocks, even when wage rates are uncertain. The reason is that flexible work hours serve as insurance against negative shocks in the financial market.

Nevertheless, after retirement (here, mandatory at age 65), the insurance-like feature of flexible work is eliminated. Moreover, older households still face substantial income uncertainty. Both factors reduce the investor's willingness to take financial risk. Accordingly, a rational response for wealthy and active investors is to reduce their equity share post-retirement.

Table 2 presents summary statistics concerning the dynamics of portfolio management methods when no delegation option is feasible. On average, investors change their portfolio management approaches 6.58 times during their lifetimes, and they elect inertia for 66.44 years overall. Years of portfolio self-management average 13.56, and the first year when people elect self-management is about 30.87 years after entering the labor force. About 6.8% of people ( $=1-[1864/2000]$ ) never choose self-management and remain inactive over their lifetimes.

*Table 2 here*

Figure 3 shows what happens when investors can delegate their investment management to a financial advisor. Panel A illustrates the pattern of portfolio inertia, self-management, and delegation by age, while Panel B shows the equity share for each subgroup. Overall, we see that access to delegation reduces inertia and self-management compared to Figure 2, and delegation grows more attractive with age. Approximately 15% of young investors (up to age 30), 10% of middle-aged investors (age 30-65), and 25%-50% of retired investors now optimally delegate to financial advisors. Self-management is adopted by only a small fraction (under 1%) of the younger investors, but many more middle-aged (around 10%) and older investors (around 15%)

do so. And access to delegation substantially reduces inertia, especially among the very young and oldest investors.

*Figure 3 here*

Next we compare equity investment profiles in Panel B of Figures 2 and 3. Compared to the no-delegation world, people who elect inertia or self-management ramp up their equity holdings younger and hold more equity until retirement. Our analysis of the various portfolio management methods (Table 3) shows that investors facing favorable stock market experience and adverse labor market shock are more likely to become active rather than inertia investors. When the delegation option is available, only investors with very favorable stock market and extremely adverse labor market shock will still elect to self-manage their portfolios, but they will invest more in equity because of favorable stock market returns. So equity share for active investors may be higher when a delegation option is available.

*Table 3 here*

When a delegation option is available, the higher equity share of inertia investors can be attributed to the financial advisors. That is, some investors who hire financial advisors early in life end up with relatively high equity shares, consistent with the normative implications of a life cycle model with flexible work supply (see Bodie et al. 1992). Later in life, they might retain these portfolios and become inactive, because sticking with the higher equity share produces an outcome sufficiently close to their optimal equity share. By contrast, those who continue to hire financial advisors until reaching retirement have intermediate equity holdings compared to the other two groups.

In retirement, the investors in the inertia subgroup hold a relatively constant equity share, while the other two groups curtail their equity holdings. In particular, self-managers reduce their equity holdings substantially. As is true with no delegation, the sharp decline of active-investors'

equity investment post-retirement can be explained by the loss of the insurance-like feature of flexible work hours to compensate for negative stock market shocks.

Table 3 also shows some of the dynamics of portfolio management methods when a financial advisor is available. On average, investors now change their portfolio management approach much more often, 15.15 times over their lives (versus 6.58 in Figure 2), and they elect inertia for a shorter period (56.55 years, versus 66.44 in Figure 2). Of most interest is the fact that people devote less than half as many years to portfolio self-management (5.81 versus 13.56 previously), and they engage advisors for more than a fourth of their adult lives ( $=17.84 \text{ years}/80$ ). Investors begin to delegate relatively soon, after only 12.46 years from starting work and virtually all elect to delegate at some point ( $99\%=1988/2000$ ). By contrast, when delegation is an option, fewer self-manage and those who do, begin much later, 45.77 years after starting work (versus 30.87 in Figure 2).

Table 4 summarizes changes by age when a delegation option becomes available, in patterns of wealth, equity share, labor income, labor supply, human capital, and consumption. All are expressed as a percent of the no-adviser base case. Here we see that having access to an advisor boosts wealth more than 20% across all age groups. This is due to the higher equity share noted above, and also the greater time devoted to work and human capital accumulation early in the career. Having access to an advisor also increases leisure in later life, and while consumption declines a bit early on, it increases rises by more than 2% after age 50 and into retirement.

*Table 4 here*

Next we summarize the factors associated with choice of portfolio management method in our simulated data using descriptive Logit regressions. Table 5 reports estimated marginal effects of lagged wealth, stock market shocks, and wage shocks when no delegation is permitted. Table 6 conducts a similar analysis when financial advisors are available. The first analysis implies that

wealthier investors are less likely to engage in inertia, possibly because sticking to a non-optimal level of equity exposure can be more expensive for them than for the less wealthy. We also see that investors experiencing negative stock market shocks and positive wage shocks are more likely to elect portfolio inertia, due to the need to invest more in human capital and avoid riskier equity. Results are similar across model specifications.

*Tables 5 and 6 here*

Table 6 presents a similar descriptive Logit analysis but this time, investors can elect inertia, self-management, or delegation. Panel A uses self-management as the reference group, while Panel B uses inertia as the reference. As before, the wealthier are less likely to engage in inertia, but now wealthier people are also more likely to delegate. Negative stock market shocks deter both inertia and delegation, and positive wage shocks boost inertia while deterring delegation. Those who elect delegation over self-management tend to have more financial assets but experience negative labor income shocks. This is sensible since, for this subgroup, devoting time to financial management does not come at a high opportunity cost in terms of foregone labor earnings, and it helps increase their wealth. Additionally, wealthier people with unfavorable stock market shocks prefer delegation over inertia, consistent with the important link between labor income and portfolio management methods. When an investor faces a positive financial market as well as high wages, he opts for ways to obtain his optimal portfolio without sacrificing the chance to work and learn by doing.

Finally, Figure 4 depicts the increase in lifetime welfare from having access to financial advisors, versus not having that option. As is conventional, we measure this in terms of a certainty equivalent (CE) consumption stream, or the stream of consumption that would afford the investor the same level of expected lifetime utility if he lacks access to the delegation option,

versus having it.<sup>20</sup> The Figure shows that providing young investors with access to a financial advisor boosts lifetime welfare substantially, by the equivalent of a 2.5% enhancement in the annual consumption stream.<sup>21</sup> This is similar in magnitude to that reported in Cocco et al. (2005), comparing welfare in a world with a fixed versus flexible equity share in the portfolio. It is also worth noting that welfare increases trace out a U-shaped profile with age. In other words, younger and older investors benefit most from having access to a financial advisor. This can be explained by the fact that the young do better by investing in their human capital instead of managing their money; the older group does better with advisors due to their declining efficiency of financial decisionmaking.

*Figure 4 here*

Figure 4 also reports the results from sensitivity analysis, where we now assume that investors are even less efficient in decisionmaking than in the baseline calibration, across all age groups. The most financially savvy middle-aged investors are assumed to spend only 5% ( $\phi_{22}$ ) of their available time, while young investors are assumed to sacrifice 12% ( $\phi_1$ ) of their available time if they invest actively. Not surprisingly, the welfare gains of having a delegation option increases, especially for young and old investors. Having the option to hire financial advisors is more valuable, the more costly is portfolio self-management.

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<sup>20</sup> As in Chai et al. (2011), the certainty equivalent constant consumption stream ( $c_{CE}$ ) is defined as:

$$V_t(W_1, H_1, \pi_1, y_1) = E \left[ \sum_{i=t-1}^T \beta^i \frac{1}{1-\gamma} (C_i(L_i)^\alpha)^{1-\gamma} \right] = \sum_{i=t-1}^T \beta^i \frac{1}{1-\gamma} (c_{CE}^t(L_*)^\alpha)^{1-\gamma}$$

where  $L_*$  is a fixed level of leisure and  $(W_1, H_1, \pi_1, y_1)$  is the initial pair of state. With some algebraic manipulation, we get:

$$c_{CE}^t = \left[ \frac{(1-\gamma)V_t}{\sum_{i=t-1}^T (L_*^\alpha)^{1-\gamma} \beta^i} \right]^{\frac{1}{1-\gamma}}$$

In calculating this measure, we set leisure  $L_*$  as time deducted from mean labor hours over 40 working years, due to retirement thereafter.

<sup>21</sup> It is noteworthy that the welfare gains introduced may be larger than measured here, if a financial advisor can provide the investor with access to lower transaction costs and possibly excess returns.

## 6 Conclusions

This paper develops and simulates a life cycle model to illustrate optimal portfolio management methods selected by finitely-lived investors who face portfolio management costs and an age-dependent inefficiency pattern for financial decisionmaking. Using a reasonable set of parameters, our model replicates observed patterns of portfolio inertia across age groups. Investors who can accumulate job-specific knowledge by working tend to devote less time to managing their money when they are young. Middle-aged both have more assets to invest and suffer less from the opportunity costs of self-managing their assets, though many still elect inertia. Declining decisionmaking efficiency later in life prompts many older investors to select portfolio inertia. When investors can delegate the portfolio management task to a financial advisor, this enables many to avoid portfolio inertia. In general, the model predicts that older investors will find financial advisors most attractive. Finally, we find rather substantial welfare gains resulting from having a delegation option.

Our findings are relevant to a variety of stakeholders including individual investors, financial advisors, retirement plan sponsors, and policymakers. Those who will value financial advisory services the most are the young and the older age groups, so making such services available can greatly enhance their well-being. Also of interest is the prediction that advisors will find that some middle-aged clients will wish to continue self-managing their own financial assets, even when a delegation option is available. Policymakers may also wish to consider the potential positive welfare gains of improving investor access to financial advisory services. In an environment where financial advisors with fiduciary responsibility can help investors manage their financial wealth optimally, this will enable more people to accrue job-specific skills, thus contributing to the economy as a whole.



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**Table 1: Parameter Values for Numerical Solution**

Parameter	Baseline
Working periods	45
Retirement periods	35
Time discounting $\beta$	0.98
Risk aversion $\gamma$	3
Leisure preference $\alpha$	1.3
Experience formulation $a$	0.2192
Elasticity of $H_t$ accumulation $\theta$	0.2954
Depreciation of Human Capital $\delta_t$	0.07% $\times$ age per annum
Inefficiency of financial decisionmaking $\phi_t$	$\frac{0.09-0.03}{30^4} (t-30)^4 + 0.03$
Wage shock drift $\eta$	0
Wage shock auto correlation $\rho$	0.85
Std. of permanent wage shock $\sigma_{wage}$ (pre-retirement)	0.2917
Std. of permanent earnings shock (post-retirement)	0.28
Replacement rate	20% of maximum earnings at age 65
Risk premium	0.04
Std. of stock return $\sigma_{stock}$	0.205
Risk free rate $\bar{R}$	1.02
Delegation annual fee: variable rate $\varphi_t$	1.3% per annum
Delegation annual fee: fixed fee	0
Correlation between wage and stock return $\rho_{\epsilon, \zeta}$	0.15
Initial wealth for simulation $W_0$	0
Initial human capital for simulation $H_0$	10
Initial equity share for simulation $\pi_0$	40%
Initial wage shock for simulation $y_0$	0.1

**Table 2: Dynamics of Portfolio Management over the Life Cycle with no Delegation Option**

	Mean	Std	Median	5%-Q	95%-Q	N
No. of switches	6.58	4.41	6	0	14	2000
Years of inertia	66.44	12.90	70.00	37.00	80.00	2000
Years of self-mgmt	13.56	12.90	10.00	0.00	43.00	2000
First year of self-mgmt	30.87	20.20	28	6	66	1864

Notes: No. of switches refers to the number of times someone alters his portfolio management method (from self-managed to inertia or vice versa) over the life cycle (from age 20 to age 100), conditional on survival. Years of inertia refers to the total length of the inertia period for specific simulated life cycle paths; and years of self-mgmt refers to the total length of the self-management period over specific simulated life cycle paths. First year of self-mgmt refers to the first year when the individual changes the portfolio management method from inertia to self-management.

**Table 3: Dynamics of Portfolio Management over the Life Cycle with a Delegation Option**

	Mean	Std	Median	5%-Q	95%-Q	N
No of Switches	15.16	10.71	9	2	35	2000
Years of inertia	56.35	15.73	56.00	29.00	78.00	2000
Years of self-mgmt	5.81	7.90	3.00	0.00	26.00	2000
Years of delegation	17.84	11.19	17.00	2.00	36.00	2000
First year change	12.49	10.71	9	2	35	1999
First year self-mgmt	45.77	25.04	48.00	12.00	73.00	1561
First year delegation	12.46	10.63	9.00	2.00	34.65	1988

Notes: See Table 2. Also years of delegation refer to the total length of the period of delegated management for specific simulated life cycle paths. First year delegation refers to the first year when people change to delegation from inertia or from self-management.

**Table 4: Average Percent Change in Outcomes by Age Group: With and Without a Delegation Option**

Age	Wealth	Equity Share	Labor Income	Labor Supply	Human Capital	Consumption
20~35	21.74	11.05	1.00	0.64	0.04	-0.51
36~50	23.03	16.20	0.49	0.28	0.08	-0.22
51~65	23.04	13.04	0.38	-0.20	0.03	2.51
66+	25.26	5.57	-	-	-	2.80

Notes: The numbers represent the percentage increase summed over the age bin of having access to a delegation option versus not having access to a delegation option.

**Table 5: Factors Associated with Portfolio Inertia (versus Self-Management) with No Delegation Option: Marginal Effects from Descriptive Logit Analysis**

	Inertia Chosen (vs Self-Management)		
	(1)	(2)	(3)
$Wealth_{\{-1\}}$	-0.154*** (0.00)		-0.154*** (0.00)
$Stock\ market\ shock_{\{-1\}}$		-0.030*** (0.00)	-0.028*** (0.00)
$Wage\ shock_{\{-1\}}$		0.066*** (0.00)	0.067*** (0.00)

Notes: Each column reports marginal effects from separate Logit regressions. Dependent variable = 1 if the individual elected inertia in that period, or 0 = self-management. The observations experiencing inertia total 83.05% and self-management 16.95% for 2000 individuals simulated over 80 years. Standard errors in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at 10%, 5%, and 1% respectively. See text for additional explanation.

**Table 6: Factors Associated with Portfolio Inertia and Delegation (versus Inertia) with a Delegation Option: Marginal Effects from Descriptive Multinomial Logit Analysis**

A. Reference Group = Self-Management

	Inertia Chosen (vs Self-Management)		
	(1)	(2)	(3)
<i>Wealth</i> <sub>{-1}</sub>	-0.233*** (0.00)		-0.233*** (0.00)
<i>Stock market shock</i> <sub>{-1}</sub>		-0.014** (0.01)	-0.013*** (0.01)
<i>Wage shock</i> <sub>{-1}</sub>		0.088*** (0.00)	0.089*** (0.00)
	Delegation Chosen (vs Self-Management)		
	(1)	(2)	(3)
<i>Wealth</i> <sub>{-1}</sub>	0.186*** (0.00)		0.186*** (0.00)
<i>Stock market shock</i> <sub>{-1}</sub>		-0.006 (0.01)	-0.008 (0.00)
<i>Wage shock</i> <sub>{-1}</sub>		-0.017*** (0.00)	-0.017*** (0.00)

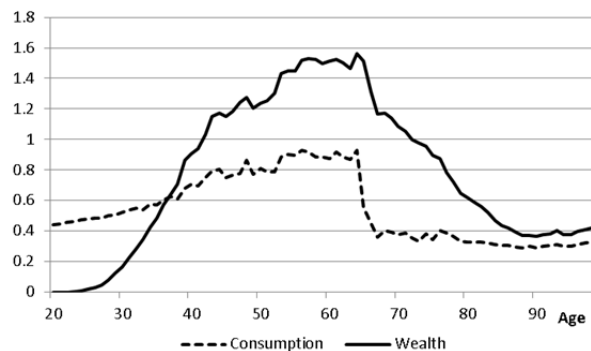
B. Reference Group = Inertia

	Self-Management Chosen (vs Inertia)		
	(1)	(2)	(3)
<i>Wealth</i> <sub>{-1}</sub>	0.048*** (0.00)		0.047*** (0.00)
<i>Stock market shock</i> <sub>{-1}</sub>		0.020*** (0.00)	0.021*** (0.00)
<i>Wage shock</i> <sub>{-1}</sub>		-0.071*** (0.00)	-0.072*** (0.00)
	Delegation Chosen (vs Inertia)		
	(1)	(2)	(3)
<i>Wealth</i> <sub>{-1}</sub>	0.186*** (0.00)		0.186*** (0.00)
<i>Stock market shock</i> <sub>{-1}</sub>		-0.006 (0.01)	-0.008 (0.00)
<i>Wage shock</i> <sub>{-1}</sub>		-0.017*** (0.00)	-0.017*** (0.00)

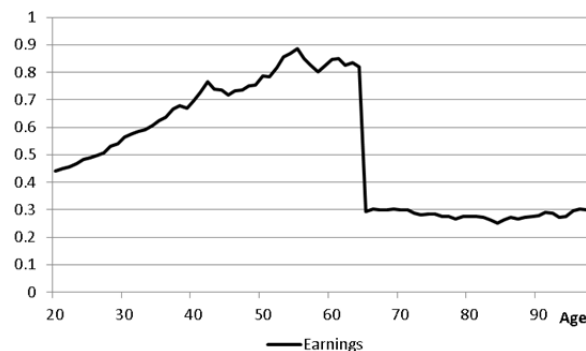
Notes: Each column reports marginal effects from separate Logit regressions. Dependent variable: = 0 active, 1 if inertia, and 2 = delegation. Observations of self-management account for 7.27% of the sample; inertia for 70.44%; and 22.29% for delegation respectively, from 2000 individuals simulated over 80 periods of life. Standard errors in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at 10%, 5%, and 1% respectively. See text for additional explanation.

**Figure 1: Life Cycle Profiles of Consumption, Earnings, Equity Share, Labor Supply, and Human Capital with No Delegation Option**

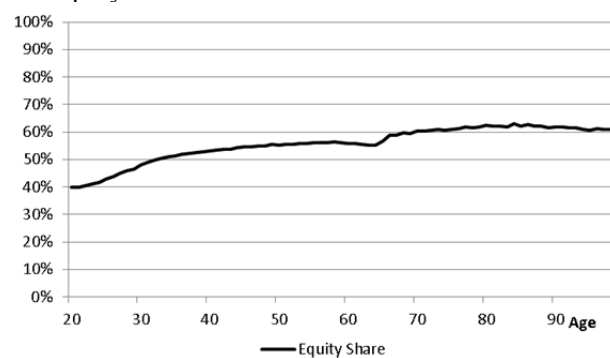
A. Consumption and Wealth



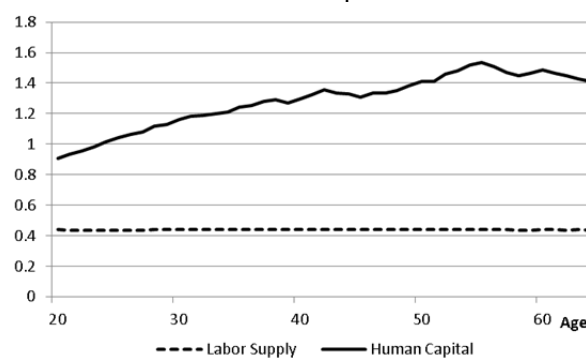
B. Earnings



C. Equity Share



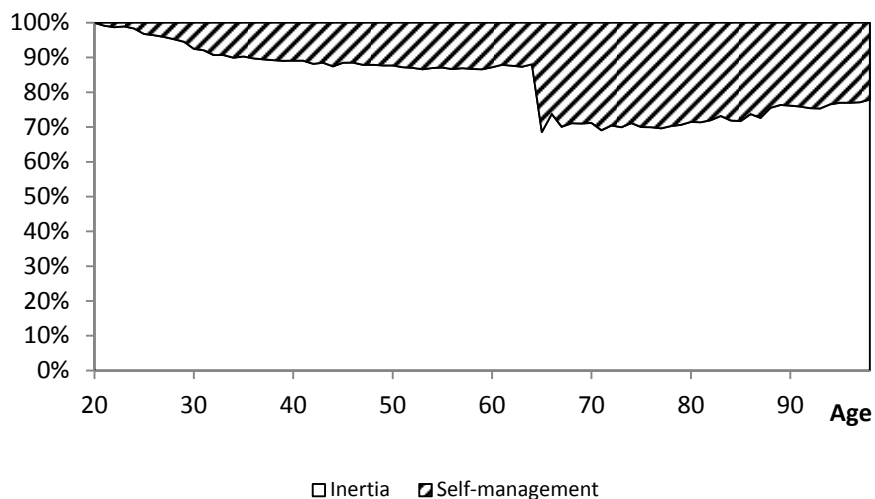
D. Work and Human Capital



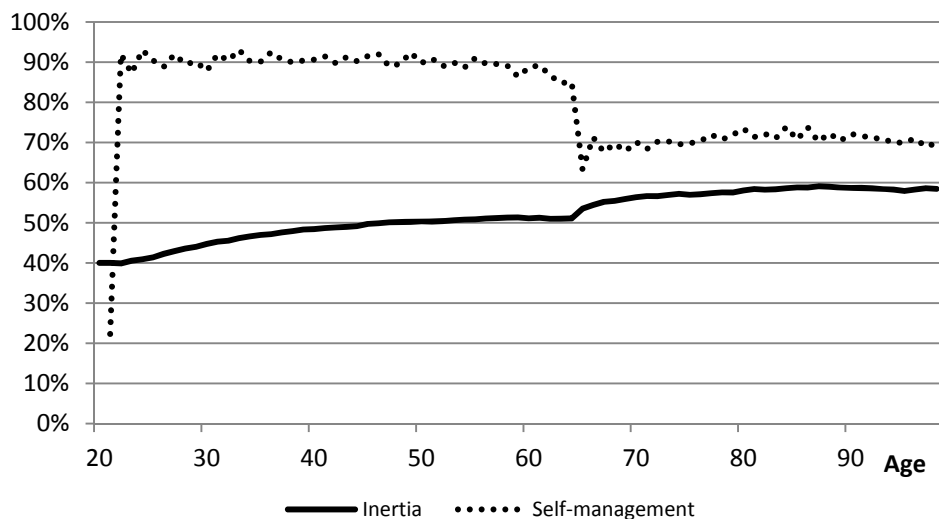
Notes: This figure shows expected life cycle profiles when only self-management or inertia are feasible, generated from 2,000 independent simulations based on optimal feedback controls from the baseline specification of the life cycle model (see Table 1). Panel A displays average consumption and wealth paths; Panel B shows average labor earnings from work or retirement benefits; Panel C shows the average fraction of wealth invested in equities; and Panel D shows average work and human capital profiles.

**Figure 2: Choice of Portfolio Management Method and Equity Exposure Over the Life Cycle: No Delegation Option**

A. Life Cycle Portfolio Management Method: No Delegation Option



B. Average Equity Share over Life Cycle Conditional on Portfolio Management Method

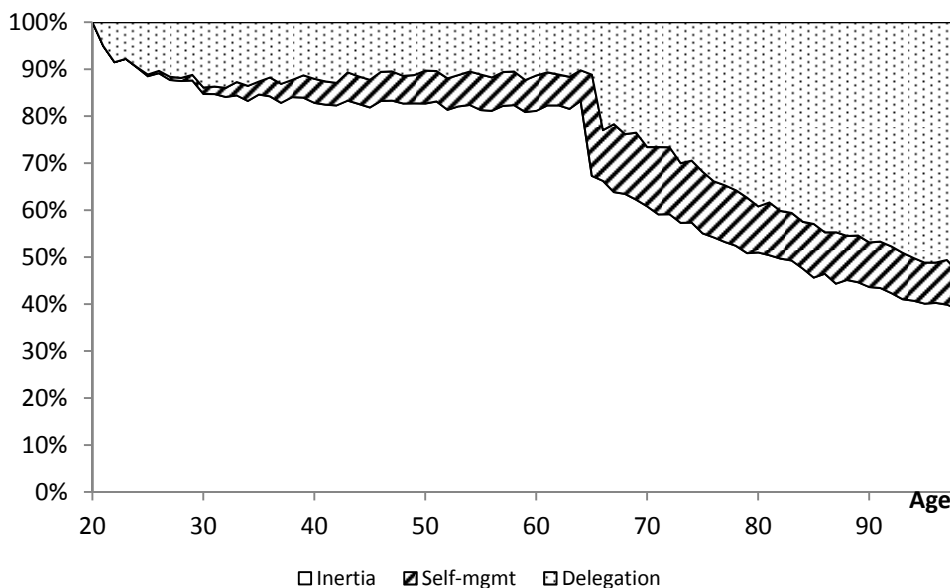


Notes: This Figure shows the life cycle pattern of management method and equity share (when no delegation is feasible). Panel A illustrates the fraction of investors selecting inertia versus self-management by age. Panel B displays the average fraction of financial wealth invested in equities, depending on whether the investor elected inertia or self-management. Averages generated from 2,000 independent simulations for individuals based on optimal feedback controls from the baseline specification of the life cycle model (see Table 1).

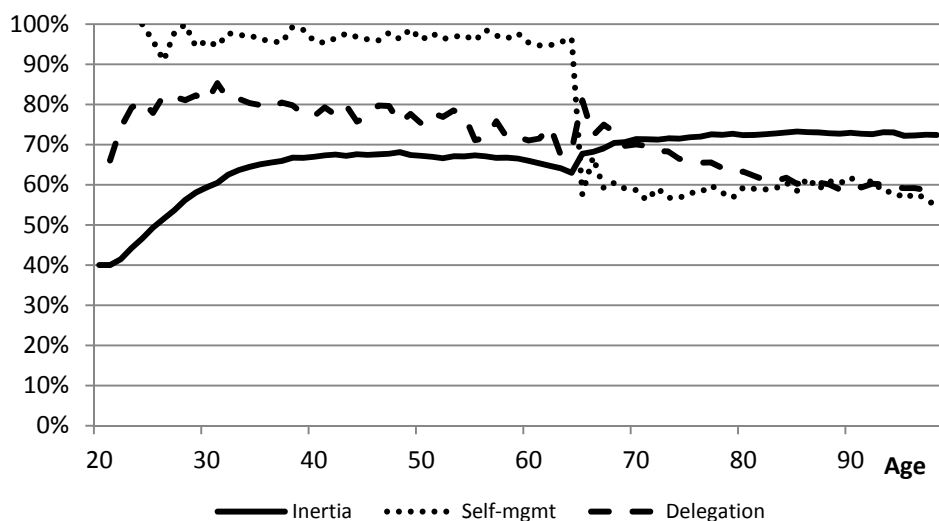


**Figure 3: Choice of Portfolio Management Method and Equity Exposure Over the Life Cycle: With Delegation Option**

A. Life Cycle Portfolio Management Method: With Delegation Option

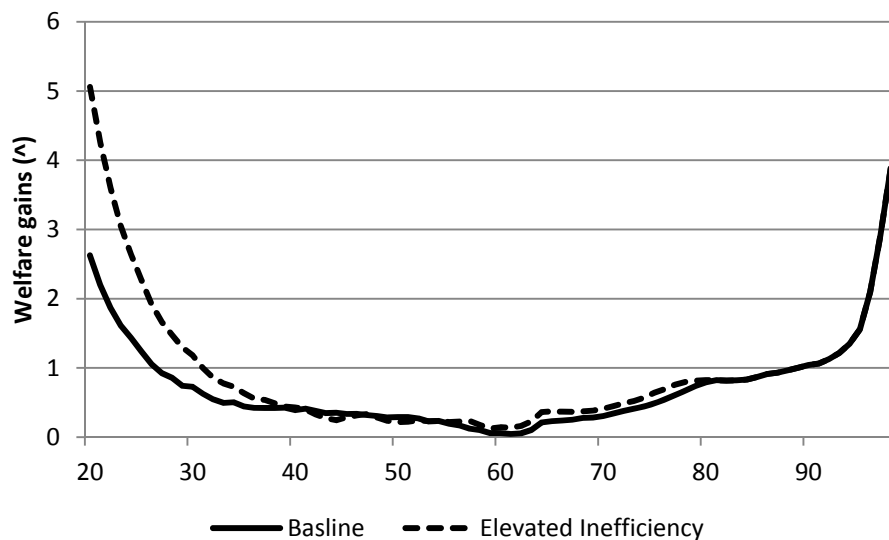


B Average Equity Share over Life-cycle Conditional on Portfolio Management Method



Notes: This Figure shows the life cycle pattern of management method and equity share when a delegation is feasible. Panel A illustrates the fraction of investors selecting inertia, self-management, or delegation by age. Panel B displays the average fraction of financial wealth invested in equities, depending on whether the investor elected inertia, self-management, or delegation. Averages generated from 2,000 independent simulations for individuals based on optimal feedback controls from the baseline specification of the life cycle model (see Table 1).

**Figure 4: Welfare Gains from Delegation Under Two Efficiency Scenarios**



Notes: This Figure plots the pattern of welfare gains over the life cycle, if the investor can delegate portfolio management for a fee. The solid line indicates welfare gains in the base case, while the dashed line refers to welfare gains under reduced financial decisionmaking efficiency (most financially savvy middle-aged with  $\phi_{22} = 0.05$  and young investors with  $\phi_1 = 0.12$ .)

## Appendix A. Sufficient Conditions for an Investor to Elect Portfolio Inertia

Let  $(\hat{l}_t^a, \hat{\pi}_{t+1}^a, \hat{c}_t^a)$  and  $(\hat{l}_t^i, \hat{\pi}_{t+1}^i = \pi_t, \hat{c}_t^i)$  be maximizers of the objective functions of a self-management method and a portfolio inertia method, respectively. Then the following implication holds:

**Proposition.** *For any  $(\hat{l}_t^a, \hat{\pi}_{t+1}^a, \hat{c}_t^a)$  with  $\max\{|\hat{l}_t^a - \hat{l}_t^i|, |\hat{c}_t^a - \hat{c}_t^i|\} < |\hat{\pi}_{t+1}^a - \pi_t|$ , there exists  $\delta^* > 0$  such that  $\forall \hat{\pi}_{t+1}^a$  with  $|\hat{\pi}_{t+1}^a - \pi_t| < \delta^*$  implies  $V_t^i(W_t, H_t, \pi_t, y_t) > V_t^a(W_t, H_t, \pi_t, y_t)$ .*

Proof:

We define the excess value of choosing inertia portfolio over self-management method as:

$$\tilde{V}_t(l_t, \pi_{t+1}, C_t; W_t, H_t, \pi_t, y_t) \equiv V_t^i(W_t, H_t, \pi_t, y_t) - \{U(C_t, 1 - l_t - \phi_t) + \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})]\}$$

The latter part of the equation represents the objective function of self-portfolio management method. Then, the excess value function is

$\tilde{V}_t(\hat{l}_t^i, \hat{\pi}_{t+1}^i, \hat{c}_t^i; W_t, H_t, \pi_t, y_t) = u(\hat{c}_t^i, 1 - \hat{l}_t^i) - u(\hat{c}_t^i, 1 - \phi_t - \hat{l}_t^i) > 0$  because the utility function  $u$  is increasing in leisure time. Since  $\tilde{V}_t(\cdot; W_t, H_t, \pi_t, y_t)$  is continuous in  $(l_t, \pi_{t+1}, C_t)$ , then  $\exists \delta > 0$  such that  $\forall (l_t, \pi_{t+1}, C_t)$  with  $d[(l_t, \pi_{t+1}, C_t), (\hat{l}_t^i, \pi_t, \hat{c}_t^i)] < \delta$ , we have  $\tilde{V}_t(l_t, \pi_{t+1}, C_t; W_t, H_t, \pi_t, y_t) > 0$ . Choose  $\delta^* = \sqrt{\frac{\delta}{3}}$ .

By the assumption that  $\max\{|\hat{l}_t^a - \hat{l}_t^i|, |\hat{c}_t^a - \hat{c}_t^i|\} < |\hat{\pi}_{t+1}^a - \pi_t|$ , the condition  $|\hat{\pi}_{t+1}^a - \pi_t| < \delta^*$  implies:

$$|\hat{l}_t^a - \hat{l}_t^i|^2 + |\hat{\pi}_{t+1}^a - \pi_t|^2 + |\hat{c}_t^a - \hat{c}_t^i| < 3(\delta^*)^2 = \delta$$

Thus,  $V_t^i(W_t, H_t, \pi_t, y_t) - \{u(\hat{c}_t^a, 1 - \hat{l}_t^a - \phi_t) + \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \hat{\pi}_{t+1}^a, y_{t+1})]\} > 0$  and the latter part is now  $V_t^a(W_t, H_t, \pi_t, y_t)$  because  $(\hat{l}_t^a, \hat{\pi}_{t+1}^a, \hat{c}_t^a)$  is the solution of self-management method. So we showed  $V_t^i > V_t^a$  for all  $\hat{\pi}_{t+1}^a$  with  $|\hat{\pi}_{t+1}^a - \pi_t| < \delta^*$ .

QED

Discussion:

If next period's labor and consumption levels resulting from portfolio self-management are very similar to those resulting from inertia, there will be a 'dominant boundary of portfolio inertia' where inertia will be preferred to self-management. In other words, if an investor expects he will end up choosing a similar consumption/labor supply pair next period, a small change in his portfolio will be costly without enhancing his discounted lifetime utility. In such a case, it will then be optimal for him not to alter his portfolio.

## Appendix B. Proof of Existence of a Solution

We use backward induction to show the existence of a solution for an investor's portfolio choice problem without a delegation option. The existence of solution for delegation option can be similarly proved. Using a discrete choice model, we define the value function as

$$V_t(W_t, H_t, \pi_t, y_t) = \max\{V_t^a(W_t, H_t, \pi_t, y_t), V_t^i(W_t, H_t, \pi_t, y_t)\}$$

for all state vectors  $\{(W_t, H_t, \pi_t, y_t)\}_{t=1}^T$ . The superscript  $a$  denotes the portfolio adjustment and  $i$  denotes inaction.

The value functions for each decision are defined as

$$\begin{aligned} V_t^a(W_t, H_t, \pi_t, y_t) &= \max_{\{C_t, \pi_{t+1}, l_t\}} u_t(C_t, L_t) + \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\ &\text{s. t. } C_t \leq W_t + l_t H_t Y_t \\ &\quad W_{t+1} = R_{t+1}^p(W_t + l_t H_t Y_t - C_t) \\ &\quad H_{t+1} = (1 - \delta_t)H_t + F_t(H_t, l_t) \\ &\quad l_t + L_t + \varphi_t = 1 \\ &\quad y_{t+1} = \eta + \rho y_t + \epsilon_{t+1} \end{aligned}$$

and for the inactivity case

$$\begin{aligned} V_t^i(W_t, H_t, \pi_t, y_t) &= \max_{\{C_t, l_t\}} u_t(C_t, L_t) + \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\ &\text{s. t. } C_t \leq W_t + l_t H_t Y_t \\ &\quad W_{t+1} = R_{t+1}^p(W_t + l_t H_t Y_t - C_t) \\ &\quad H_{t+1} = (1 - \delta_t)H_t + F_t(H_t, l_t) \\ &\quad l_t + L_t = 1 \\ &\quad y_t = \eta + \rho y_{t-1} + \epsilon_t \end{aligned}$$

Next we use backward induction:

1. In the last period  $T$ , an investor makes no portfolio decision ( $\pi_{T+1} = \pi_T$ ) and consumes all his wealth.

$$C_T = R_T^p(W_T + l_T^* H_T Y_T)$$

where  $l_T^*$  is determined by static optimal decision between  $l_t$  and  $L_t$  with  $l_t + L_t = 1$  ( $\varphi_t = 0$ ). Now,  $V_T(W_T, H_T, \pi_T, y_T)$  for each state is well defined and we can find  $V_{T-1}^a(\cdot)$  and  $V_{T-1}^i(\cdot)$  using their definitions.

2. With known  $V_{T-1}^a(\cdot)$  and  $V_{T-1}^i(\cdot)$ , we can find  $V_{T-1}(\cdot)$  as

$$V_{T-1}(\cdot) = \max\{V_{T-1}^a(\cdot), V_{T-1}^i(\cdot)\}$$

We know there exists a solution for  $V_{T-1}^a$  and  $V_{T-1}^i$  because the constraint sets are compact and the objective function is continuous [The Weierstrass Theorem].

3. Repeat step 1 and 2 until the first period.
4. After finding the value functions at every period, we can derive policy functions for portfolio adjustment decisions each period.

## Appendix C. Numerical Solution Procedure

Here we describe the procedure for obtaining the numerical solution to the investor's problem.<sup>22</sup> In the last period of life  $T$ , assuming  $V_{T+1} = 0$  and  $a_T = 0$ , the investor maximizes his utility over  $c_T$  and  $l_T$  at every pair of state variables  $(W_T, H_T, \pi_T, y_T)$ . Thus,  $V_T(W_T, H_T, \pi_T, y_T) = \max_{\{c_T, l_T\}} u(C_T, 1 - l_T)$ . This maximization problem is solved by the Nelder-Mead simplex method. We approximate  $\hat{V}_T$  by the polynomial regression of the maximized value  $V_T$  over the pairs of state variable  $(W_T, H_T, \pi_T, y_T)$ . In period  $T-1$ , we calculate  $V_{T-1}^i, V_{T-1}^a, V_{T-1}^d$  using their definitions and the Monte Carlo integration (based on 100 runs) of  $E_{T-1}[\hat{V}_T(W_T, H_T, \pi_T, y_T)]$ , as well as Nelder-Mead optimization over  $(l_{T-1}, \pi_T, c_T)$ . Of course,  $\pi_T = \pi_{T-1}$  in calculating  $V_{T-1}^i$ .

Then we get  $V_{T-1}(W_{T-1}, H_{T-1}, \pi_{T-1}, y_{T-1}) = \max \{V_{T-1}^i, V_{T-1}^a, V_{T-1}^d\}$ . When  $V_{T-1}^i = \max \{V_{T-1}^i, V_{T-1}^a, V_{T-1}^d\}$ , portfolio inertia is optimal. A different choice of management method is similarly derived. We approximate  $\hat{V}_{T-1}$  by the polynomial regression of  $V_{T-1}$  over the pair of state variables  $(W_{T-1}, H_{T-1}, \pi_{T-1}, y_{T-1})$ . Iterating these steps until the first period, we get the approximate value functions  $\{\hat{V}_t\}_{t=1}^T$  which completely characterize the solution to the investor's problem. Last, we generate 2,000 sample paths for individual investors using variations of the wage shock and uncertain stock market returns

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<sup>22</sup> This numerical procedure is implemented with FORTRAN90 and the GNU Gfortran compiler on the Wharton Grid System.