Company Stock and Pension Plan Diversification

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Defined contribution (DC) plans are an important and growing form of private retirement system in the United States,[1] and they are growing increasingly popular in the rest of the world as well (see Walliser, forthcoming and Turner and Rajnes, forthcoming). In many of these plans, the employee's contribution to the plan is matched by the employer, so there is a strong incentive to participate. A notable feature of such plans is that the employee takes charge of his own investment decisions, thereby bearing the risk of fluctuating returns to the chosen investments. By contrast, the defined benefit (DB) plan specifies a promised benefit formula for the employee, and the employer funds the plan and selects its investment portfolio. In this latter case, the risk of DB plan asset returns is borne by company shareholders, together with the Pension Benefit Guaranty Corporation in the U.S. context.

A key feature of a DC plan, often perceived as a plus, is that it empowers the beneficiary to take charge of his retirement planning in accordance with his income and preferences. Nevertheless recent market volatility and corporate bankruptcies have underscored the lack of adequate diversification in the portfolios of many DC plan participants. DC participants with lopsided portfolios, holding a great deal of company stock, have suffered losses when their employers experience financial distress. Sometimes employees hold undiversified positions because the firm’s matching contribution is made in company stock, and this investment may not be altered until they attain a certain age (typically 50 or 55). Even when an employee is permitted to direct his employer’s matching contribution, the data indicate that employees tend to invest substantial amounts in company stock, and hence they are inadequately diversified.[2] Note, however, that there is no insurance currently provided to DC plan participants to protect them against the avoidable decline in their portfolios due to overweighted company stock. By contrast, DB plan sponsors have access to insurance against decline in plan assets, and furthermore they are restricted from owning more than 10 percent own company stock in the plan portfolio (see Mitchell and Utkus, forthcoming).

What causes employees to sometimes ignore the sensible advice that they should diversify, given that they are surely provided such advice from many sources? The fact that this question is hard to answer points out that academics may have failed to educate practitioners and professionals who dispense investment advice to the typical DC plan participant.[3] Finance textbooks, of course, show that dividing a portfolio's wealth equally among an increasing number of (randomly chosen) equities lowers portfolio variance. Furthermore, Modern Portfolio Theory shows that knowledge of the means, variances and covariances can help find a portfolio that minimizes risk at every level of expected return. Surprisingly, these ideas have apparently not been harnessed to assess the diversification level of a typical 401(k) plan participant's portfolio.[4]

This chapter makes two contributions. First, it develops a measure of the diversification level in a DC plan participant's portfolio. This measure computes how much additional risk reduction can be had by reallocating investments among the choices permitted *within* the DC plan, without changing the expected return of the currently chosen portfolio. (Of course, the participant may be constrained from reallocating that part of his portfolio held in company stock.) I call this an “efficiency” measure, related to the closeness to the frontier discussed in Kandel and Stambaugh (1995). It computes the reduction in risk available by moving to the mean-variance efficient frontier, at the participant's chosen level of expected return. It should be emphasized that in the current context, we confine ourselves to examining the diversification of the individual's 401(k) plan assets; thus we ignore the possibility that he might have sizable assets and achieve diversification outside the plan.

Second, the chapter shows that the plan participant can privately avail himself of insurance against the decline in his wealth attributable to his undiversified position within the DC plan. This insurance takes the form of an option contract that gives the recipient the higher of the return to company stock or a diversified (suggestively, index) portfolio over a given future term: the resulting return would be applied to the dollar amount invested in company stock. This insurance gives the participant a rate of return at least as great as on the diversified index, when applied to the amount invested in company stock. Indeed, if the employer so chooses, this insurance can be attached to the matching contribution made in company stock. The cost of the insurance can be borne by (or shared between) the plan participant and the employer, providing thereby the proper incentive to both to realize the benefits of a diversified portfolio. Even if the DC plan participant elects to self-insure and avoid the purchase of this option, the cost of the insurance can serve as a “monetized” version of the diversification measure.

In an early and closely-related study directed at separating the components of *ex post* portfolio performance, Fama (1972: 559) calculates a measure he calls *diversification,* defined as the “extra portfolio return the manager's winners have to produce in order to make concentration of resources in them worthwhile”. This quantity is precisely the difference in return (using an *ex ante* interpretation) that is required to compensate for having to take an undiversified position. Brennan and Torous (1999) have looked at the cost (in terms of loss of certainty equivalence, using specific preference assumptions) to investors in choosing an inadequately diversified position. Meulbroek (2002) evaluates the cost to an employee of the grant of company stock within a DC plan; her assessment of this cost relates to the value that is lost due to the lowered level of diversification, an assessment closely related to Fama's computation of the foregone return. This lost value serves as a measure of a dollar discount to the share price at which the employee would have the same Sharpe-ratio (expected excess return to total portfolio risk, or standard deviation of return) as the market portfolio. The efficiency measure described in the present chapter at the plan participant's chosen level of expected return computes the fraction of total risk that the employee takes, that is rewarded, from the menu of assets within the DC Plan. In contrast to Fama's and Meulbroek's analysis, which is embedded in a capital market equilibrium, I have embedded the problem within the more narrow context of a set of DC plan menu assets.

In what follows, I first show how the measure of diversification efficiency is computed. Next, I show how one can use Margrabe's formula to find the cost of private insurance for a DC plan portfolio that has a fraction allocated to company stock, where I specialize that insurance to apply to the company stock holdings, although it can be regarded in a more general context. A final section concludes.

DC Plans and Portfolio Diversification

A typical participant in a DC plan is permitted to allocate his contribution, as well as his company's matching contribution (if and when that is permitted under plan rules), between at least four to five professionally managed investment alternatives (Mitchell and Schieber, 1998). One of these alternatives may be a money market fund; one may be a bond fund; one of the funds might be a balanced fund, combining equities and bonds in an active mix; and the remaining alternatives tend to be equity funds, of which one might be a low-cost passive vehicle. Employees are typically offered a company stock fund as an investment choice, and especially for larger firms, the company's matching contribution is made in company stock. In some plans, the employer's matching contribution in company stock is not subject to the employee's self-directed asset allocation decision until the worker attains a certain age. It is possible then for the portfolio allocation of the employee's assets within the DC plan to become lopsided and inadequately diversified, and especially so if the employee allocates part of his own contribution towards company stock.

The benefits of diversification are by now very well known. In an early paper, Samuelson (1967) showed that the outcome of diversification ─ equal division of investment among N alternatives, sometimes called naive diversification ─ follows, whenever the joint distribution of returns to securities shows symmetry in the interdependence among them. Finance textbooks usually depict the variance (risk) reduction available from dividing a portfolio's wealth equally among an increasing number of securities, the conclusion being drawn either from a simulation or from portfolios with successively increasing numbers of randomly-chosen equities. Benefits from international diversification are also shown to depend on the strength of correlation between domestic and foreign market indices. Indeed, more sophisticated models for risk decomposition permit examination of the exposure of a chosen portfolio to particular risk factors, to industry sectors, and to investment styles.

Bernheim (1998) notes that the average employee may have other financial assets and private savings elsewhere, outside 401(k) plan assets, but those savings typically will not guarantee an adequate level of post-retirement income. One could argue that even though the employee's 401(k) investments are not diversified, his overall savings may well be, although there is little evidence supporting such conclusion. In addition, as is often observed, the employee's human capital is at least partly dependent on the company's fortunes, which might make a further tilt towards company stock within the DC plan questionable.

Is it possible to design a measure that assesses the level of diversification of an individual's portfolio in the DC plan context? Conventional measures of portfolio diversification rely on numbers of securities within the portfolio and the strength of the average correlation between these securities (Goetzman and Kumar, 2001). Using these might lead an observer to conclude that there is a sufficient number within the participant's portfolio, even though the weighting of one particular security (company stock) might be several multiples of the weight in any other security.

Another measure relates to the familiar mean-variance efficient frontier, which is the locus of all minimum risk (variance of return) positions at different levels of reward (expected return). Given an individual's particular portfolio chosen from his set of opportunities; one may seek a feasible portfolio chosen from the same set with less risk, at the same level of expected return as the chosen portfolio. It would tell investors how “close” their actual portfolios are to an efficient choice. The measure itself is not new, having been proposed by Kandel and Stambaugh (1995) as a measure of closeness to the mean-variance efficient frontier.[5]

In the familiar mean-variance framework, this involves moving from the point (*Z*) representing the individual's portfolio due west to the point (*X*) on the efficient frontier, as shown in Figure 1. Then the ratio of the variance of *X* to that of *Z*

$$\eta_z = \frac{\sigma_X^2}{\sigma_Z^2} \tag{1}$$

represents a measure of how diversified *Z* is relative to *X*. At one extreme, the measure η_z approaches 0: this happens when $\sigma_X^2 >> \sigma_Z^2$, or when the individual's chosen portfolio *Z* is “far” from the frontier and extremely undiversified given the investment alternatives available to him. At the other extreme, η_z is equal to 1.0, when *Z* is on the frontier and coincides with *X*.

Here the individual has chosen an efficient portfolio, and no further reduction in risk is possible at the chosen level of expected return.

Figure 1 here

In the current context, when investment allocations among risky assets within a DC plan are of interest, we can adapt the computation of the frontier in useful ways. First, we make the set of investment choices into which a plan participant allocates his 401(k) wealth as the primitive assets with which the mean variance frontier is generated. One may properly call such a frontier one that embodies “constrained” mean-variance efficiency with respect to the assets in the DC plan menu, which is itself a subset of the very large universe of equity securities and portfolios on offer in the capital markets. As described above, the typical DC plan menu includes at least three to four mutual funds, including a stable value fund and a balanced fund, and some menus include a passive Index fund as well. Of course, the DC plan menus that are of interest to us are those that include company stock as an investment alternative in which the company's contribution or some self-directed allocation is made.

Second, we make a strong but simplifying assumption that enables computation of the constrained mean-variance efficient frontier and the “closeness” measure, *without* requiring knowledge of expected returns to the investment choices within the plan menu. The assumption implies that, in addition to knowledge of the covariance matrix of returns to the menu of assets within the plan, we can identify one asset or one portfolio of plan menu assets that is on the efficient portion of the mean variance frontier. We denote this portfolio as “*S*”, which for example, might be a blend of the S&P500 and style and sector funds; or it could be a passive extended Index fund. We simply require *S* to be a feasible portfolio chosen from the plan's menu. If a plan participant elects a portfolio *Z* from the permitted menu of plan assets (including some company stock, held perhaps involuntarily) one can find the feasible portfolio *X*, also chosen from the plan menu, that is on the mean-variance efficient frontier and that has the same expected return as her chosen portfolio *Z*. The efficiency measure of how well portfolio *Z* is diversified can then be computed. This measure varies between zero and one, where a value of one indicates the ideal ─ an efficiently chosen investment at its level of expected return. This may not be achievable if the plan participant cannot reallocate the company's matching contribution made in company stock.

In order to compute this measure η_z, we must know the location of *Z* and *X*. Essentially, one must know the locus of the mean-variance frontier, given the set of investment opportunities. This requires knowledge of the vector of expected returns and the variance-covariance matrix of the returns and the investment alternatives, and access to a standard optimization program that will trace the frontier. As long as there is a sufficient history of the returns to these investment alternatives, the covariance matrix can be estimated: indeed there are several commercially available risk measurement services that can be used in this context.

Estimation of the vector of expected returns given to the list of available plan assets is more difficult. Assessing the *ex ante* return for company stock requires forecasting future earnings, which is difficult. Worse still, this forecast might be the subject of disagreement when analyzing the influence of company stock contributed into the plan participant's portfolio. Forecasting the expected returns to other investment alternatives within the plan menu is equally difficult, whether we use a top-down or bottom-up approach. A model-based approach requires forecasts of market risk premiums in the context of an equilibrium pricing model.

It is also well known that the average returns computed from historical data (as estimates of expected returns) have low precision, and that their use in constructing mean-variance efficient portfolios can lead to extreme weights. Indeed, in practice, restrictions must be placed to constrain these weights to acceptable levels, or some form of shrinkage must be employed in adjusting the mean vector of returns.

The objective is to compute the η_z measure for any individual's portfolio chosen from a menu of DC plan, restricting the information available to only the covariance matrix of returns and without recourse to a forecast of the expected returns to each of the assets offered in the DC plan menu. This appears clearly impossible in practice, for with that restricted information set, one can only identify the global minimum variance portfolio *V* in Figure 1. It turns out, however, that with one additional assumption, one can compute the η_z measure for any portfolio *Z* chosen from that DC plan menu. This assumption requires that:

(A) *One known portfolio combination of the DC plan assets is on the efficient segment of the minimum variance frontier constructed from the DC plan menu*

Figure 1 indicates the portfolio referred to in Assumption (A) as *S*; we require it to be on the positively-sloped portion (the efficient segment) of the frontier.[6] This ensures that we know another point on the mean-variance frontier, or equivalently one of the portfolios other than *V*, chosen from the DC plan menu, that is on the mean-variance efficient frontier. Notice that one need not forecast the expected returns to portfolio *S* or even portfolio *Z*.

If the plan menu offers a low cost passive index fund, then many may find the assumption that that passive vehicle was chosen as portfolio *S* to be reasonable. If, in addition, the plan menu offers other style or sector funds that provide exposure to value or growth stocks, or international assets, for example, then a predefined and suitable mixture of these assets can be assumed to be point *S* on the frontier.

Assumption (A) is strong: it supplants the need to forecast expected returns to all the assets in the DC plan menu. Because the DC plan sponsor usually chooses from available investment funds to put into the menu of plan choices, it is typically the case that each of these choices is reasonably diversified in terms of its own holdings, on a stand-alone basis. In some instances, where the plan participant has not elected to self-direct an allocation across the plan menu, the employer employs a default allocation with an acceptable diversification level and an acceptable trade-off of risk and return to some risk-averse investor (Choi et al., 2002).[7] Nevertheless, it need not follow that this choice, or a particular combination of available choices, is on the minimum variance frontier. Indeed, individuals with heterogeneous beliefs might disagree as to the values of the expected returns, so that they may not agree that a particular portfolio S is on the efficient frontier.[8]

Computation of the Efficiency Measure

We next demonstrate how assumption (A) permits us to compute our efficiency measure η_z for a given portfolio Z. We define $\tilde{r}_j$ as the return to any asset (or portfolio) j, and Ω as the ($N \times N$ positive definite) covariance matrix of the returns to the $i = 1, 2, \ldots, N$ risky investment alternatives within the DC plan's menu of offerings. Let the N-th investment alternative be company stock. The individual's total wealth in the 401(k) or DC plan is comprised of investments made with his own contributions, as well as matching company contributions. If individual Z elects to direct part or all of those amounts, the vector $w_Z = \{ w_{Z_i}\ i = 1, 2, \ldots N\}$ represents the resulting investment proportions in each of the N investment alternatives, with

$$\sum_{i=1}^{N} w_{Z_i} = 1 \qquad (2)$$

These investment proportions are computed using the aggregate wealth (including all company match contributions) in the plan.

The variance of the return on his portfolio is given by:

$$\sigma_Z^2 = w_Z' \Omega \omega_Z \tag{3}$$

The *global* minimum variance portfolio *V* has an associated vector of investment proportions w_Z which is the solution to

$$\underset{\{w_{V_i}, i=1,2..,N\}}{Min} \quad w_V' \Omega w_V \qquad \text{subject to} \qquad \sum_{i=1}^{N} w_{V_i} = 1 \tag{4}$$

Notice that short sales are permitted, so the resulting global minimum variance portfolio may have some negative weights. Of course, the global minimum variance portfolio wouldn't be optimal for any one unless he is “infinitely” risk-averse. These weights can be computed with the knowledge of the covariance matrix alone, so given this information we can compute the variance of *V* as:

$$\sigma_V^2 = w_V' \Omega w_V \tag{5}$$

To compute the η_Z measure, we need to find the portfolio *X* that is on the frontier with the same mean as the individual's chosen portfolio *Z*. The following three well-known properties of the mean-variance frontier (see Huang and Litzenberger, 1988), in addition to Assumption A, are sufficient to locate the investment weights in portfolio *X*:

Property 1: The investment proportions in any portfolio on the minimum variance frontier are a weighted sum (with weights that sum to unity) of the investment proportions of any two distinct portfolios that are also on the minimum variance frontier.

Property 2: The covariance of the return on any portfolio with the global minimum variance portfolio V is equal to the variance of the global minimum portfolio's return, σ_V^2.

Property 3: The covariance of any portfolio Z with the portfolio X that is on the minimum variance frontier and that has the same expected return as Z is equal to the variance of the return to the frontier portfolio X.

The first property is well-known: it says that the mean-variance frontier is spanned by any two portfolios that are on the frontier.[9] It enables us to identify X as a weighted average of the investment proportions in two portfolios that are known to be on the frontier. In terms of our notation, if portfolios S, X, and V are known to be on the frontier, then there is a number λ such that:

$$\lambda w_{S_i} + (1 - \lambda)\, w_{V_i} = w_{X_i} \qquad \text{for} \qquad i = 1, 2, \ldots N \tag{6}$$

The second and the third properties follow from the fact that the frontier portfolios V and X have minimum variance, so that the separate portfolio combinations of either Z and V or of Z and X, respectively, attain their minimum variances when the weight on Z is set to zero. In other words Properties 2 and 3 state that:

$$\sigma_{ZV} \equiv \mathrm{Cov}(\tilde{r}_Z, \tilde{r}_V) = \sigma_V^2 \text{ and } \sigma_{ZX} \equiv \mathrm{Cov}(\tilde{r}_Z, \tilde{r}_X) = \sigma_X^2 . \tag{7}$$

To find the value of λ we use these properties to compute

$$\mathrm{Cov}(\tilde{r}_Z, \tilde{r}_X) = Cov(\tilde{r}_Z, \lambda\tilde{r}_S + (1-\lambda)\tilde{r}_V) \quad . \tag{8}$$

Using Property 3, the above relation is set equal to

$$\mathrm{Var}(\tilde{r}_X) = \mathrm{Var}(\lambda\tilde{r}_S + (1-\lambda)\tilde{r}_V) \tag{9}$$

The solution for λ from the above two relations is

$$\hat{\lambda} = \frac{\sigma_{SZ} - \sigma_V^2}{\sigma_S^2 - \sigma_V^2}. \tag{10}$$

Given this value for $\hat{\lambda}$ we can show that the efficiency measure η_Z for portfolio *Z* is

$$\eta_Z = \frac{\sigma_X^2}{\sigma_Z^2} = \hat{\lambda}^2 \frac{\sigma_S^2}{\sigma_Z^2} + \{1 - \hat{\lambda}^2\} \frac{\sigma_V^2}{\sigma_Z^2} \tag{11}$$

It is easy to verify that if the investor chooses *S* as her optimal portfolio so that *Z* = *S*, then $\hat{\lambda} = \eta_Z = 1$; and if he chose a portfolio on the mean variance efficient frontier, such as *Z* = *X*, then by construction $\eta_Z = 1$.[10]

It is natural to ask, in this context, whether it is possible to find a portfolio *Y* which has the same variance as *Z* but a higher expected return. That frontier portfolio would be due north of *Z* in the mean-variance diagram, as shown in Figure 1. Then the portfolios in the segment *XY* would be preferred (by the preferences of investors using mean variance analysis) to portfolio *Z*, as they would all offer either a higher mean return or a lower variance, or both, relative to *Z*. In order to compute the investment proportions in portfolio *Y*, we would use Property 1 to find a number γ such that *Y* is a combination of *V* and *S*, satisfying the variance condition

$$\sigma_Z^2 = \mathrm{Var}(\tilde{r}_Y) = \mathrm{Var}\,(\gamma\tilde{r}_S + (1-\gamma)\tilde{r}_V). \tag{12}$$

Solving the above relation for $\hat{\gamma}$,

$$\hat{\gamma} = \sqrt{\frac{\sigma_Z^2 - \sigma_V^2}{\sigma_S^2 - \sigma_V^2}}, \tag{13}$$

so that the weights in portfolio *Y* can be recovered from

$$w_{Yi} = \hat{\gamma} w_{Si} + (1-\hat{\gamma}) w_{Vi}, \qquad i = 1, 2, \ldots N. \tag{14}$$

Note that if $\sigma_Z^2 = \sigma_S^2$ then $\hat{\gamma} = 1$ and $Y = S$.

Portfolios along the segment *XY* on the efficient frontier in Figure 1 will be strictly preferred to portfolio *Z* by all risk-averse plan participants with preferences that are described by the mean and variance of their portfolio returns. It should be emphasized that it is not possible to fix a measure of "closeness" of the chosen portfolio *Z* to portfolio *Y* without having information on the vector of mean returns to the available assets.

A more general analysis that examines the diversification level of a portfolio that is chosen from the larger universe of all capital assets ─ not just the DC plan menu ─ would find that the frontier would offer opportunities for even further risk reduction at every level of expected return; the efficient frontier would be to the left of the frontier constrained to DC plan choice is shown in Figure 1. If we then specify a different portfolio, *S′,* on that frontier, then we could find the westward frontier portfolio, *X′,* and recompute the efficiency measure η_Z in the same way as shown above. Those computed measures would generally be smaller (reflecting the possibly additional risk reduction obtainable from the larger universe of assets) than computed from the constrained frontier. If, in addition, we were to estimate expected returns and risks to the larger universe of assets, and we assumed the existence of a riskless asset, then the frontier would collapse[11] to the familiar Capital Market Line (CML). In this case, we could compute both the reduction in risk to the CML at the same level of expected return as *Z*, and the foregone expected return at the same level of risk as *Z* (Meulbroek, 2002).

The efficiency measure η_Z is equal to the square of the correlation coefficient between portfolios *Z* and *X*; it is therefore the same as the diversification measure corresponding to the R^2 of a market model regression of the returns to portfolio *Z* regressed on a chosen benchmark *X*. Such a measure is discussed in Sharpe (1970). Many performance measurement services (for example, Morningstar) report the R^2 measure in the context of a regression of fund returns on the returns to a chosen market index. Here we have found portfolio *X* from the knowledge of the DC plan menu's choices and designed it to have the same mean as the participant's chosen portfolio *Z*.

Sample Calculations

Suppose now that the 401(k) portfolio *Z* chosen by a plan participant has a fraction w_{ZN} of the portfolio wealth invested in company stock, the *N*-th asset, either by virtue of the company contribution made in locked-up company stock or due to a self-directed contribution. The remaining fraction (1-w_{ZN}) is distributed among the other choices within the plan menu. Then the η_Z measure for his portfolio would indicate the extent to which his portfolio was undiversified.

We can easily compute η_Z for different values of the fraction invested in company stock. For simplicity, assume that the remaining fraction is invested in the mean-variance efficient portfolio *S*; the efficiency numbers therefore correspond to a "best" case, and in practice, participant portfolios are likely to be less efficient at each level of company stock holding. The following parameters are used: $\sigma_V = 0.1$, $\sigma_S = 0.18$, and we use cases with low, average and high risk company whose "market" betas β_N computed with respect to *S* are 0.8, 1.0 and 1.2 respectively.

Table 1 shows values for the efficiency measure η_Z for holdings of company stock from 10 percent to 90 percent. Mitchell & Utkus (forthcoming) report that the fraction of self-directed wealth in 401(k) plans averages nearly 30 percent; when the company match contribution is included, it averages approximately 53 percent. The table shows that for these values of the holdings of company stock the efficiency measures are, in the best case, 0.64 and 0.39

respectively. In some larger firms (Purcell 2002), participants have holdings of company stock as high as 90 percent, and for these portfolios, the η_Z measure is the least in each case.

Table 1 here

It is noteworthy that DB plans are restricted to holding no more than 10 percent in company stock; at that level of holding, and assuming that the balance is in an efficient portfolio, the efficiency measure is in the 90 percent range for the three cases shown in Table 1.

Insurance Against a Decline in Portfolio Wealth due to Company Stock Investment

The diversification measure discussed in the previous section has the potential to be a useful tool, especially to those familiar with Modern Portfolio Theory. To others, it may appear take on the aspect of an amulet, with no easily comprehensible benefit to increasing the diversification level within their portfolios. Nevertheless, it is possible to provide the DC plan participant with a more immediate and tangible measure of his undiversified stance by monetizing this measure into a price.

Suppose the DC plan participant were offered insurance, for a fee, that would give him the return on the better performing of two assets: company stock, or a well-diversified efficient portfolio *S*, both feasible choices within his DC plan menu. This return guarantee would be applied to the dollar value of his chosen investment in company stock, at the expiration of the term of the insurance. This insurance contract is equivalent to providing him with an exchange option, first analyzed by Margrabe (1977). The exchange option here permits the DC plan participant to exchange his ownership of shares in company stock for a fixed number of units of the efficiently-diversified portfolio *S* at the option's expiry date. The right to "swap" his ownership of the company stock into a fixed number of units of the diversified portfolio *S* would be exercised if the value of the latter were greater than the value of the shares invested in company stock, on the final maturity date of the option.

It is possible for competitive market makers to provide this insurance in a more general setting. For example, it is theoretically possible to provide the plan participant insurance against the “bad” outcome that his investment in a portfolio (Z) chosen from the plan menu, including company stock, might decline *relative* to the performance of the more efficient portfolio X, just as in the previous section. This form of insurance has the property that it must be tailor-made to every participant's chosen portfolio. Rather than discuss the private provision of diverse insurance to a heterogeneous pool of investors with different needs, we confine the discussion here to insurance and the related exchange options that apply to company stock and an efficient portfolio as the benchmark. This focuses attention on the main reason a typical plan participant's portfolio becomes prone to substantial and precipitous declines: overweighting in company stock. By making available the option to swap that investment for a diversified alternative, we would provide him with an insured position at the termination of the option ─ in the event that company stock declined ─ providing, thereby, education on the benefits of diversification. Furthermore, a firm that provides a matching contribution in company stock would then recognize the cost incurred in protecting the employee's retirement savings from declines due to the presence of the company stock granted to the employee, at least for period during which the company stock remained locked in the participant's portfolio.[12]

It is well-known that the Margrabe exchange option's value can be found without resorting to investors' aversion to risk or knowledge of expected returns, by using the assumptions and ideas that underlie the Black-Scholes analysis. The exchange option's price depends upon the volatilities of company stock and the efficient portfolio S, and on the correlation between them. In particular, we assume that the employer's matching contribution is \$1,000 in company stock; that the volatilities of the continuously-compounded rate of return on company stock and diversified benchmark portfolio *S* are given by σ and σ_S respectively; that the correlation coefficient between the returns on these assets is given by ρ, and that these assets pay a continuous dividend at rate q and q_S respectively. Then the current (date t) value of the exchange option that permits the participant to exchange the company stock for the future (date *T*) value of \$1,000 initially invested in the diversified benchmark *S* is given by:

$$1{,}000 \exp\{-q_S(T-t)\}N(d_1) - 1{,}000\exp\{-q(T-t)\}N(d_1 - \hat{\sigma}\sqrt{T-t}), \tag{15}$$

where:

$$\hat{\sigma} = \sqrt{\sigma^2 + \sigma_s^2 - 2\rho\sigma\sigma_S}, \qquad d_1 = \frac{(q_S - q + 0.5\hat{\sigma}^2)(T-t)}{\hat{\sigma}(T-t)}, \tag{16}$$

and the function $N(x)$ represents the standard cumulative normal probability evaluated at x. The volatility $\hat{\sigma}$ is the standard deviation of a position that is effectively long \$1 in the benchmark asset *S* and short \$1 in company stock.

By virtue of a self-directed allocation or due to the company's matching contribution, an investor who owns \$1,000 worth of company stock and acquires the exchange option will have a dollar amount at the option's expiry date *T* that guarantees a return that is the greater of the return on company stock or the benchmark asset *S*.

Sample Valuations

Next we compute the cost of such an insurance policy and show that it becomes very expensive for longer terms. We assume typical parameters for the company stock ($\beta = 1$, $\sigma =$ 48 percent), and we further assume that the volatility of the benchmark asset *S* is σ_S = 18 percent. For simplicity, we posit that neither the stock nor the benchmark asset *S* pays dividends. The cost of insurance to obtain the better performing return between company stock and portfolio *S* on every $1,000 invested in company stock turns out to be $178, or 17.8 percent for a one-year term, a cost that will appear prohibitive to most investors.[13] Administration proposals suggest a three-year term over which a company's matching contribution may not be reallocated. If an employee wished to purchase such insurance for three years, the cost would rise to $303, or 30.3 percent.[14] If we were to use stocks with varying volatilities, then Figures 2 and 3 show the cost of the option for one and three year terms, respectively; in these graphs we have retained the β of the company stock at one.

Figures 2 and 3 here

Clearly, an undiversified position can involve a substantial implicit insurance cost. For cases where the employer's matching contribution is made in company stock, the price of the exchange option is equivalent to a cost imposed on the employee (who might otherwise hold an efficient, well-diversified alternative) for the term that the granted stock remains untradeable.

Implementation

A provider of this form of portfolio insurance will typically seek to hedge by buying a number of units in portfolio *S* and shorting a certain number of shares in company stock, both these numbers corresponding to the hedge ratios dictated by Margrabe's formula. In actual implementation, however, it is possible (for example) for the grant of company stock to be coupled with exchange options, in which case no explicit short position must be held. Here the insurance is effected by shifting funds between company stock and the feasible efficient portfolio *S*, such that at the terminal date, the funds are totally in the better performing of the two assets. Of course, such shifting of funds implicitly assumes that the company stock is tradeable.

One way in which the insurance can be effected would be to grant the matching contribution in company stock and in the efficient portfolio *S*, in equal dollar amounts. Then instructions would be given to trade out of the underperforming asset and into the better-performing asset in incremental amounts each period. If such a procedure were implemented with the incremental trades corresponding to the changing hedge ratios in Margrabe's formula, then the portfolio will implicitly replicate the insurance option. Transactions costs in these cases will be sizable, the more volatile the stock and the weaker is the correlation between the stock and the efficient portfolio.

Conclusion

The lack of diversification found in privately-managed DC pension accounts has important ramifications. A wealthy and well-informed investor whose position is not well-diversified can take action quickly to avoid serious declines to his retirement wealth. For an ill-informed investor, whose 401(k) plan represents the bulk of his savings for retirement, the consequences of a badly-diversified position loaded in company stock, especially when part of that stock is frozen, are very grave indeed.

Although much has been done to educate and guide investors, it is still the case that they often end up eliciting badly-diversified positions. In the case of 401(k) portfolios, much of this might be avoidable. A first step in this direction is to design a measure that reveals to an investor how efficiently-chosen his 401(k) portfolio is, on a stand-alone basis, ignoring his non-DC plan wealth. Most measures designed to answer such questions must account for heterogeneous investor preferences, and they therefore rely on estimates about future risks and returns. The measure proposed in this chapter uses standard mean-variance analysis, so it avoids the difficult problem of forecasting the mean returns to investments within the DC plan menu. It does require us to make a strong assumption about the frontier, namely that we know at least one efficient portfolio on it, but this may be an assumption that is more palatable than attempting to obtain agreement on expected returns to company stock and the other choices within the plan.

Companies typically emphasize the incentive effects of stock ownership by their employees (both inside and outside their pension accounts), which has not been addressed in this paper. It is noteworthy, however, that DB plans have stricter diversification rules, and that in the U.S. at least, DB plan participants have access to government-mandated pension insurance. Our research shows that a privately obtainable insurance policy would be very costly, if it were to assure that 401(k) investments in company stock will do at least as well as a diversified position, even in the short term.

I am grateful to Olivia Mitchell and Steve Utkus for their comments, to Ron Stambaugh and Craig MacKinlay for helpful discussions, and to Choong-Tze Chua and Alexander Grantcharov for research assistance.

Table 1. Values of the Diversification Measure η_Z

Company Weighting w_{ZN}	Low Risk $\sigma_N = 0.35, \beta = 0.8$		Average Risk $\sigma_N = 0.48, \beta = 1.0$		High Risk $\sigma_N = 0.60, \beta = 1.2$	
	σ_Z	η_Z	σ_Z	η_Z	σ_Z	η_Z
0.1	0.1793	0.97	0.1855	0.94	0.1920	0.91
0.2	0.1843	0.88	0.2012	0.80	0.2182	0.74
0.3	0.1945	0.76	0.2250	0.64	0.2544	0.56
0.4	0.2093	0.63	0.2546	0.50	0.2968	0.43
0.5	0.2277	0.51	0.2881	0.39	0.3432	0.33
0.6	0.2489	0.41	0.3245	0.31	0.3922	0.27
0.7	0.2723	0.33	0.3628	0.25	0.4429	0.22
0.8	0.2973	0.26	0.4025	0.20	0.4948	0.18
0.9	0.3236	0.21	0.4432	0.16	0.5475	0.15

Note: These computations assume that the DC plan participant has a fraction w_{ZN} of his portfolio *Z*, including the company matching contribution, in company stock, the *N*-th asset; and the balance of the portfolio investment proportion $(1 - w_{ZN})$ in a feasible portfolio *S* which is on the efficient portion of the minimum variance efficient frontier, constructed with the assets within the DC plan menu. The measure η_Z is the ratio of the variance of his portfolio *Z* to the variance of a portfolio *X* that is on the efficient frontier and that has the same expected return as *Z*.

In constructing the table, we assume the standard deviation of the return to the global minimum variance portfolio, $\sigma_V = 0.1$; and the standard deviation of the return to portfolio *S*, $\sigma_S = 0.18$.
Source: Author's computations

Figure 1. Portfolio Efficient Frontier
Source: Author's computations.

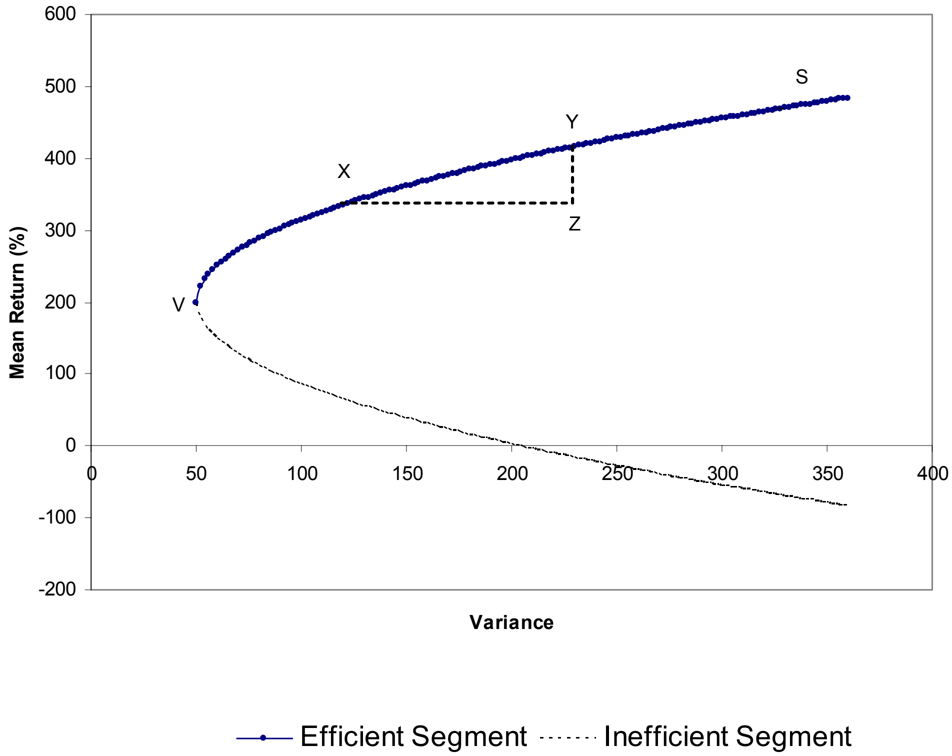


Figure 2. Exchange Option Value (1 year)
Source: Author's computations

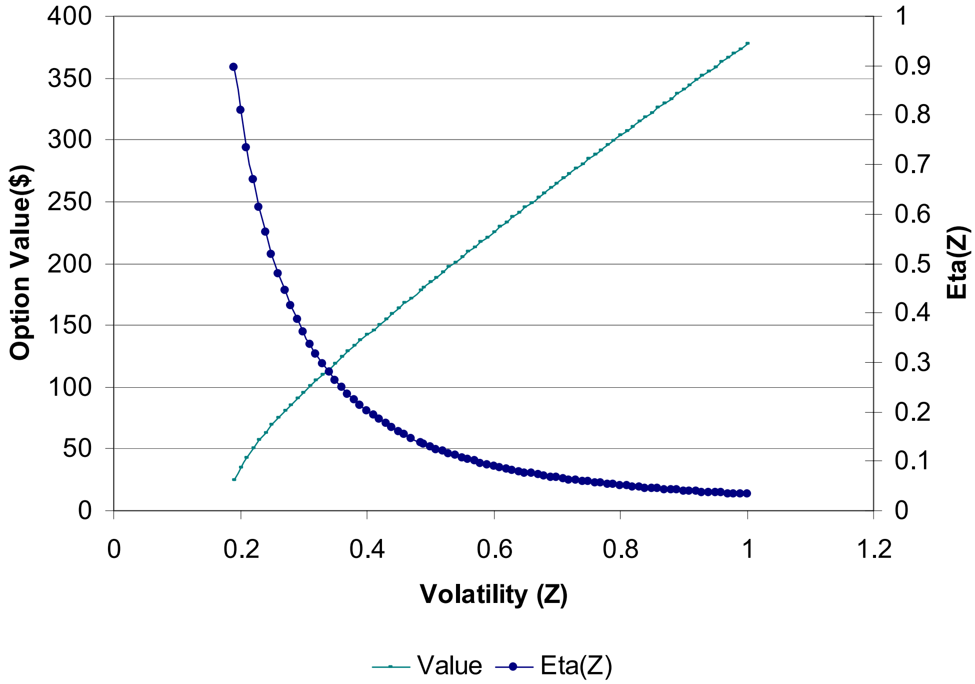
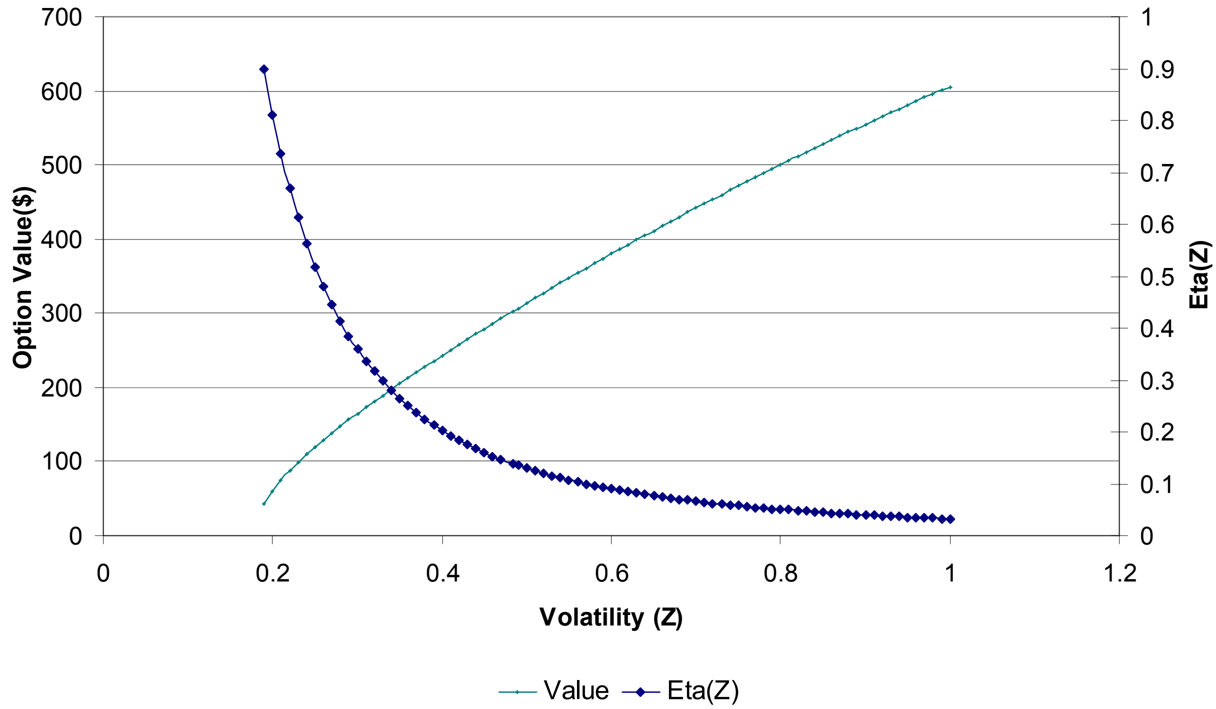


Figure 3. Exchange Option Value (3 year)
Source: Author's computations



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Endnotes

[1] For a comprehensive discussion of these issues, see Mitchell and Schieber (1998).

[2] This finding is not new, for the allocations by retail investors in equity portfolios have been distributed across a handful (typically three to four) stocks. Early evidence on this point is provided by Blume and Friend (1975) and more recent analysis by Goetzman and Kumar (2001).

[3] Several studies have documented the allocation and participation behavior of DC plan participants and suggested explanations; see Benartzi (2001); Benartzi and Thaler (2001); and Choi et al (2001).

[4]Many online services provide a simulation platform where investors may evaluate the impact of various allocation alternatives on retirement wealth (see Bodie, forthcoming). These platforms typically employ forecasts of expected returns to the investment choices (including company stock) which may be the source of substantial difference of opinion when advocating a shift in allocation. This procedure for assessing how efficiently a plan participant has diversified his 401(k) portfolio requires more information than the measure described below.

[5] The connection between mean variance efficiency of a portfolio and the diversification level characterized by the smallness of weights in the portfolio is studied by Green and Hollifield (1992). They show that the existence of a well-diversified portfolio on the frontier depends on a bound that relates expected returns on the portfolio to its covariance with other assets.

[6] The figure is an example drawn with a level of $\sigma_s > \sigma_z$. The arguments in the body of the text are general and apply even when $\sigma_s \leq \sigma_z$.

[7] One could argue that that default allocation sometimes invests in money funds or bonds and that it *ought* to have a larger allocation to equities, but that is not the thrust of the present chapter.

[8] Indeed, it is well known that a test of whether portfolio *S* is on the efficient frontier is equivalent to a test of whether expected returns to assets are linearly related to their betas with respect to *S (*see Fama, 1976; Roll, 1977). Kandel and Stambaugh (1995) show that the closeness of a portfolio to the frontier need not imply a nearly linear relationship between expected returns and betas.

[9] Property 1 in conjunction with Assumption A (and knowledge of the covariance matrix of plan asset returns) says in effect that the investment proportions of all portfolios that are frontier portfolios are known. This means that agreement as to *S* being on the frontier is equivalent to agreement as to the investment proportions of all frontier portfolios, but what we cannot specify is their location as to scale along the *Y*-axis in the traditional mean-variance diagram.

[10] If the employee chooses the global minimum variance portfolio so that $Z = V$ then $\hat{\lambda} = 0$ and $\eta_Z = 1$; but this would be a sub-optimal choice. It is possible that the value of $\hat{\lambda}$ is negative: this occurs only if the investor's chosen portfolio has a lower expected return than the minimum variance portfolio *V*.

[11] Here the plot referred to is that of mean returns *versus* standard deviation of returns.

[12] We can always interpret our computations and Figure 1 to portfolio *Z* in the general case; it is possible to consider the cost of the insurance as applying to either portfolio *Z* or to company stock.

[13] Plan participants and company managers might disagree as to the volatility levels and correlation assumed in computing the cost of the insurance. I have chosen parameter values that are representative; the correlation between company stock and the index portfolio *S* is given by $\rho = \beta \times \frac{\sigma_S}{\sigma} = \frac{0.18}{0.48} = 0.375$ The higher the correlation coefficient, the lower the cost of the insurance, *ceteris paribus.* It should be noted however that even favorable estimates for volatilities and correlations will give prohibitive expensive premiums, as shown in the figures that follow.

[14] Not surprisingly, a 25-year old employee who wanted to buy an insurance policy on company stock that he cannot reallocate until he is 50 years old, would have to pay $739 per $1,000 of stock, retaining the assumption that this is an average beta company stock.