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Abstract

We develop a dynamic portfolio choice model which incorporates anticipated regret and pride in individual's preferences and show that those preferences can cause investors to sell winning stocks and hold on to losing stocks; that is, anticipating regret and pride can help explain the disposition effect.

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1 Introduction

In financial markets, there is a unique phenomenon where investors appear reluctant to realize losses and eager to realize gains; that is, investors seem to have a preference for selling winning stocks too early and holding losing stocks too long. This pattern has been labeled the disposition effect by Shefrin and Statman (1985) and cannot be explained by traditional trading explanations. For instance, Odean (1998) found this effect even after accounting for portfolio rebalancing and trading costs. Similarly, Lakonishok and Smidt (1986) and Ferris et al. (1987) consider trading volume and find that the disposition effect dominates tax-related motives for selling stocks at a loss.

The disposition effect has also been discovered in the Finnish stock market (Grinblatt and Keloharju, 2001), Finnish apartment market (Einio and Puttonen, 2006), and in the sale of residential housing (Genesove and Mayer, 2001). Furthermore, it has been found for professional investors at an Israeli brokerage house (Shapira and Venezia, 2001); although, Dhar and Zhu (2002) find that investors with less trading experience exhibit a stronger disposition effect. Experimental evidence has further supported the disposition effect (Weber and Camerer, 1998; Andreassen, 1988). We refer to Barber and Odean (2005) for a more in-depth review of the disposition effect.

Several explanations for the disposition effect were proposed by Shefrin and Statman (1985), including loss aversion, mental accounting, seeking pride and avoiding regret, and self control. Much of the literature to date on the disposition effect has concentrated on loss aversion, which we explain in further detail below. In this paper, we focus on how anticipating regret and pride in a dynamic setting may cause investors to optimally follow a strategy in which they sell winning stocks and hold losing stocks; that is, we model how anticipating regret and pride can help explain the disposition effect.

As mentioned, loss aversion has been suggested as one explanation for the disposition effect by Shefrin and Statman (1985) and also by several of the empirical papers which document the disposition effect in data (Odean, 1998; Lakonishok and Smidt, 1986; Ferris et al 1987; Grinblatt

and Keloharju, 2001; Shapira and Venezia, 2001; Dhar and Zhu, 2002). Loss aversion as part of prospect theory was proposed by Kahneman and Tversky (1979) and argues that people make decisions considering gains and losses relative to some reference point rather than wealth levels. Individuals who are loss-averse have preferences which are risk-seeking over losses and risk-averse over gains. The intuition behind how loss aversion can explain the disposition effect is that a winning stock is considered a gain, and as individuals are risk-averse in this domain, they will sell the stock. On the other hand, a losing stock would be considered a loss and being risk-seeking in this domain would cause the investor to hold the stock.

Most previous studies that consider the disposition effect are empirical and list loss aversion as an explanation for the effect. More recently, a few papers have formally modeled loss averse preferences in portfolio choice problems. Gomes (2005) finds that the optimal portfolio choice with loss-averse investors would be consistent with the disposition effect. Kyle et al. (2006) examine the liquidation decision of a project with loss-averse preferences and also find optimal behavior that is consistent with the disposition effect. Yet, both papers do not consider the initial decision. That is, the investor is endowed with the stock or the project and the issue of whether a loss-averse investor would even buy the stock or invest in the project initially is not considered. In contrast, Hens and Vlcek (2005) and Barberis and Xiong (2006) take the initial decision into consideration and find that loss aversion cannot explain the disposition effect with short time horizons. The equity premium must be so high for loss averse investors to initially invest in the stock that subsequent optimal behavior is not consistent with the disposition effect. In fact, Barberis and Xiong (2006) show that this often implies momentum trading by the investor, i.e. the opposite of the disposition effect: keeping winning and selling losing stocks. Only when the number of trading periods is large, Barberis and Xiong (2006) show that the equity premium can be sufficiently low to both make the investor willing to invest initially in the stock and exhibit optimal behavior which is consistent with the disposition effect.

Another explanation for the disposition effect suggested by Shefrin and Statman (1985) and examined in this paper is regret and pride, which has recently been supported with experimental

evidence (O'Curry Fogel and Berry, 2006). The idea is that if the stock has gone down one regrets the investment, and in hoping that the stock price will rise in the next period and thereby avoid regret, holds the stock. If the stock has gone up, however, an individual wants to feel pride for having made such a good investment and therefore sells the stock; if he had held it and then the price fell, he would have foregone feeling pride. Wanting to feel pride and delaying regret is what causes investors to realize gains more quickly than losses.

Although the explanation seems intuitive, as it seems with loss aversion, it is not as straightforward to argue that preferences which include regret and pride would give rise to the disposition effect in a dynamic setting. For instance, if the stock rose over one period and the investor sells it, but then the stock rises again over the following period, the investor would feel regret from having sold the stock. Therefore, anticipating regret over both periods, in this instance, could cause the investor to hold the stock after the first period.

As far as we know, no work to date has been done on formally analyzing how preferences with regret and pride could predict the disposition effect. In this paper, such a model is developed. Yet, considering a dynamic setting with regret and pride raises some interesting questions and thus requires certain assumptions to be made. For instance, does the investor experience future regret or pride only for the current investment decision or including all decisions already made in the past? When does the investor experience regret - at the final period or during intermediate periods? In a dynamic setting, some decisions will elicit regret and others pride. How do these feelings interact and compound over time? Furthermore, if the investor does not hold the stock, does he know how it did and can he experience regret then from not holding it if it does well (or pride if it performs poorly)?

In what follows, we will state and explain the assumptions with respect to those questions under which anticipating regret and pride causes individuals to sell stocks that have gained recently and hold stocks that have lost. Therefore, we conclude that the disposition effect can occur if investors experience regret and pride with regard to their investment decisions.

The paper is structured as follows. In the next section, we introduce the model, the as-

assumptions, and preferences that allow individuals to consider regret and pride. In Section 3, we examine the optimal portfolio choice problem and provide necessary and sufficient conditions for the investor's optimal strategy to be consistent with the disposition effect. In Section 4, we discuss the robustness of our assumptions. Finally, we conclude in Section 5.

2 Model and Preferences

Regret is the ex-post feeling of an individual that his ex-ante decision turned out to be suboptimal with respect to the resolved uncertainty; that is, the individual's ex-post level of wealth could have been higher with an foregone alternative decision. Equivalently, pride is the ex-post feeling that the ex-ante decision turned out to be better than some foregone alternative decision. In this setting, an individual makes a decision considering the anticipated disutility or additional utility derived from regret or pride.

Regret theory was initially formulated by Bell (1982) and Loomes and Sugden (1982) and has been shown in both the theoretical and empirical literature to explain individual behavior. Bell (1982) depicted how regret could explain preferences for both insurance and gambling and Braun and Muermann (2004) found that preferences which include regret can explain the preference for low deductibles in personal insurance markets. In a static framework, regret has also been incorporated more recently into asset pricing and portfolio choice models by Muermann et al. (2006) and Gollier and Salanié (2005). We contribute to this literature by considering a portfolio choice problem in a dynamic setting. In addition to the effect that the possibility of intermediate portfolio adjustment has on the portfolio allocation, regret and pride raises some interesting questions with respect to the dynamic nature of those feelings. In the following, we introduce a model that is simple yet rich enough to capture those issues.

There are two assets: a risk-free asset (bond) with a zero normalized return and a risky asset (stock) with a stochastic return \tilde{x}_t per period. We consider only one risky asset to be consistent with the mental accounting framework noted by Thaler (1985) and supported by Gross (1982); the

idea is that decision makers differentiate gambles into separate accounts, applying their preferences to each account, and ignoring the interaction between them. In this manner, investors would view each stock they hold individually and therefore we only consider one here.

We assume that the risky returns are independent and identically distributed across periods and take the two values $x^+ > 0 > x^-$ with probability p and $1 - p$ in each period. The individual is endowed with initial wealth w_0 and can only invest all of his wealth in one of the two assets. There are two periods. At $t = 0$ the investor decides whether to invest his wealth, w_0 , into the stock or bond. At $t = 1$ the investor observes his realized level of wealth, w_1 , and decides again whether to invest it into the stock or bond. At $t = 2$ all assets are liquidated and the investor observes and consumes his final level of wealth, w_2 .

The restriction that the individual cannot split his wealth between the two assets is consistent with the discussion and analysis in Shefrin and Statman (1985) and Odean (1998) which is based on stock trading records of individual investors. Alternative settings include the purchase and sale of an indivisible asset such, e.g. housing, or the investment in and liquidation of a project as in Kyle et al. (2006) who consider such decisions with loss-averse preferences. Our model would thus speak to the empirical evidence of the disposition effect in the real estate market provided by Genesove and Mayer (2001) and Einio and Puttonen (2006).

We follow Bell (1982, 1983) and Loomes and Sugden (1982) by implementing the following two-attribute utility function to incorporate regret and pride in investor's preferences

$$v(w) = u(w) - g(u(w^{alt}) - u(w)). \quad (1)$$

The first attribute represents the risk-aversion of the individual and is characterized by the individual's utility function of actual level of wealth, w . We assume that the utility function $u(\cdot)$ exhibits CRRA preferences, i.e.

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma} \quad (2)$$

for $\gamma \neq 1$ and $u(w) = \ln(w)$ for $\gamma = 1$ where γ is the coefficient of relative risk aversion. This implies that the time horizon has no effect on the optimal portfolio allocation of an individual who does not consider regret and pride in his decision. That is, the individual makes his decision as if he was myopic. Additionally, we assume that stock returns satisfy

$$p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} > 1 \quad (3)$$

for $\gamma \neq 1$ and $(1+x^+)^p(1-x^-)^{1-p} > 1$ for $\gamma = 1$. This assumption implies that the risk premium is high enough such that an individual who does not consider regret and pride finds it optimal to invest in the stock in all periods. Therefore, portfolio rebalancing cannot explain the disposition effect. This allows us to focus on how regret and pride influence the optimal portfolio allocation.

The second attribute represents the individual's feeling of regret or pride towards the "fictitious" level of wealth, w^{alt} , the individual would have obtained from a foregone alternative. If the actual level of wealth, w , falls below the alternative level of wealth, w^{alt} , the individual regrets his decision; otherwise the individual feels pride. The function $g(\cdot)$ measures the amount of regret and pride that the investor experiences and we assume that it is increasing and convex with $g(0) = 0$; that is, the individual weighs the disutility incurred from regret relatively more than the additional utility derived from pride. This assumption is supported in the literature (Thaler, 1980; Kahneman and Tversky, 1982) and has recently found experimental support by Bleichrodt et al., 2006.

We assume that the individual incurs the disutility or additional utility from regret or pride only at the final period. Similar to the assumption that there is no intermediate consumption, we assume that the individual does not incur regret or pride in intermediate periods. The investor thus makes his portfolio choice by maximizing his expected utility of terminal wealth using the value function $v(\cdot)$ given in (1).

We make the following two additional assumptions which turn out to be crucial for predicting that regret and pride causes individuals to behave according to the disposition effect. In Section 4, we will discuss how deviations from these assumptions impact our results.

Assumption 1 The individual only observes the realized stock return if he holds the stock.

This assumption is relevant for regret-averse individuals as foregone alternatives and their resolution can impact decisions. In our setup, it implies that the individual has the option to avoid regret or forego pride by investing in the bond and not observing the realized return of the stock; e.g., by not reading the newspaper. This relates to Bell (1983) who shows that it can be optimal for a regret-averse individual to not have a foregone alternative lottery resolved. In fact, we will show in Section 4 that observing stock returns after selling the stock leads to a lower level of expected utility. This implies that if the individual has the choice to observe stock returns or not then it is optimal in our setting for him not to observe them.

Assumption 2 If the individual's decisions turn out to be ex-post optimal, i.e. they imply the maximum level of wealth with respect to the realized returns, then he experiences pride towards the foregone worst alternative (FWA), i.e. towards the lowest level of wealth he could have obtained with respect to the realized returns. If the individual's choices turn out to be ex-post suboptimal, then he incurs regret towards the foregone best alternative (FBA), i.e. towards the level of wealth he would have obtained from the ex-post optimal choices. We assume the investor feels regret/pride for all past decisions including the current one; that is, the FWA and FBA is derived with respect to all decisions up to and including the current one.

This assumption addresses the issue of how the feeling of regret and pride interact and accumulate over time. A decision rule might turn out to be optimal over the first period but suboptimal over the second period. Here, we assume that the feeling of regret is stronger than pride in the sense that the individual incurs regret as long as one decision turns out to be ex-post sub-optimal. In other words, the individual incurs pride only if all decisions turn out to be ex-post optimal. In that case, we assume that his additional utility from pride is measured in reference to the FWA.

3 Optimal Portfolio Choice and the Disposition Effect

In this section, we examine how an individual who is prone to feelings of regret and pride makes decisions in a dynamic portfolio choice problem. In the first subsection, we investigate the optimal decision at $t = 1$ under the assumption that the individual invested into the stock at $t = 0$. We show that the disposition effect can emerge as the optimal strategy; conditional on a positive stock return over the first period, it is optimal to sell the stock at $t = 1$ and vice versa. We follow Hens and Vlcek (2005) by calling this the “ex-post” disposition effect as it presumes that the individual bought the stock in the first place. In the second subsection, we then solve for the optimal choice at $t = 0$ and show that the “true” disposition effect can emerge as an optimal strategy. That is, it can be optimal for the investor to buy the stock at $t = 0$, and then sell it at $t = 1$ if it went up or hold it if it went down over the first period. Regret and pride can therefore help explain the true disposition effect as opposed to loss aversion which has been shown to only explain the ex-post disposition effect (Hens and Vlcek, 2005, Barberis and Xiong, 2006).

3.1 The Ex-Post Disposition Effect

In this section, we assume that the investor bought the stock at $t = 0$, i.e. his level of wealth at $t = 1$ is given by $\tilde{w}_1 = w_0(1 + \tilde{x}_1)$ which can take the two values $w_1^+ = w_0(1 + x^+) > w_0$ or $w_1^- = w_0(1 + x^-) < w_0$ depending on whether the stock went up or down over the first period. The following proposition determines the condition under which it is optimal for the individual follow the disposition strategy.

Proposition 1 *Suppose the individual bought the stock at $t = 0$. It is then optimal for the individual with $\gamma \neq 1$ at $t = 1$ to sell the stock if it went up and to keep the stock if it went down*

over the first period if and only if stock returns satisfy the following two conditions

$$\begin{aligned} & \frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1 \right) + g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{1-\gamma} \right) \right) \\ < & pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{2(1-\gamma)} \right) \right) + (1-p)g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \end{aligned} \quad (4)$$

and

$$\begin{aligned} & \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1 \right) + g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \\ > & pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) + (1-p)g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)} \right) \right). \end{aligned} \quad (5)$$

For $\gamma = 1$, i.e. $u(w) = \ln(w)$, the conditions are

$$\ln \left((1+x^+)^p (1+x^-)^{1-p} \right) + g(-\ln(1+x^+)) < pg(-2\ln(1+x^+)) + (1-p)g(-\ln(1+x^-)) \quad (6)$$

and

$$\ln \left((1+x^+)^p (1+x^-)^{1-p} \right) + g(-\ln(1+x^-)) > pg(-\ln(1+x^-)) + (1-p)g(-2\ln(1+x^-)). \quad (7)$$

Proof. See Appendix A.1. ■

Condition (4) implies that selling the stock after it went up over the first period is optimal. It depends on the equity premium and the relative strengths of the certain feeling of pride when selling the stock versus the uncertain feelings of additional pride or regret when holding the stock over the second period. Condition (5) assures that keeping the stock after it went down over the first period is optimal. Again, this condition depends on the equity premium and the relative strengths of the certain feeling of regret when selling the stock versus the uncertain feelings of additional or less regret when holding the stock over the second period. Optimal behavior is thus consistent

with the disposition effect if both conditions hold.

We provide intuition for why both conditions can hold by focusing on the effects of the following two interrelated changes on the optimal choice: increasing the equity premium and adding anticipated regret and pride to the decision making, i.e. the second attribute of the utility function $v(\cdot)$ in (1). Considering the first attribute of the utility function, $u(\cdot)$, increasing the equity premium makes keeping the stock at $t = 1$ more attractive, independent of the stock's movement over the first period. For a fixed equity premium, adding the second attribute, $-g(u(w^{alt}) - u(w))$, makes selling the stock at $t = 1$ more attractive, again independent of the stock's movement over the first period. This is due to the convexity of $g(\cdot)$ as the individual prefers a certain level of regret and pride by selling the stock to an uncertain exposure of regret and pride by keeping the stock. In other words, an individual who considers anticipated regret and pride in his decision making requires a higher equity premium for keeping the stock than an individual whose preferences do not include these psychological factors.

The crucial effect that implies different behavioral responses to the stock's movement over the first period arises from the different effects that a marginal increase in the equity premium has on regret and pride, i.e. on the second attribute of the utility function. If the stock went down over the first period, then the individual will only feel regret but no pride as he already made a suboptimal decision at $t = 0$ (see Assumption 2). If the stock went up, however, then the individual will either only feel pride if he sells the stock or expose himself to either regret or more pride if he keeps the stock. Marginally increasing the equity premium will impact the differences in the marginal effects between selling and keeping the stock. If this difference is larger after an up-move of the stock over the first period than after a down-move, then selling the stock would be marginally more attractive after an up-move which can imply optimal consistent with the disposition effect.

To focus on those effects, suppose that $\gamma = 1$, i.e. $u(w) = \ln(w)$, $p = 1/2$, and that we increase the equity premium by marginally increasing the positive return, x^+ , while keeping the negative

return, x^- , fixed.¹ If the stock went down over the first period, there is no effect on the impact of the second attribute as it solely depends on x^- (see lower limit in (7)). Conversely, if the stock went up, the convexity of $g(\cdot)$ implies that the marginal increase in pride when selling the stock is larger than the potential marginal increase in pride when keeping the stock (see (6)). Thus increasing the equity premium makes selling more attractive than buying when the stock has gone up over the first period while having no effect when the stock has gone down. As a result, there arises situation in which anticipated regret and pride induces behavior which is consistent with the disposition effect.

These marginal effects on the psychological factors have to be traded-off against the marginal effect that an increase in the equity premium has on the first attribute, i.e. the increased attractiveness of keeping the stock. Condition (4) implies that the benefit of securing pride at $t = 1$ by selling the stock outweighs the cost and benefit of regret or additional pride and the risk premium when keeping the stock. Condition (5) implies that the risk premium is high enough to compensate the individual for the additional spread in regret incurred when keeping the stock. In Section 3.3, we will show with an illustrative example that both conditions (4) and (5) can be satisfied.

3.2 The True Disposition Effect

In this section, we examine the dynamically optimal behavior of the individual with the preferences specified above. We thus endogenize the decision at $t = 0$ compared to the section above. Let us emphasize again that this proved to be crucial for the attempt to explain the disposition effect by loss aversion. Although loss aversion can explain the ex-post disposition effect (Gomes, 2005, Kyle et al, 2006), it cannot explain the true disposition effect (Hens and Vlcek, 2005, Barberis and Xiong, 2006). In contrast to loss aversion, we show in the following proposition that regret and pride can explain the true disposition effect which is buying the stock at $t = 0$ and behaving according to the ex-post disposition effect at $t = 1$. Furthermore, the necessary and sufficient

¹Equivalent results obtain for $\gamma \neq 1$ and/or increasing the equity premium by marginally increasing x^- while keeping x^+ fixed.

conditions on stock returns for the true disposition effect to hold are equivalent to the necessary and sufficient conditions for the ex-post disposition effect to hold, i.e. conditions (4) and (5).

Proposition 2 *It is optimal for the individual at $t = 0$ to buy the stock and at $t = 1$ to sell the stock if it went up and to keep the stock if it went down over the first period if and only if stock returns satisfy conditions (4) and (5).*

Proof. See Appendix A.2. ■

Therefore, in a dynamic portfolio choice problem, for a certain range of stock returns, i.e. under conditions (4) and (5), it is optimal for an investor who is prone to feeling regret and pride to follow the disposition effect strategy. That is, when the stock value rises, the investor sells the stock and when the stock value decreases, he holds the stock. The range of stock returns for this strategy to be optimal is the same for both the ex-post and true disposition effect. This implies that the individual's behavior is time-consistent. Under conditions (4) and (5), the investor optimally plans at $t = 0$ to follow the disposition effect strategy at $t = 1$ (Proposition 2) and at $t = 1$ optimally executes this strategy (Proposition 1).

3.3 An Illustrative Example

The objective of providing an illustrative example is to show that the set of stock returns that satisfy the necessary and sufficient conditions (4) and (5) is non-empty. Suppose $\gamma = 1$, $p = \frac{1}{2}$ and that the function $g(\cdot)$ is given by $g(x) = \exp(x) - 1$. Then conditions (4) and (5) are equivalent to

$$\frac{-x^-}{(1+x^-)^2} < \ln((1+x^+)(1+x^-)) < \frac{x^{+2} - 2x^+x^- - x^-}{(1+x^+)^2(1+x^-)}.$$

In Figure 1, the lower line represents all level of stock returns $y = x^+$ and $x = x^-$ such that the lower constraint is binding. Analogously, the upper line represents the upper constraint. Thus, for any pair of stock returns (x^+, x^-) that falls between those two lines the individual optimally follows the disposition strategy. Otherwise, for any pair of stock returns (x^+, x^-) that is below

the lower line it is optimal at $t = 1$ to sell the stock independent of the stock's movement over the first period. Equivalently, for any pair of stock returns (x^+, x^-) that is above the upper line it is optimal at $t = 1$ to buy the stock independent of the stock's movement over the first period.

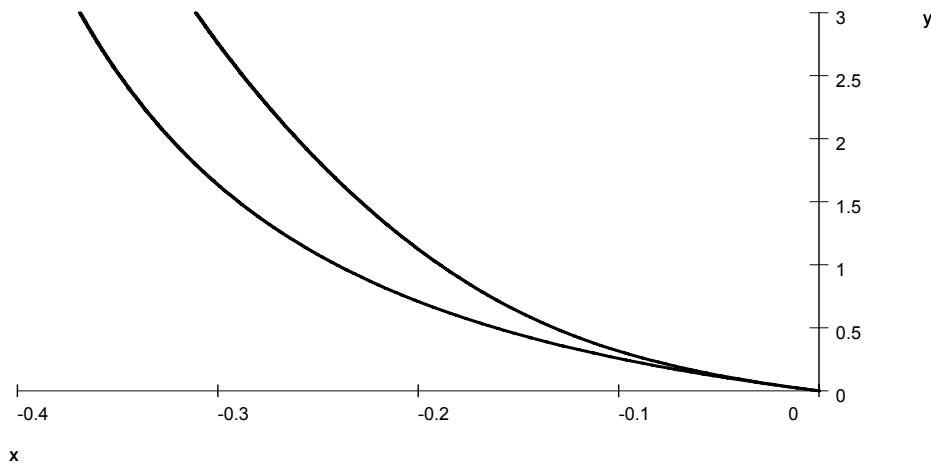


Figure 1: This graph plots for $\gamma = 1$, $p = 1/2$, and $g(x) = \exp(x) - 1$ the constraints on stock returns in conditions (4) and (5) which are necessary and sufficient for the disposition effect to hold.

4 Discussion of Assumptions

In this section, we discuss the importance of the assumptions made to explain the disposition effect and give intuition about why deviations from those assumptions change the predictions. We focus on the ex-post disposition effect as this is a necessary step in explaining the true disposition effect. The ex-post disposition effect would be reinforced by changes in the assumptions that would make selling the stock more attractive after it went up and make holding the stock more attractive after it went down over the first period.

The first assumption considers whether the individual observes stock returns even if he does

not hold it in his portfolio.

Assumption 1 The individual only observes the realized stock return if he holds the stock.

By comparing the levels of expected utility as in the proof of Proposition 1, it can be shown that observing stock returns implies the opposite optimal decision after the stock went up, i.e. it becomes optimal for the individual to keep the stock. This holds for any deviations in Assumption 2 that we discuss below.

The intuition behind this result is as follows. Suppose the individual observes stock returns after selling the stock. Since he will observe the realization of the foregone alternative, he is exposed to a spread in feelings of regret and pride over the next period. As the function $g(\cdot)$ is convex, the individual's level of expected utility is lower when being exposed to this spread compared to the situation in which he does not observe stock returns after selling and is thereby not exposed to this spread. Note that when holding the stock the individual necessarily observes stock returns as they impact his level of wealth. Hence, observing stock returns makes selling less attractive and leads to the opposite optimal decision after the stock went up over the first period, i.e. it is not optimal to follow the disposition strategy.

This also implies that if the individual has the choice to observe stock returns or not, then it is optimal in our setting for him to not observe them and follow the disposition strategy under conditions (4) and (5). This relates to the result of Bell (1983) who shows that it can be optimal for a regret-averse individual, i.e. with a convex function $g(\cdot)$, to not have a foregone alternative lottery resolved.

The second assumption relates to the “reference” level of wealth, w^{alt} , towards which the individual feels regret or pride.

Assumption 2 If the individual's decisions turn out to be ex-post optimal, i.e. they imply the maximum level of wealth with respect to the realized returns, then he experiences pride towards the foregone worst alternative (FWA), i.e. towards the lowest level of wealth he

could have obtained with respect to the realized returns. If the individual's choices turn out to be ex-post suboptimal, then he incurs regret towards the foregone best alternative (FBA), i.e. towards the level of wealth he would have obtained from the ex-post optimal choices. We assume the investor feels regret/pride for all past decisions including the current one; that is, the FWA and FBA is derived with respect to all decisions up to and including the current one.

In a dynamic setting, some decisions will elicit regret and others pride. This raises the interesting question how those feelings interact and aggregate. We assume that the individual only incurs the feelings of pride if he has made choices that are all optimal after the fact. He then feels pride towards the FWA which includes all decisions in the past and the current one. If one decision, either in the past or the current one, is sub-optimal then the individual incurs regret towards the FBA. We discuss the following two deviations from Assumption 2 under both Assumption 1 and its deviation.

First, suppose the individual only considers regret in his decision making but not pride. Sugden (1993) and Quiggin (1994) provide an axiomatic foundation for regret in which the individual's disutility from regret depends only on the actual level of wealth and the level of wealth associated to the FBA. This change in assumption only potentially effects the decision after the stock went up over the first period as only then the individual can incur pride. By comparing the levels of expected utility, it can be shown that by not considering pride, selling the stock becomes relatively less attractive compared to keeping the stock. Furthermore, this effect implies that it is then never optimal to follow the ex-post disposition strategy.

The intuition is that when keeping the stock, the individual only incurs pride if the stock went up over the second period. When selling the stock the individual incurs a certain level of pride (if he does not observe returns) or he incurs pride if the stock goes down over the second period (if he observes returns). In both cases, the convexity of $g(\cdot)$ implies that the ex-ante value of foregone pride is smaller when keeping the stock compared to the ex-ante value when selling the

stock. Note that in the latter case in which the individual observes all stock returns, it is more valuable incurring pride when the stock goes down compared to when it goes up over the second period. Not considering pride makes therefore selling the stock relatively less attractive compared to keeping it.

Second, suppose that past decisions do not matter with respect to the anticipated feeling of regret or pride, i.e. at $t = 1$ the individual only considers the current decision when evaluating those feelings and not his decision at $t = 0$. By comparing the levels of expected utility, it can be shown that by only considering the current decision selling the stock becomes relatively less attractive after it went up but relatively more attractive after it went down over the first period compared to keeping the stock. Furthermore, this effect implies that it is then never optimal to follow the ex-post disposition strategy.

The intuition behind this result is similar to above. After the stock went up over the first period, not considering the pride from the initial decision at $t = 0$ takes relatively more pride away when selling the stock compared to keeping it. As argued above, this is implied by the convexity of $g(\cdot)$. However, after the stock went down, the disutility from regret is larger when keeping the stock compared to selling it. Not considering regret from the initial decision at $t = 0$ thus makes selling relatively more attractive.

We conclude that these deviations from Assumptions 1 and 2 make selling the stock less attractive after it went up and potentially make keeping the stock less attractive after it went down over the first period. Those effects work against the disposition strategy and imply its non-optimality. Assumptions 1 and 2 are thus crucial for explaining the disposition effect with investors' feelings of regret and pride.

5 Conclusion

Prior empirical analyses have shown that trading patterns in capital markets exhibit the disposition effect, and current theoretical work seems to suggest that loss aversion does not explain this effect.

In this paper, we show that investors who feel regret and pride may exhibit trading behavior that is consistent with the disposition effect.

Understanding how regret and pride affect investors' trading behavior and the disposition effect enables us to learn more about the potential "costs" these investors may incur, which is especially relevant for the current debate about introducing Personal Retirement Accounts (PRAs) to the Social Security system. Dhar and Zhu (2002) have shown that investors with less trading experience and/or lower income exhibit a stronger disposition effect, which may lead to lower after tax returns. The introduction of PRAs would thus lead to a much more pronounced disposition effect in capital markets and provides a rationale for policymakers to protect investors with such demographic characteristics. It is therefore important to understand individuals' trading behavior and the factors that affect it, which we do here with regard to regret and pride.

Further extensions include generalizations of the model shown here. Considering multiple time periods and a general probability distribution of stock returns are avenues we aim to explore. Also, it would be interesting to allow the investor to divide his wealth between the stock and bond instead of examining an indivisible asset. Even though those extensions will add other effects, we believe that the basic result of this paper still holds: avoiding regret and seeking pride can help explain the disposition effect.

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A Appendix: Proofs

A.1 Proof of Proposition 1

Suppose the stock went up over the first period such that the individual's level of wealth at $t = 1$ is $w_1^+ = w_0(1 + x^+) > w_0$. If he sells the stock then Assumptions 1 and 2 imply that the individual incurs additional utility at $t = 2$ from pride about his decision at $t = 0$. Note that Assumption 1 implies that the individual does not observe the realization of the stock at $t = 2$ and thereby foregoes potential regret or additional pride over the second period. The FWA would have been to not invest in the stock at $t = 0$ which yields $w^{alt} = w_0$. His final level of utility from selling the stock is thus

$$\frac{(w_0(1 + x^+))^{1-\gamma}}{1-\gamma} - g\left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1 + x^+))^{1-\gamma}}{1-\gamma}\right).$$

If the individual keeps the stock at $t = 1$ he either incurs additional pride if the stock went up again over the second period or regret if it went down. In the first case, Assumption 2 implies that the individual incurs pride towards the FWA which is not to have invested at all, i.e. $w^{alt} = w_0$. In the latter case, the individual made the optimal choice at $t = 0$ but the sub-optimal choice at $t = 1$. Assumption 2 implies that, in aggregate, the individual incurs regret towards the FBA which is to have invested in the stock at $t = 0$ and sold it at $t = 1$ yielding $w^{alt} = w_0(1 + x^+)$. His final level of expected utility is then

$$\begin{aligned} & p \left(\frac{(w_0(1 + x^+))^2)^{1-\gamma}}{1-\gamma} - g\left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1 + x^+))^2)^{1-\gamma}}{1-\gamma}\right) \right) \\ & + (1-p) \left(\frac{(w_0(1 + x^+)(1 + x^-))^{1-\gamma}}{1-\gamma} - g\left(\frac{(w_0(1 + x^+))^{1-\gamma}}{1-\gamma} - \frac{(w_0(1 + x^+)(1 + x^-))^{1-\gamma}}{1-\gamma}\right) \right) \end{aligned}$$

Selling the stock at $t = 1$ is preferred by the individual if and only if

$$\begin{aligned} & \frac{(w_0(1 + x^+))^{1-\gamma}}{1-\gamma} \left(p(1 + x^+)^{1-\gamma} + (1-p)(1 + x^-)^{1-\gamma} - 1 \right) + g\left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1 + x^+)^{1-\gamma} \right)\right) \\ & < pg\left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1 + x^+)^{2(1-\gamma)} \right)\right) + (1-p)g\left(\frac{(w_0(1 + x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1 + x^-)^{1-\gamma} \right)\right). \end{aligned} \quad (8)$$

Now suppose the stock went down over the first period. If the individual sells the stock at $t = 1$ he incurs regret about his decision at $t = 0$ which leads to a final level of utility

$$\frac{(w_0(1 + x^-))^{1-\gamma}}{1-\gamma} - g\left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1 + x^-))^{1-\gamma}}{1-\gamma}\right).$$

If he keeps the stock then Assumption 2 implies that he will incur regret independent of the stock movement over the second periods as he made a sub-optimal choice once at $t = 0$. The level of expected utility is then

$$p \left(\frac{(w_0 (1 + x^+) (1 + x^-))^{1-\gamma}}{1-\gamma} - g \left(\frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^+) (1 + x^-))^{1-\gamma}}{1-\gamma} \right) \right) \\ + (1-p) \left(\frac{(w_0 (1 + x^-)^2)^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^-)^2)^{1-\gamma}}{1-\gamma} \right) \right).$$

Keeping the stock at $t = 1$ is preferred to selling it if and only if

$$\frac{(w_0 (1 + x^-))^{1-\gamma}}{1-\gamma} \left(p (1 + x^+)^{1-\gamma} + (1-p) (1 + x^-)^{1-\gamma} - 1 \right) + g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1 + x^-)^{1-\gamma} \right) \right) \\ > pg \left(\frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1 + x^-)^{1-\gamma} \right) \right) + (1-p) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1 + x^-)^{2(1-\gamma)} \right) \right). \quad (9)$$

Note that the right-hand side of inequality (9) is positive as g is increasing. Both conditions (8) and (9) must be satisfied for the ex-post disposition strategy to be optimal.

A.2 Proof of Proposition 2

Following the true disposition strategy yields a level of expected utility

$$p \left(\frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} \right) \right) \\ + p(1-p) \left(\frac{(w_0 (1 + x^+) (1 + x^-))^{1-\gamma}}{1-\gamma} - g \left(\frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^+) (1 + x^-))^{1-\gamma}}{1-\gamma} \right) \right) \\ + (1-p)^2 \left(\frac{(w_0 (1 + x^-)^2)^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^-)^2)^{1-\gamma}}{1-\gamma} \right) \right).$$

Next, examine all possible other strategies and compare their level of expected utility with the one derived from the true disposition strategy.

1. The individual invests in the stock only once which yields a level of expected utility

$$p \left(\frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^+))^{1-\gamma}}{1-\gamma} \right) \right) \\ + (1-p) \left(\frac{(w_0 (1 + x^-))^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0 (1 + x^-))^{1-\gamma}}{1-\gamma} \right) \right).$$

The true disposition strategy is preferred to this strategy if and only if

$$\begin{aligned} & \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1 \right) + g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \\ > pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) + (1-p)g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)} \right) \right) \end{aligned} \quad (10)$$

which is equivalent to condition (5).

2. The individual invests twice into the stock which yields a level of expected utility

$$\begin{aligned} & p^2 \left(\frac{(w_0(1+x^+)^2)^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1+x^+)^2)^{1-\gamma}}{1-\gamma} \right) \right) \\ & + (1-p)^2 \left(\frac{(w_0(1+x^-)^2)^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1+x^-)^2)^{1-\gamma}}{1-\gamma} \right) \right) \\ & + 2p(1-p) \left(\frac{(w_0(1+x^+)(1+x^-))^{1-\gamma}}{1-\gamma} - g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} - \frac{(w_0(1+x^+)(1+x^-))^{1-\gamma}}{1-\gamma} \right) \right). \end{aligned}$$

The true disposition strategy is preferred to this strategy if and only if

$$\begin{aligned} & \frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1 \right) + g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{1-\gamma} \right) \right) \\ < pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{2(1-\gamma)} \right) \right) + (1-p)g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \end{aligned} \quad (11)$$

which is equivalent to condition (4).

3. The individual does not invest in the stock at all which yields a level of utility

$$\frac{w_0^{1-\gamma}}{1-\gamma}.$$

The true disposition strategy is preferred to this strategy if and only if

$$\begin{aligned} & \frac{w_0^{1-\gamma}}{1-\gamma} \left(1 + (1-p)(1+x^-)^{1-\gamma} \right) \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1 \right) \\ > pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{1-\gamma} \right) \right) + (1-p)^2 g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)} \right) \right) \\ & + p(1-p)g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \end{aligned} \quad (12)$$

which is equivalent to

$$\begin{aligned}
& \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} \left(1 + (1-p)(1+x^-)^{1-\gamma}\right) \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1\right) \\
> & (1+x^-)^{1-\gamma} \cdot \left[pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{1-\gamma}\right) \right) + (1-p)^2 g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)}\right) \right) \right. \\
& \left. + p(1-p) g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \right].
\end{aligned}$$

Condition (10) is equivalent to

$$\begin{aligned}
& \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} \left(1 + (1-p)(1+x^-)^{1-\gamma}\right) \left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} - 1\right) \\
> & \left(1 + (1-p)(1+x^-)^{1-\gamma}\right) \cdot \left[pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \right. \\
& \left. + (1-p) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)}\right) \right) - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \right]
\end{aligned}$$

Condition (10) is thus stronger than condition (12) if

$$\begin{aligned}
& \left(1 + (1-p)(1+x^-)^{1-\gamma}\right) \cdot \left[pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \right. \\
& \left. + (1-p) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)}\right) \right) - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \right] \\
> & (1+x^-)^{1-\gamma} \cdot \left[pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{1-\gamma}\right) \right) + (1-p)^2 g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)}\right) \right) \right. \\
& \left. + p(1-p) g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \right]
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
& pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) + (1-p) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)}\right) \right) \\
& - \left(1 - (1-p)(1+x^-)^{1-\gamma}\right) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma}\right) \right) \\
> & p(1+x^-)^{1-\gamma} g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^+)^{1-\gamma}\right) \right). \tag{13}
\end{aligned}$$

The RHS of this inequality is negative. If $1 - (1-p)(1+x^-)^{1-\gamma} < 0$ then the LHS is positive and the inequality (13) is satisfied. Suppose now that $1 - (1-p)(1+x^-)^{1-\gamma} > 0$. The convexity of g

implies

$$\begin{aligned}
& pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) + (1-p)g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{2(1-\gamma)} \right) \right) \\
& - \left(1 - (1-p)(1+x^-)^{1-\gamma} \right) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \\
> & g \left(\left(p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} + 1-p \right) \left(1 - (1+x^-)^{1-\gamma} \right) \frac{w_0^{1-\gamma}}{1-\gamma} \right) \\
& - \left(1 - (1-p)(1+x^-)^{1-\gamma} \right) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \\
> & g \left(\left(1 - (1-p)(1+x^-)^{1-\gamma} \right) \left(1 - (1+x^-)^{1-\gamma} \right) \frac{w_0^{1-\gamma}}{1-\gamma} \right) \\
& - \left(1 - (1-p)(1+x^-)^{1-\gamma} \right) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right).
\end{aligned}$$

The last inequality follows from the condition

$$p(1+x^+)^{1-\gamma} + (1-p)(1+x^-)^{1-\gamma} > 1$$

and the monotonicity of g . Take $c \geq 1$ such that

$$\frac{1}{c(1 - (1-p)(1+x^-)^{1-\gamma})} < 1.$$

Then the convexity of g yields

$$\begin{aligned}
& g \left(\left(1 - (1-p)(1+x^-)^{1-\gamma} \right) \left(1 - (1+x^-)^{1-\gamma} \right) \frac{w_0^{1-\gamma}}{1-\gamma} \right) \\
& - \left(1 - (1-p)(1+x^-)^{1-\gamma} \right) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \\
> & c \left(1 - (1-p)(1+x^-)^{1-\gamma} \right) \cdot \left[g \left(\frac{1}{c} \left(1 - (1+x^-)^{1-\gamma} \right) \frac{w_0^{1-\gamma}}{1-\gamma} \right) \right. \\
& \left. - \frac{1}{c} g \left(\frac{w_0^{1-\gamma}}{1-\gamma} \left(1 - (1+x^-)^{1-\gamma} \right) \right) \right] \\
> & 0.
\end{aligned}$$

The LHS of inequality (13) is therefore positive and thus satisfied which implies that condition (10) is stronger than condition (12).

4. The individual invests in the stock at $t = 0$ and at $t = 1$ keeps the stock if it went up or sells the stock

if it went down over the first period. This strategy yields a level of expected utility

$$\begin{aligned}
& p^2 \left(\frac{(w_0(1+x^+)^2)^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1+x^+)^2)^{1-\gamma}}{1-\gamma} \right) \right) \\
& + (1-p) \left(\frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} - \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} \right) \right) \\
& + p(1-p) \left(\frac{(w_0(1+x^+)(1+x^-))^{1-\gamma}}{1-\gamma} - g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} - \frac{(w_0(1+x^+)(1+x^-))^{1-\gamma}}{1-\gamma} \right) \right).
\end{aligned}$$

The true disposition strategy is preferred to this strategy if and only if

$$\begin{aligned}
& p \frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} - p^2 \frac{(w_0(1+x^+)^2)^{1-\gamma}}{1-\gamma} \\
& - (1-p) \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} + (1-p)^2 \frac{(w_0(1+x^-)^2)^{1-\gamma}}{1-\gamma} \\
& > pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^+)^{1-\gamma}) \right) - p^2 g \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^+)^{2(1-\gamma)}) \right) \\
& + (1-p)^2 g \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^-)^{2(1-\gamma)}) \right) - (1-p) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^-)^{1-\gamma}) \right) \quad (14)
\end{aligned}$$

Conditions (10) implies

$$\begin{aligned}
& - (1-p) \frac{(w_0(1+x^-))^{1-\gamma}}{1-\gamma} + (1-p)^2 \frac{(w_0(1+x^-)^2)^{1-\gamma}}{1-\gamma} \\
& > (1-p) \cdot \left[pg \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} (1 - (1+x^-)^{1-\gamma}) \right) - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^-)^{1-\gamma}) \right) \right] \\
& + (1-p) g \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^-)^{2(1-\gamma)}) \right) - p \frac{(w_0(1+x^+)(1+x^-))^{1-\gamma}}{1-\gamma}
\end{aligned}$$

and (11) implies

$$\begin{aligned}
& p \frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} - p^2 \frac{(w_0(1+x^+)^2)^{1-\gamma}}{1-\gamma} \\
& > -p \cdot \left[pg \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^+)^{2(1-\gamma)}) \right) - g \left(\frac{w_0^{1-\gamma}}{1-\gamma} (1 - (1+x^+)^{1-\gamma}) \right) \right] \\
& + (1-p) g \left(\frac{(w_0(1+x^+))^{1-\gamma}}{1-\gamma} (1 - (1+x^-)^{1-\gamma}) \right) - (1-p) \frac{(w_0(1+x^+)(1+x^-))^{1-\gamma}}{1-\gamma}
\end{aligned}$$

Combining these two inequalities yields condition (14).

Conditions (10) and (11) are thus necessary and sufficient for the true disposition effect to hold and are equivalent to those for the ex-post disposition effect to hold, i.e. conditions (5) and (4).