

PENSION MATHEMATICS
with Numerical Illustrations
Second Edition

Pension Research Council

Pension Research Council Publications

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PENSION MATHEMATICS **with Numerical Illustrations**

Second Edition

Howard E. Winklevoss, Ph.D., MAAA, EA
President
Winklevoss Consultants, Inc.

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*To my parents,
Marian and Howard Winklevoss,
my wife,
Carol,
and our children,
Amanda, Cameron, and Tyler*

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The Pension Research Council of the Wharton School of the University of Pennsylvania was created in 1952 for the purpose of sponsoring objective research in the area of private pensions. It was formed in response to the urgent need for a better understanding of the private pension movement. Private pensions have experienced a phenomenal growth during the last three decades, but their economic, political, and social implications are yet to be explored. They seem destined to play a major role in the quest for old-age economic security, but the nature of that role can be ascertained only on the basis of more enlightened evaluation of the capabilities and limitations of the private pension mechanism. It was to conduct an impartial study into the facts and basic issues surrounding private pensions, under the auspices of an academic and professional group representing leadership in every phase of the field, that the Council was organized.

Projects undertaken by the Council are broad in scope and predominantly interpretive rather than technical in nature. In general, attention is concentrated on areas which are not the object of special investigation by other research groups. Its research studies are conducted by mature scholars drawn from both the academic and business spheres. Research results are published from time to time in a series of books and monographs.

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Foreword from the First Edition

This book is the culmination of ten years of intensive inquiry by Howard Winklevoss into the intricacies of pension costs and actuarial liabilities. It is a welcome and much needed addition to pension literature.

When I began my pension research activities in the early 1950s, I sought in vain for explanations of pension cost behavior of the type that abound in this book. I suspect that many of these relationships were known, perhaps intuitively, by actuaries actively engaged in pension consulting or the pension operations of life insurance companies. If so, there insights and perceptions were apparently treated as proprietary information, to be confined to internal communications or the inner recesses of their minds. With some notable exceptions, the ideas did not find their way into published pension literature.

To a considerable degree, knowledge of pension cost behavior was constrained by the technology of the day. Pension cost calculations were laborious and many shortcuts and approximations were used. With the development of sophisticated, high-speed computers, it has become feasible to refine pension actuarial techniques and assumptions. In particular, it has become feasible through the construction of computer models to simulate the experience of a pension plan over many years, with a changing population mix and a variety of actuarial assumptions, especially those of an economic nature. The fruit of this improved technology and refined actuarial approaches is a keener perception of pension cost behavior under varied circumstances.

To a large extent this book simply quantifies and provides new or improved actuarial notation for long recognized pension cost concepts and procedures. In certain areas, however, new insights and techniques have been developed. Both types of contributions are useful and innovative. The book should serve the

needs of pension actuaries of all persuasions, pension consultants, management and labor pension specialists, governmental officials with pension responsibilities, students, and others interested in the dynamics of pensions. Dr. Winklevoss is to be commended for having completed this prodigious and scholarly task.

On a more personal note, it has been an intellectually rewarding experience to work with Dr. Winklevoss on this project and other academic undertakings.

Philadelphia, PA
December 1976

Dan M. McGill

Preface

Although this book is entitled *Pension Mathematics with Numerical Illustrations*, Second Edition, it is more like the first edition of a new book. With the exception of the first few chapters, the text is a virtual rewrite of the original book. Two topics have been trimmed back from the first edition, namely, the analysis of ancillary benefits and early retirement. This edition covers all of the relevant material on these topics, but the presentation is more concise. The major additions to the book are chapters on (1) statutory funding requirements, (2) pension accounting, (3) funding policy analysis, (4) asset allocation, and (5) retiree health benefits.

The pension industry has changed considerably since the first edition was published in 1977. At that time, ERISA had just been enacted and was being implemented throughout the pension industry. Just prior to its passage, I was involved in several government-sponsored research assignments on the expected cost of alternative vesting provisions, and the Business Roundtable asked me to share my findings at one of their meetings. After my talk, one of the members cornered me and asked, "Why are you so enthusiastic about the pending pension legislation? Don't you know that once they get the first law passed they'll keep on passing them and eventually do more harm than good?" Little did I know how accurate this statement was.

Many of us had such great expectations for ERISA and, while many of its provisions are clearly beneficial, the avalanche of regulations and new pension legislation since its passage has been terrifying. As an example of its complexity, the reader need only turn to page 156 and glance at the series of equations needed to determine the minimum required contributions to a qualified pension plan. Unless Congress simplifies the various statutory and regulatory requirements for defined benefit pension plans,

and does it quickly, only historians will be interested in this book, for it will provide the mathematics of a subject no longer relevant to corporate America.

Another major change since the first edition was the promulgation of SFAS 87, which sets forth a comprehensive set of procedures to follow in accounting for pension plans. Corporate America vigorously fought against the FASB requiring such a rigid set of accounting rules but has hardly said a word since its implementation in 1987. One of the reasons for this silence may be that many companies have experienced negative pension expense (i.e., pension income) since the implementation of SFAS 87, which no doubt came as a pleasant surprise. This resulted from strong capital markets in the 1980's and the FASB's requirement that the discount rate used in calculating pension expense be tied to the spot rate on long-term corporate bonds. These rates, in recent years, have been uncharacteristically high relative to inflation, a result that no doubt occurred because of the poor performance of fixed-income investments when inflation was high in the early 1980's. In any event, it will be interesting to see the reaction among plan sponsors if long-term rates decrease at a time when the market value of plan assets drop. Complacency with the accounting standard may turn, once again, to vigorous complaints.

The third area of change in the pension field since the mid-1970's is among practitioners. I recall speaking at an actuarial meeting in the late 1970's where I was asked to defend the use of explicit best estimate assumptions. It did not seem like much of an issue because using such assumptions appeared logical. Conservatism, I argued, could be achieved by contributing more to the plan than the amount determined using best estimate assumptions. To my surprise, only a few individuals at the meeting seemed to embrace the use of explicit best estimate assumptions. This has all changed now, with best estimate assumptions being required for both statutory and accounting calculations. In fact, if anything, the assumptions may have become a little too "best estimate." For example, the first edition of this book used a 7 percent interest rate in all of the illustrations, a rate that was on the high side at the time. This edition uses an 8 percent rate, which, ironically, is on the low side. Has the actuarial community, egged on by SFAS 87 as well as the IRS, become a little too aggressive, with 9 and 10 percent rates being common? Plan

sponsors have no doubt enjoyed the effects of using higher interest rates which, in many cases, caused their contributions to be zero and accounting expense to be negative. On the other hand, if future interest rates must be lower than the rates currently used because of sustained low levels of inflation, the adjustment to increased costs may be painful for many corporations.

The author would like to thank all of the individuals who assisted in this edition. I owe special thanks to Dan M. McGill, Emeritus Professor, Wharton School. Nearly 25 years ago Dan hired me to teach at the Wharton School, and my association with him over the ensuing 12 years while I was teaching there was a richly rewarding experience. He gave me considerable encouragement and assistance in completing the first edition as well as the second edition of this book.

Steven R. Strake, senior actuary at Winklevoss Consultants, performed all of the numerical calculations in the book. As readers will soon discover, this represents a huge amount of work for which I am very grateful. This work was performed by Glenn D. Allison in the first edition and the author would like to recognize that contribution in this edition as well. Jing Cheng Liu, Winklevoss Consultants, provided considerable assistance in editing the book, especially with respect to the statutory and regulatory aspects of pension funding.

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Any remaining errors, omissions, or other shortcomings in the book are the sole responsibility of the author. Readers are invited to assist with the next edition by advising the author of any errors and offering suggestions.

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Howard E. Winklevoss