PENSION MATHEMATICS
with Numerical Illustrations

Second Edition

Howard E. Winklevoss, Ph.D., MAAA, EA
President
Winklevoss Consultants, Inc.

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<td>$\bar{a}_{n\rVert}$</td>
<td>Present value of an $n$-year annuity certain, with payments made at the beginning of the year (p. 47).</td>
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<td>Present value, at age $x$, of a life annuity, with payments made at the beginning of the year (p. 46).</td>
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<td>$\bar{a}_{x:n\rVert}$</td>
<td>Present value of an annuity payable until age $n$ or the annuitant's death, whichever occurs first, with payments made at the beginning of the year (p. 47).</td>
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<tr>
<td>$\bar{a}_{x:n\rVert}$</td>
<td>Present value of an $n$-year period certain life annuity, with payments made at the beginning of the year (p. 47).</td>
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<td>$n\bar{a}_x$</td>
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<td>Present value of a joint and survivor annuity, paying $1$ while both annuitants are alive and $k$ to the survivor, with payments made at the beginning of the year (p. 47).</td>
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</table>
$^{k} \dot{a}_{xz} = \text{Present value of a joint life annuity paying $1 per year while the life age } x \text{ is alive and } $k \text{ per year to the life age } y \text{ if the life age } x \text{ dies first (p. 48).}$

$^H B \dot{a}_x = \text{Economic liability for health benefits for retiree age } x \text{ (p. 261).}$

$k^{H B} \dot{a}_x = \text{Deferred health benefits annuity at the point of retirement (p. 261).}$

$^{M C R} \dot{a}_r = \text{Present value of a modified cash refund annuity with lump sum death payment equal to the difference, if any, between the employee's pension contributions and the benefits received at date of death (p. 49).}$

$^{M I R} \dot{a}_r = \text{Present value of a modified installment refund annuity with payments at least until the employee's pension contributions are returned and thereafter until the annuitant's death (p. 49).}$

$\ddot{a}^d_x = \text{Present value, at age } x, \text{ of a life annuity based on disabled-life mortality (p. 116).}$

$\ddot{a}^T_{x:n} = \text{Present value of an } n \text{-year employment-based annuity from age } x, \text{ with payments made at the beginning of the year (p. 51).}$

$s^{T} \ddot{a}_{x:n} = \text{Present value of an } n \text{-year employment-based annuity from age } x, \text{ with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age } x \text{ (p. 52).}$

$r^{T} \ddot{a}_{x:n} = \text{Present value of an } n \text{-year employment-based annuity from age } x, \text{ with payments made at the beginning of the year and assuming multiple retirement ages (p. 133).}$
Present value of an n-year employment-based annuity from age \( x \), with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age \( x \), and assuming multiple retirement ages (p. 133).

\[
\frac{r^{x-n} \cdot r}{a_{x:n}} = \text{present value of an } n\text{-year employment-based annuity from age } x, \text{with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age } x, \text{and assuming multiple retirement ages (p. 133).}
\]

\((ABO)\) = Accumulated benefit obligation for all plan members at time \( t \) (p. 179).

\((ABO)_x\) = Accumulated benefit obligation for employee age \( x \) (p. 178).

\((AC)_k\) = Amortization cost during year \( k \) for prior service cost (p. 189).

\((AFC)_t\) = Additional funding contribution for year \( t \) (p. 151).

\((AFS)_t\) = Average future service of plan participants expected to receive benefits (p. 188).

\((AL)_t\) = Actuarial liability for plan at beginning of year \( t \) (p. 98).

\((AL)_x\) = Actuarial liability under a specified actuarial cost method for individual age \( x \) assumed to retire at age \( r \) (p. 72).

\((AL)_x^{AB}\) = Actuarial liability under the accrued benefit method for individual age \( x \) assumed to retire at age \( r \) (p. 74).

\((AL)_x^{BD}\) = Actuarial liability under the benefit prorate (constant dollar) method for individual age \( x \) assumed to retire at age \( r \) (p. 74).

\((AL)_x^{BD}\) = Actuarial liability under the benefit prorate (constant dollar) method for individual age \( x \) assuming multiple retirement ages (p. 132).
\[ b_x = \text{Benefit accrual during age } x \text{ (p. 40).} \]
\[ b_{x,y} = \text{Benefit accrual during age } x \text{ for an age-}y\text{-entrant (p. 84).} \]
\[ b_x^T = \text{Total benefit allocated during age } x \text{ under benefit-based cost methods that include implicit supplemental costs (pp. 106–108).} \]
\[ CD b_x = \text{Constant dollar benefit accrual during age } x \text{ (p. 42).} \]
\[ CP b_x = \text{Constant percent benefit accrual during age } x \text{ (p. 43).} \]
\[ B_n = \text{Benefits paid to participants during year } n \text{ (p. 98).} \]
\[ B_x = \text{Accrued benefit at beginning of age } x \text{ (p. 40).} \]
\[ B_{r,y} = \text{Accrued benefit at age } r \text{ for an age-}y\text{-entrant (p. 84).} \]
\[ CD B_x = \text{Constant dollar accrued benefit at beginning of age } x \text{ (p. 43).} \]
\[ CE B_t = \text{Current year's benefit payments associated with all contingent events (p. 155).} \]
\[ CP B_x = \text{Constant percent accrued benefit at beginning of age } x \text{ (p. 43).} \]
\[ (BV)_t = \text{Book value of assets at beginning of year } t \text{ (p. 172).} \]
\[ C_r = \text{Accumulated employee contributions at retirement (p. 48).} \]
\[ Dr_{gs} C_x = \text{Expected prescription drug costs at age } x \text{ (p. 255).} \]
\[ BP \text{ } r(\text{AL})_x = \text{Actuarial liability under the benefit prorate (constant percent) method for individual age } x \text{ assumed to retire at age } r \text{ (p. 74)}. \]

\[ BP \text{ } r'(\text{AL})_x = \text{Actuarial liability under the benefit prorate (constant percent) method for individual age } x \text{ assuming multiple retirement ages (p. 132)}. \]

\[ CD \text{ } r(\text{AL})_x = \text{Actuarial liability under the cost prorate (constant dollar) method for individual age } x \text{ assumed to retire at age } r \text{ (p. 75)}. \]

\[ CD \text{ } r'(\text{AL})_x = \text{Actuarial liability under the cost prorate (constant dollar) method for individual age } x \text{ assuming multiple retirement ages (p. 133)}. \]

\[ CD \text{ } r''(\text{AL})_x = \text{Actuarial liability under the cost prorate (constant dollar) method for individual age } x \text{ assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134)}. \]

\[ CD \text{ } T(\text{AL})_x = \text{Actuarial liability, including ancillary benefits, under the cost prorate (constant dollar) method for individual age } x \text{ (p. 121)}. \]

\[ CP \text{ } r(\text{AL})_x = \text{Actuarial liability under the cost prorate (constant percent) method for individual age } x \text{ assumed to retire at age } r \text{ (p. 75)}. \]

\[ (\text{Assets})_t = \text{Plan assets at beginning of year } t \text{ (p. 98)}. \]

\[ (\text{Assets})_x = \text{Plan assets allocated to employee age } x \text{ (p. 110)}. \]

\[ (ATI)_t = \text{After-tax income in year } t \text{ (p. 265)}. \]

\[ (AV)_t = \text{Actuarial value of assets at beginning of year } t \text{ (p. 172)}. \]

\[ '(\text{AVPNC})_x = \text{Accumulated value of past normal costs for employee age } x \text{ (p. 82)}. \]
$b_x = \text{Benefit accrual during age } x \text{ (p. 40).}$

$b_{x,y} = \text{Benefit accrual during age } x \text{ for an age-}y \text{ entrant (p. 84).}$

$b^T_x = \text{Total benefit allocated during age } x \text{ under benefit-based cost methods that include implicit supplemental costs (pp. 106–108).}$

$CD_b_x = \text{Constant dollar benefit accrual during age } x \text{ (p. 42).}$

$CP_b_x = \text{Constant percent benefit accrual during age } x \text{ (p. 43).}$

$B_n = \text{Benefits paid to participants during year } n \text{ (p. 98).}$

$B_x = \text{Accrued benefit at beginning of age } x \text{ (p. 40).}$

$B_{r,y} = \text{Accrued benefit at age } r \text{ for an age-}y \text{ entrant (p. 84).}$

$CDB_x = \text{Constant dollar accrued benefit at beginning of age } x \text{ (p. 43).}$

$CEB_t = \text{Current year's benefit payments associated with all contingent events (p. 155).}$

$CPB_x = \text{Constant percent accrued benefit at beginning of age } x \text{ (p. 43).}$

$(BV)_t = \text{Book value of assets at beginning of year } t \text{ (p. 172).}$

$C_r = \text{Accumulated employee contributions at retirement (p. 48).}$

$D_{rgs}C_x = \text{Expected prescription drug costs at age } x \text{ (p. 255).}$
\[ EE_{C_x} = \text{Employee's expected cost and/or contributions at age } x \text{ (p. 256)}. \]

\[ ER_{C_x} = \text{Employer's expected health benefit cost for employee age } x \text{ (p. 256)}. \]

\[ ER_{iC_x} = \text{Employer's expected health benefit cost at age } x + t \text{ for a retiree currently age } x \text{ (p. 261)}. \]

\[ Hosp_{C_x} = \text{Expected hospital costs at age } x \text{ (p. 255)}. \]

\[ Lab_{C_x} = \text{Expected laboratory costs at age } x \text{ (p. 255)}. \]

\[ Other_{C_x} = \text{Expected other charges at age } x \text{ (e.g., nursing home costs) (p. 255)}. \]

\[ Phy_{C_x} = \text{Expected physician costs at age } x \text{ (p. 255)}. \]

\[ Total_{C_x} = \text{Total expected health benefit costs at age } x \text{ (p. 255)}. \]

\[ QttC_{t-1} = \text{Prior year's quarterly contributions (p. 165)}. \]

\[ ScC_{t-1} = \text{Portion of prior year's total contribution applied to the maximum supplemental cost limits (p. 165)}. \]

\[ ScnC_{t-1} = \text{Portion of prior year's contribution applied to } n\text{th supplemental cost for determining maximum contributions (p. 166)}. \]

\[ (CC)_t = \text{Cumulative contributions from plan inception to year } t \text{ (p. 188)}. \]

\[ (CE)_t = \text{Cumulative expense from plan inception to year } t \text{ (p. 188)}. \]

\[ (CG)_t = \text{Capital gains (or losses), both realized and unrealized, during year } t \text{ (p. 173)}. \]
(CL)_x = Current liability for employee age x (pp. 151-152).

(Cont)_n = Employer contributions during year n (p. 99).

\( E^R (CP)_t \) = Employer copayment fraction (p. 257).

d = Rate of discount (i.e., \( i \nu \)) (p. 99).

\( d_x^{(d)} \) = Number of employees becoming disabled during age x from a service table (p. 34).

\( d_x^{(m)} \) = Number of employees dying during age x from a service table (p. 34).

\( d_x^{(r)} \) = Number of employees retiring at age x from a service table (p. 34).

\( d_x^{(t)} \) = Number of employees terminating during age x from a service table (p. 34).

\( d_x^{(T)} \) = Number of employees leaving service during age x from a service table (p. 32).

\( D_t \) = Employee deductible in year t (p. 257).

\( (DC)_t \) = Tax deductible contributions in year t (p. 265).

\( e_x \) = Curtate life expectancy (i.e., based on whole years only) at age x (p. 46).

\( \Delta^A e_t \) = Error term for asset class in year t (p. 245).

\( \Delta^{inf} e_t \) = Error term for unexpected inflation in year t, (p. 244).

\( E_x^{(k)} \) = Function to determine if employee age x is eligible for benefit-type k (p. 187).
\[ E[(AL)_{n+1}] = \text{Expected actuarial liability at year end, or the beginning of year } n + 1 \text{ (p. 98).} \]

\[ E[(\text{Assets})_{n+1}] = \text{Expected assets at year end, or the beginning of year } n + 1 \text{ prior to any contributions (p. 98).} \]

\[ E(B) = \text{Expected early retirement benefit based on multiple retirement ages (p. 131).} \]

\[ E(B)_t = \text{Expected benefit payments during year } t \text{ (p. 157).} \]

\[ E(C)_t = \text{Expected employer contributions during year } t \text{ ((p. 185).} \]

\[ \text{Min}(EC)_{t+1} = \text{Minimum required contribution payable at the end of year } t \text{ (p. 140).} \]

\[ Q^r(\text{EC})_t = \text{Minimum required quarterly contributions in year } t \text{ (p. 141).} \]

\[ E(I_t) = \text{Expected inflation in year } t \text{ (p. 245).} \]

\[ H_B(EL)_x = \text{Economic liability for health benefits for employee age } x \text{ (p. 261).} \]

\[ (ERB)_x = \text{Function denoting whether the employee is expected to receive benefits (p. 188).} \]

\[ k(ERCR)_x = \text{Early retirement cost ratio: the cost (or liability) of an early retirement benefit to the cost (or liability) of a normal retirement benefit (p. 135).} \]

\[ k(ERCR)_x^* = \text{Early retirement cost ratio with actuarially equivalent early retirement benefits: the cost (or liability) of an early retirement benefit to the cost (or liability) of a normal retirement benefit (p. 135).} \]

\[ (EROA)_t = \text{Expected return on the market-related value of assets (p. 185).} \]
\[ E[(UL)_{n+1}] = \text{Expected unfunded liability at year end, or the beginning of year } n + 1 \text{ prior to any contributions (p. 98).} \]

\[ dF_k = \text{Value of disability benefits payable at age } k \text{ (p. 120).} \]

\[ rF_r = \text{Value of retirement benefits payable at age } r \text{ (p. 120).} \]

\[ sF_k = \text{Value of surviving spouse benefits payable at age } k \text{ (p. 120).} \]

\[ vF_k = \text{Value of vested termination benefits payable at age } k \text{ (p. 120).} \]

\[ AL(FFL)_t = \text{Full funding limit based on the statutory funding method's actuarial liability for year } t \text{ (p. 157).} \]

\[ CL(FFL)_t = \text{Full funding limit based on the current liability for year } t \text{ (p. 159).} \]

\[ QAA(FFL)_{t+1} = \text{Full funding limit based on qualified asset account, applicable at end of year } t \text{ (p. 268).} \]

\[ (FR)_t = \text{Funded ratio equal to actuarial value of assets less the FSA credit balance, all divided by the current liability (p. 154).} \]

\[ (FS)_x = \text{Future service for employee age } x \text{ (p. 187).} \]

\[ (FSA)_t = \text{Funding standard account balance from end of prior year (i.e., at beginning of year } t), \text{ with a credit balance representing a positive value and a funding deficiency representing a negative value (p. 140).} \]

\[ CB(FSA)_t = \text{Credit balance in the funding standard account at beginning of year } t, \text{ i.e., from the end of prior year (p. 153).} \]
$FD_{(FSA)}_t = \text{Funding deficiency in funding standard account at end of year, determined without regard to prior year credit balance and current year contribution (p. 162).}$

$g_x^{(d)} = \text{Grading function equal to the proportion of accrued benefit provided if disability occurs during age } x \text{ (p. 116).}$

$g_x^{(r)} = \text{Grading function equal to the proportion of accrued benefit payable if retirement occurs at the beginning of age } k \text{ (p. 129).}$

$g_k^{(r)} = \text{Grading function which, when applied to the participant's accrued benefit, produces actuarially equivalent benefits (p. 126).}$

$g_x^{(s)} = \text{Grading function equal to the proportion of accrued benefit provided to a surviving spouse if death occurs during age } x \text{ (p. 117).}$

$g_x^{(v)} = \text{Grading function equal to the proportion of accrued benefit vested at age } x \text{ (p. 115).}$

$I = \text{Rate of inflation (p. 38).}$

$I_t = \text{Inflation in year } t \text{ (p. 244).}$

$I_\infty = \text{Long-term rate of inflation (p. 244).}$

$i = \text{Interest rate (p. 34).}$

$i' = \text{Current liability interest rate for statutory funding requirements (p. 158) and expected return on assets for accounting requirements (p. 186).}$

$(IC)_t = \text{Interest cost component of net periodic pension cost (p. 185).}$

$l_x^{(T)} = \text{Number of employees in active service at age } x \text{ from a service table (p. 32).}$
\( I_{x,y} \) = Number of age-\( y \) entrants currently age \( x \) (p. 84).

\( M \) = Probability that the participant has a surviving spouse at death (p. 117).

\( (MC)_t \) = Miscellaneous credits, primarily reflecting full funding limits (p. 162).

\( (MRA)_t \) = Market-related value of assets for year \( t \) (p. 185).

\( (MV)_t \) = Market value of assets at beginning of year \( t \) (p. 172).

\( r'(NC)_x \) = Normal cost under a specified actuarial cost method for individual age \( x \) assumed to retire at age \( r \) (p. 80).

\( (NC)_t \) = Normal cost under statutory funding method, including any implicit supplemental costs, for year \( t \) (p. 140).

\( AAB r'(NC) \) = Normal cost under the aggregate accrued benefit method for all participants, based on retirement at age \( r \) (p. 84).

\( AB r'(NC)_x \) = Normal cost under the accrued benefit method for individual age \( x \) assumed to retire at age \( r \) (p. 84).

\( AB r'(NC)_x \) = Normal cost under the accrued benefit method for individual age \( x \) assuming multiple retirement ages (p. 131).

\( AB *r'(NC)_x \) = Normal cost under the accrued benefit method for individual age \( x \) assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 131).

\( A^B T(NC)_x \) = Normal cost, including ancillary benefits, under the accrued benefit method for individual age \( x \) (p. 120).
$ABD^r(NC) = \text{Normal cost under the aggregate benefit prorate (constant dollar) method for all participants, based on retirement at age } r \text{ (p. 85).}$

$ABP^r(NC) = \text{Normal cost under the aggregate benefit prorate (constant percent) method for all participants, based on retirement at age } r \text{ (p. 86).}$

$ACD^r(NC) = \text{Normal cost under the aggregate cost prorate (constant dollar) method for all participants, based on retirement at age } r \text{ (p. 88).}$

$ACP^r(NC) = \text{Normal cost under the aggregate cost prorate (constant percent) method for all participants, based on retirement at age } r \text{ (p. 88).}$

$BD^r(NC)_x = \text{Normal cost under the benefit prorate (constant dollar) method for individual age } x \text{ assumed to retire at age } r \text{ (p. 85).}$

$BD^r' (NC)_x = \text{Normal cost under the benefit prorate (constant dollar) method for individual age } x \text{ assuming multiple retirement ages (p. 132).}$

$BD^*r(NC)_x = \text{Normal cost under the benefit prorate (constant dollar) method for individual age } x \text{ assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 132).}$

$BD^T(NC)_x = \text{Normal cost, including ancillary benefits, under the benefit prorate (constant dollar) method for individual age } x \text{ (p. 120).}$

$BP^r(NC)_x = \text{Normal cost under the benefit prorate (constant percent) method for individual age } x \text{ assumed to retire at age } r \text{ (p. 85).}$

$BP^r'(NC)_x = \text{Normal cost under the benefit prorate (constant percent) method for individual age } x \text{ assuming multiple retirement ages (p. 132).}$
\[ BP\ T(NC)_x = \] Normal cost, including ancillary benefits, under the benefit prorate (constant percent) method for individual age \( x \) (p. 121).

\[ CD\ r(NC)_y = \] Normal cost under the cost prorate (constant dollar) method for individual age \( x \) assumed to retire at age \( r \) (p. 86).

\[ CD\ r'(NC)_x = \] Normal cost under the cost prorate (constant dollar) method for individual age \( x \) assuming multiple retirement ages (p. 133).

\[ CD\ *r'(NC)_x = \] Normal cost under the cost prorate (constant dollar) method for individual age \( x \) assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

\[ CD\ T(NC)_x = \] Normal cost, including ancillary benefits, under the cost prorate (constant dollar) method for individual age \( x \) (p. 121).

\[ CL(NC)_t = \] Current liability normal cost at beginning of year \( t \) (p. 158).

\[ CP\ r(NC)_x = \] Normal cost under the cost prorate (constant percent) method for individual age \( x \) assumed to retire at age \( r \) (p. 87).

\[ CP\ r'(NC)_x = \] Normal cost under the cost prorate (constant percent) method for individual age \( x \) assuming multiple retirement ages (p. 133).

\[ CP\ *r'(NC)_x = \] Normal cost under the cost prorate (constant percent) method for individual age \( x \) assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

\[ PT(NC)_x = \] Normal cost under the plan termination method for individual age \( x \) assumed to retire at age \( r \) (p. 95).

\[ (NR)_t = \] Nominal return in year \( t \) (p. 245).
\[(NRI)_t = \text{Net realized income in year } t \text{ (p. 265).}\]

\[P = \text{Rate of productivity reflected in salary increases (p. 38).}\]

\[p_x^{(d)} = \text{Probability of an employee not terminating from service from age } x \text{ to age } x + 1, \text{ excluding consideration of other decrements (p. 21).}\]

\[nP_x^{(d)} = \text{Probability of an employee not terminating from service from age } x \text{ to age } x + n, \text{ excluding consideration of other decrements (p. 21).}\]

\[p_x^{(m)} = \text{Probability of a life age } x \text{ living to age } x + 1 \text{ (p. 16).}\]

\[nP_x^{(m)} = \text{Probability of a life age } x \text{ living to age } x + n \text{ (p. 16).}\]

\[d p_x^{(m)} = \text{Probability of a disabled life age } x \text{ living to age } x + 1 \text{ (p. 21).}\]

\[p_x^{(r)} = \text{Probability of an employee retiring at the beginning of age } x, \text{ excluding consideration of other decrements (p. 31).}\]

\[p_x^{(t)} = \text{Probability of an employee not terminating employment from age } x \text{ to age } x + 1, \text{ excluding consideration of other decrements (p. 18).}\]

\[nP_x^{(t)} = \text{Probability of an employee not terminating employment from age } x \text{ to age } x + n, \text{ excluding consideration of other decrements (p. 18).}\]

\[p_x^{(T)} = \text{Probability of an employee surviving in service from age } x \text{ to age } x + 1 \text{ (p. 31).}\]

\[nP_x^{(T)} = \text{Probability of an employee surviving in service from age } x \text{ to age } x + n \text{ (p. 32).}\]
\[ nP_x^{(T)} = \text{Probability of surviving in employment } n \text{ years, where retirement decrements are included with mortality, termination, and disability decrements (p. 129).} \]

\[(PBO)_x = \text{Projected benefit obligation for employee age } x \text{ (p. 181).} \]

\[ AB \cdot (PCL)_x = \text{Plan continuation liability for accrued benefits for employee age } x \text{ (p. 70).} \]

\[(PSC)_x = \text{Prior service cost created at an employee's age } x \text{ (p. 189).} \]

\[(PTL)_x = \text{Plan termination liability for employee age } x \text{ (p. 69).} \]

\[ d(PVFB)_x = \text{Present value of future disability benefits for employee age } x \text{ (p. 117).} \]

\[ HB(PVFB)_x = \text{Present value of future health care benefits for employee age } x \text{ (p. 261).} \]

\[ HB' (PVFB)_x = \text{Present value of future health benefits (without health care inflation) for employee age } x \text{ (p. 267).} \]

\[ r(PVFB)_x = \text{Present value of future benefits for employee age } x \text{ assumed to retire at age } r \text{ (p. 72).} \]

\[ r(PVFB)_{x,y} = \text{Present value of future benefits at age } x \text{ for an age-} \gamma \text{ entrant (p. 84).} \]

\[ r'(PVFB)_x = \text{Present value of future benefits for employee age } x \text{ under multiple retirement ages (p. 129).} \]

\[ r'(PVFB)_x \quad \text{Present value of future benefits for employee age } x \text{ under multiple retirement ages with actuarially equivalent early retirement benefits (p. 130).} \]
\( s(PVFB)_x \) = Present value of future surviving spouse benefits for employee age \( x \) (p. 118).

\( v(PVFB)_x \) = Present value of future vested termination benefits for employee age \( x \) (p. 115).

\( r(PVFNC)_y \) = Present value of future normal costs for employee age \( y \) (p. 80).

\( q_x^{(d)} \) = Disability rate at age \( x \) (p. 21).

\( q_x^{(d)} \) = Probability of decrementing from active service due to disability during age \( x \) (p. 31).

\( q_x^{(m)} \) = Mortality rate at age \( x \) (p. 16).

\( q_x^{(m)} \) = Probability of decrementing from active service due to death during age \( x \) (p. 32).

\( d q_x^{(m)} \) = Mortality rate for a disabled life at age \( x \) (p. 21).

\( q_x^{(r)} \) = Retirement rate at age \( x \) (p. 25).

\( q_x^{(r)} \) = Probability of retiring at the beginning of age \( x \) (p. 31).

\( q_x^{(t)} \) = Termination rate at age \( x \) (p. 18).

\( q_x^{(t)} \) = Probability of decrementing from active service due to termination during age \( x \) (p. 31).

\( q_x^{(T)} \) = Probability of decrementing from active service during age \( x \) (p. 32).

\( (\Delta QAA)_t \) = Addition to a qualified asset account in year \( t \) (p. 265).

\( (QDC)_t \) = Qualified direct cost in year \( t \) (p. 265).

\( r \) = Normal retirement age (p. 5).
\begin{align*}
r' &= \text{First age at which an employee becomes eligible for early retirement (p. 5)}. \\
r'' &= \text{Age by which all employees are assumed to be retired (p. 5)}. \\
(RA)_t &= \text{Reconciliation account balance (p. 163)}. \\
(RR)_t &= \text{Real return in year } t \text{ (p. 245)}. \\
^MCR_x &= \text{Expected reimbursements from Medicare (p. 256)}. \\
^Other R_x &= \text{Expected reimbursements from other governmental programs and/or other private insurance programs (p. 256)}. \\
s_x &= \text{Current dollar salary for a participant age } x \text{ (p. 38)}. \\
s_{x,y} &= \text{Salary at age } x \text{ for an age-y entrant (p. 86)}. \\
S_x &= \text{Cumulative salary from entry age } y \text{ up to, but not including, age } x \text{ (p. 38)}. \\
s_y &= \text{Entry age dollar salary (p. 38)}. \\
S_{r,y} &= \text{Cumulative salary from entry age } y \text{ to retirement age } r \text{ (p. 86)}. \\
\left(s^{(d)}_{11}\right) &= \text{Factor for accumulating quarterly contributions with interest to end of year (p. 141)}. \\
(SS)_x &= \text{Merit salary scale at age } x \text{ (p. 38)}. \\
(\Sigma SC)_t &= \text{Sum of all explicit supplemental costs associated with prior increases or decreases in the unfunded liability that are not yet fully amortized (p. 140)}. \\
(SC)_x &= \text{Service cost for employee age } x \text{ (p. 184)}. 
\end{align*}
\((SC_n)_j\) = jth supplemental cost for the nth unfunded liability increment (p. 101).

\(AB(SC_n)_x\) = Implicit supplemental cost at age \(x\) for \(n\)th unfunded liability change under the accrued benefit method (p. 105).

\(CD(SC_n)_x\) = Implicit supplemental cost at age \(x\) for \(n\)th unfunded liability change under the cost pro-rate (constant dollar) method (p. 109).

\(CE(SC)_t\) = Supplemental costs associated with contingent events (p. 151).

\(CE(SC)_t\) = Supplemental costs associated with amortizing contingent event liabilities over 7 years (p. 155).

\(CP(SC_n)_x\) = Implicit supplemental cost at age \(x\) for \(n\)th unfunded liability change under the cost pro-rate (constant percent) method (p. 109).

\(IPA(SC)_t\) = Sum of all explicit supplemental costs for the initial unfunded actuarial liability and plan amendments that are not yet fully amortized (p. 151).

\(New(SC)_t\) = Supplemental cost associated with the new unfunded current liability as of the current year, excluding any remaining old unfunded current and any unfunded contingent event liabilities (p. 151).

\(Old(SC)_t\) = Supplemental cost associated with old unfunded current liability established in 1988 and amortized over the succeeding 18 years (p. 151).

\(AAB(TC)_t\) = Total cost (normal plus supplemental cost) under the aggregate accrued benefit method with implicit supplemental cost (p. 107).
\[ ABD r(TC)_t = \text{Total cost (normal plus supplemental cost) under the aggregate benefit prorate (constant dollar) method with implicit supplemental cost (p. 108).} \]

\[ ABP r(TC)_t = \text{Total cost (normal plus supplemental cost) under the aggregate benefit prorate (constant percent) method with implicit supplemental cost (p. 108).} \]

\[ ACD r(TC)_t = \text{Total cost (normal plus supplemental cost) under the aggregate cost prorate (constant dollar) method with implicit supplemental cost (p. 111).} \]

\[ ACP r(TC)_t = \text{Total cost (normal plus supplemental cost) under the aggregate cost prorate (constant percent) method with implicit supplemental cost (p. 111).} \]

\[ CD(TC)_x = \text{Total cost (normal plus supplemental cost) under the cost prorate (constant dollar) method with implicit supplemental cost (p. 109).} \]

\[ d(TC)_x = \text{Term cost of disability benefits for an employee age } x \text{ (p. 116).} \]

\[ s(TC)_x = \text{Term cost of surviving spouse benefits for an employee age } x \text{ (p. 117).} \]

\[ v(TC)_x = \text{Term cost of vested termination benefits for an employee age } x \text{ (p. 115).} \]

\[ (TP)_t = \text{Transition percentage, equal to 20 percent for 1993 and increasing by 10 percent per year to 100 percent by 2001 (p. 155).} \]

\[ Tx = \text{Unrelated business income tax rate (p. 266).} \]

\[ u = \text{Number of years (positive or negative) that, when added to the participant's age, yields an assumed age for the surviving spouse (p. 117).} \]
(UCL)_t = Unfunded current liability during year t (p. 152).

(UL)_t = Unfunded liability for plan at beginning of year t (p. 98).

(Δ_nUL) = Unfunded liability (positive or negative) developed during year n (p. 98).

(Δ_nUL)_x = nth unfunded liability developed during a prior year (p. 101).

CE(UL)_t = Unfunded liability associated with prior contingent events (p. 154).

Old(UL)_t = Balance of unfunded old current liability (p. 153).

(ULB)_t = Unamortized liability balance in year t (p. 162).

(Δ_nULB)_j = Unfunded liability balance at the beginning of the jth year for the nth unfunded liability change (p. 101).

v^n = Present value of one dollar due in n years at an annual compound rate of interest equal to i (p. 36).

w = Waiting period before disability benefits commence (p. 116) and serial correlation of successive year's inflation (p. 244).

FD(Waiver)_t = Funding deficiency waiver (p. 162).

x = Employee's attained age.

y = Employee's entry age.

(Yrs)_n = Amortization years remaining for the nth supplemental cost (p. 167).
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