PENSION MATHEMATICS
with Numerical Illustrations

Second Edition

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Glossary of Mathematical Notation

Notation		Definition
$\ddot{a}_{\overline{n}\rceil}$	=	Present value of an n-year annuity certain, with payments made at the beginning of the year (p. 47).
${}^{s}\ddot{a}_{\overline{m}\rceil}$	=	m-year period certain annuity, with payments increasing by the inflation and productivity components of the salary assumption (p. 102).
$\ddot{a}_x$	=	Present value, at age x, of a life annuity, with payments made at the beginning of the year (p. 46).
$\ddot{a}_{x:\overline{n}\rceil}$	=	Present value of an annuity payable until age n or the annuitant's death, whichever occurs first, with payments made at the beginning of the year (p. 47).
$\ddot{a}_{\overline{x:\overline{n}\rceil}}$	=	Present value of an n-year period certain life annuity, with payments made at the beginning of the year (p. 47).
${}_{n\mid}\ddot{a}_x$	=	Present value of an annuity payable for life, with the first payment deferred n years (p. 47).
${}^{k}\ddot{a}_{xy}$	=	Present value of a joint and survivor annuity, paying \$1 while both annuitants are alive and \$$k$ to the survivor, with payments made at the beginning of the year (p. 47).

${}^{k}\ddot{a}_{\underset{1}{xz}}$ = Present value of a joint life annuity paying \$1 per year while the life age x is alive and \$$k$ per year to the life age y if the life age x dies first (p. 48).

${}^{HB}a_x$ = Economic liability for health benefits for retiree age x (p. 261).

${}_{k-x|}^{HB}a_x$ = Deferred health benefits annuity at the point of retirement (p. 261).

${}^{MCR}\ddot{a}_r$ = Present value of a modified cash refund annuity with lump sum death payment equal to the difference, if any, between the employee's pension contributions and the benefits received at date of death (p. 49).

${}^{MIR}\ddot{a}_r$ = Present value of a modified installment refund annuity with payments at least until the employee's pension contributions are returned and thereafter until the annuitant's death (p. 49).

$\ddot{a}_x^d$ = Present value, at age x, of a life annuity based on disabled-life mortality (p. 116).

$\ddot{a}_{x:\overline{n}|}^T$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year (p. 51).

${}^{s}\ddot{a}_{x:\overline{n}|}^T$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age x (p. 52).

${}^{r'}\ddot{a}_{x:\overline{n}|}^T$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year and assuming multiple retirement ages (p. 133).

${}^{r's}\ddot{a}^{T}_{x:\overline{n}|}$ = Present value of an n-year employment-based annuity from age x, with payments made at the beginning of the year equal in value to the employee's attained age salary, based on a unit salary at age x, and assuming multiple retirement ages (p. 133).

$(ABO)_t$ = Accumulated benefit obligation for all plan members at time t (p. 179).

$(ABO)_x$ = Accumulated benefit obligation for employee age x (p. 178).

$(AC)_k$ = Amortization cost during year k for prior service cost (p. 189).

$(AFC)_t$ = Additional funding contribution for year t (p. 151).

$(AFS)_t$ = Average future service of plan participants expected to receive benefits (p. 188).

$(AL)_t$ = Actuarial liability for plan at beginning of year t (p. 98).

${}^{r}(AL)_x$ = Actuarial liability under a specified actuarial cost method for individual age x assumed to retire at age r (p. 72).

${}^{AB\,r}(AL)_x$ = Actuarial liability under the accrued benefit method for individual age x assumed to retire at age r (p. 74).

${}^{BD\,r}(AL)_x$ = Actuarial liability under the benefit prorate (constant dollar) method for individual age x assumed to retire at age r (p. 74).

${}^{BD\,r'}(AL)_x$ = Actuarial liability under the benefit prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 132).

b_x = Benefit accrual during age x (p. 40).

$b_{x,y}$ = Benefit accrual during age x for an age-y entrant (p. 84).

b_x^T = Total benefit allocated during age x under benefit-based cost methods that include implicit supplemental costs (pp. 106–108).

${}^{CD}b_x$ = Constant dollar benefit accrual during age x (p. 42).

${}^{CP}b_x$ = Constant percent benefit accrual during age x (p. 43).

B_n = Benefits paid to participants during year n (p. 98).

B_x = Accrued benefit at beginning of age x (p. 40).

$B_{r,y}$ = Accrued benefit at age r for an age-y entrant (p. 84).

${}^{CD}B_x$ = Constant dollar accrued benefit at beginning of age x (p. 43).

${}^{CE}B_t$ = Current year's benefit payments associated with all contingent events (p. 155).

${}^{CP}B_x$ = Constant percent accrued benefit at beginning of age x (p. 43).

$(BV)_t$ = Book value of assets at beginning of year t (p. 172).

C_r = Accumulated employee contributions at retirement (p. 48).

${}^{Drgs}C_x$ = Expected prescription drug costs at age x (p. 255).

${}^{BP\,r}(AL)_x$ = Actuarial liability under the benefit prorate (constant percent) method for individual age x assumed to retire at age r (p. 74).

${}^{BP\,r'}(AL)_x$ = Actuarial liability under the benefit prorate (constant percent) method for individual age x assuming multiple retirement ages (p. 132).

${}^{CD\,r}(AL)_x$ = Actuarial liability under the cost prorate (constant dollar) method for individual age x assumed to retire at age r (p. 75).

${}^{CD\,r'}(AL)_x$ = Actuarial liability under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 133).

${}^{CD\,*r'}(AL)_x$ = Actuarial liability under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

${}^{CD\,T}(AL)_x$ = Actuarial liability, including ancillary benefits, under the cost prorate (constant dollar) method for individual age x (p. 121).

${}^{CP\,r}(AL)_x$ = Actuarial liability under the cost prorate (constant percent) method for individual age x assumed to retire at age r (p. 75).

$(Assets)_t$ = Plan assets at beginning of year t (p. 98).

$(Assets)_x$ = Plan assets allocated to employee age x (p. 110).

$(ATI)_t$ = After-tax income in year t (p. 265).

$(AV)_t$ = Actuarial value of assets at beginning of year t (p. 172).

${}^{r}(AVPNC)_x$ = Accumulated value of past normal costs for employee age x (p. 82).

b_x = Benefit accrual during age x (p. 40).

$b_{x,y}$ = Benefit accrual during age x for an age-y entrant (p. 84).

b_x^T = Total benefit allocated during age x under benefit-based cost methods that include implicit supplemental costs (pp. 106–108).

${}^{CD}b_x$ = Constant dollar benefit accrual during age x (p. 42).

${}^{CP}b_x$ = Constant percent benefit accrual during age x (p. 43).

B_n = Benefits paid to participants during year n (p. 98).

B_x = Accrued benefit at beginning of age x (p. 40).

$B_{r,y}$ = Accrued benefit at age r for an age-y entrant (p. 84).

${}^{CD}B_x$ = Constant dollar accrued benefit at beginning of age x (p. 43).

${}^{CE}B_t$ = Current year's benefit payments associated with all contingent events (p. 155).

${}^{CP}B_x$ = Constant percent accrued benefit at beginning of age x (p. 43).

$(BV)_t$ = Book value of assets at beginning of year t (p. 172).

C_r = Accumulated employee contributions at retirement (p. 48).

${}^{Drgs}C_x$ = Expected prescription drug costs at age x (p. 255).

${}^{EE}C_x$ = Employee's expected cost and/or contributions at age x (p. 256).

${}^{ER}C_x$ = Employer's expected health benefit cost for employee age x (p. 256).

${}^{ER}_{t}C_x$ = Employer's expected health benefit cost at age $x + t$ for a retiree currently age x (p. 261).

${}^{Hosp}C_x$ = Expected hospital costs at age x (p. 255).

${}^{Lab}C_x$ = Expected laboratory costs at age x (p. 255).

${}^{Other}C_x$ = Expected other charges at age x (e.g., nursing home costs) (p. 255).

${}^{Phy}C_x$ = Expected physician costs at age x (p. 255).

${}^{Total}C_x$ = Total expected health benefit costs at age x (p. 255).

${}^{Qrt}C_{t-1}$ = Prior year's quarterly contributions (p. 165).

${}^{SC}C_{t-1}$ = Portion of prior year's total contribution applied to the maximum supplemental cost limits (p. 165).

${}^{SC_n}C_{t-1}$ = Portion of prior year's contribution applied to nth supplemental cost for determining maximum contributions (p. 166).

$(CC)_t$ = Cumulative contributions from plan inception to year t (p. 188).

$(CE)_t$ = Cumulative expense from plan inception to year t (p. 188).

$(CG)_t$ = Capital gains (or losses), both realized and unrealized, during year t (p. 173).

$(CL)_x$ = Current liability for employee age x (pp. 151–152).

$(Cont)_n$ = Employer contributions during year n (p. 99).

${}^{ER}(CP)_t$ = Employer copayment fraction (p. 257).

d = Rate of discount (i.e., $i\,v$) (p. 99).

$d_x^{(d)}$ = Number of employees becoming disabled during age x from a service table (p. 34).

$d_x^{(m)}$ = Number of employees dying during age x from a service table (p. 34).

$d_x^{(r)}$ = Number of employees retiring at age x from a service table (p. 34).

$d_x^{(t)}$ = Number of employees terminating during age x from a service table (p. 34).

$d_x^{(T)}$ = Number of employees leaving service during age x from a service table (p. 32).

D_t = Employee deductible in year t (p. 257).

$(DC)_t$ = Tax deductible contributions in year t (p. 265).

e_x = Curtate life expectancy (i.e., based on whole years only) at age x (p. 46).

${}^{AC}e_t$ = Error term for asset class in year t (p. 245).

${}^{Inf}e_t$ = Error term for unexpected inflation in year t, (p. 244).

$E_x^{(k)}$ = Function to determine if employee age x is eligible for benefit-type k (p. 187).

$E[(AL)_{n+1}]$ = Expected actuarial liability at year end, or the beginning of year $n + 1$ (p. 98).

$E[(Assets)_{n+1}]$ = Expected assets at year end, or the beginning of year $n + 1$ prior to any contributions (p. 98).

$E(B)$ = Expected early retirement benefit based on multiple retirement ages (p. 131).

$E(B)_t$ = Expected benefit payments during year t (p. 157).

$E(C)_t$ = Expected employer contributions during year t ((p. 185).

${}^{Min}(EC)_{t+1}$ = Minimum required contribution payable at the end of year t (p. 140).

${}^{Qrt}(EC)_t$ = Minimum required quarterly contributions in year t (p. 141).

$E(I_t)$ = Expected inflation in year t (p. 245).

${}^{HB}(EL)_x$ = Economic liability for health benefits for employee age x (p. 261).

$(ERB)_x$ = Function denoting whether the employee is expected to receive benefits (p. 188).

${}_k(ERCR)_x$ = Early retirement cost ratio: the cost (or liability) of an early retirement benefit to the cost (or liability) of a normal retirement benefit (p. 135).

${}_k^*(ERCR)_x$ = Early retirement cost ratio with actuarially equivalent early retirement benefits: the cost (or liability) of an early retirement benefit to the cost (or liability) of a normal retirement benefit (p. 135).

$(EROA)_t$ = Expected return on the market-related value of assets (p. 185).

$E[(UL)_{n+1}]$ = Expected unfunded liability at year end, or the beginning of year $n + 1$ prior to any contributions (p. 98).

${}^{d}F_k$ = Value of disability benefits payable at age k (p. 120).

${}^{r}F_r$ = Value of retirement benefits payable at age r (p. 120).

${}^{s}F_k$ = Value of surviving spouse benefits payable at age k (p. 120).

${}^{v}F_k$ = Value of vested termination benefits payable at age k (p. 120).

${}^{AL}(FFL)_t$ = Full funding limit based on the statutory funding method's actuarial liability for year t (p. 157).

${}^{CL}(FFL)_t$ = Full funding limit based on the current liability for year t (p. 159).

${}^{QAA}(FFL)_{t+1}$ = Full funding limit based on qualified asset account, applicable at end of year t (p. 268).

$(FR)_t$ = Funded ratio equal to actuarial value of assets less the FSA credit balance, all divided by the current liability (p. 154).

$(FS)_x$ = Future service for employee age x (p. 187).

$(FSA)_t$ = Funding standard account balance from end of prior year (i.e., at beginning of year t), with a credit balance representing a positive value and a funding deficiency representing a negative value (p. 140).

${}^{CB}(FSA)_t$ = Credit balance in the funding standard account at beginning of year t, i.e., from the end of prior year (p. 153).

${}^{FD}(FSA)_t$ = Funding deficiency in funding standard account at end of year, determined without regard to prior year credit balance and current year contribution (p. 162).

$g_x^{(d)}$ = Grading function equal to the proportion of accrued benefit provided if disability occurs during age x (p. 116).

$g_x^{(r)}$ = Grading function equal to the proportion of accrued benefit payable if retirement occurs at the beginning of age k (p. 129).

${}^{*}g_k^{(r)}$ = Grading function which, when applied to the participant's accrued benefit, produces actuarially equivalent benefits (p. 126).

$g_x^{(s)}$ = Grading function equal to the proportion of accrued benefit provided to a surviving spouse if death occurs during age x (p. 117).

$g_x^{(v)}$ = Grading function equal to the proportion of accrued benefit vested at age x (p. 115).

I = Rate of inflation (p. 38).

I_t = Inflation in year t (p. 244).

I_∞ = Long-term rate of inflation (p. 244).

i = Interest rate (p. 34).

i' = Current liability interest rate for statutory funding requirements (p. 158) and expected return on assets for accounting requirements (p. 186).

$(IC)_t$ = Interest cost component of net periodic pension cost (p. 185).

$l_x^{(T)}$ = Number of employees in active service at age x from a service table (p. 32).

$l_{x,y}$ = Number of age-y entrants currently age x (p. 84).

M = Probability that the participant has a surviving spouse at death (p. 117).

$(MC)_t$ = Miscellaneous credits, primarily reflecting full funding limits (p. 162).

$(MRA)_t$ = Market-related value of assets for year t (p. 185).

$(MV)_t$ = Market value of assets at beginning of year t (p. 172).

${}^{r}(NC)_x$ = Normal cost under a specified actuarial cost method for individual age x assumed to retire at age r (p. 80).

$(NC)_t$ = Normal cost under statutory funding method, including any implicit supplemental costs, for year t (p. 140).

${}^{AAB\,r}(NC)$ = Normal cost under the aggregate accrued benefit method for all participants, based on retirement at age r (p. 84).

${}^{AB\,r}(NC)_x$ = Normal cost under the accrued benefit method for individual age x assumed to retire at age r (p. 84).

${}^{AB\,r'}(NC)_x$ = Normal cost under the accrued benefit method for individual age x assuming multiple retirement ages (p. 131).

${}^{AB\,*r'}(NC)_x$ = Normal cost under the accrued benefit method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 131).

${}^{AB\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the accrued benefit method for individual age x (p. 120).

${}^{ABD}\,{}^{r}(NC)$ = Normal cost under the aggregate benefit prorate (constant dollar) method for all participants, based on retirement at age r (p. 85).

${}^{ABP}\,{}^{r}(NC)$ = Normal cost under the aggregate benefit prorate (constant percent) method for all participants, based on retirement at age r (p. 86).

${}^{ACD}\,{}^{r}(NC)$ = Normal cost under the aggregate cost prorate (constant dollar) method for all participants, based on retirement at age r (p. 88).

${}^{ACP}\,{}^{r}(NC)$ = Normal cost under the aggregate cost prorate (constant percent) method for all participants, based on retirement at age r (p. 88).

${}^{BD}\,{}^{r}(NC)_x$ = Normal cost under the benefit prorate (constant dollar) method for individual age x assumed to retire at age r (p. 85).

${}^{BD}\,{}^{r'}(NC)_x$ = Normal cost under the benefit prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 132).

${}^{BD}\,{}^{*r'}(NC)_x$ = Normal cost under the benefit prorate (constant dollar) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 132).

${}^{BD}\,{}^{T}(NC)_x$ = Normal cost, including ancillary benefits, under the benefit prorate (constant dollar) method for individual age x (p. 120).

${}^{BP}\,{}^{r}(NC)_x$ = Normal cost under the benefit prorate (constant percent) method for individual age x assumed to retire at age r (p. 85).

${}^{BP}\,{}^{r'}(NC)_x$ = Normal cost under the benefit prorate (constant percent) method for individual age x assuming multiple retirement ages (p. 132).

${}^{BP\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the benefit prorate (constant percent) method for individual age x (p. 121).

${}^{CD\,r}(NC)_y$ = Normal cost under the cost prorate (constant dollar) method for individual age x assumed to retire at age r (p. 86).

${}^{CD\,r'}(NC)_x$ = Normal cost under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages (p. 133).

${}^{CD\,*r'}(NC)_x$ = Normal cost under the cost prorate (constant dollar) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

${}^{CD\,T}(NC)_x$ = Normal cost, including ancillary benefits, under the cost prorate (constant dollar) method for individual age x (p. 121).

${}^{CL}(NC)_t$ = Current liability normal cost at beginning of year t (p. 158).

${}^{CP\,r}(NC)_x$ = Normal cost under the cost prorate (constant percent) method for individual age x assumed to retire at age r (p. 87).

${}^{CP\,r'}(NC)_x$ = Normal cost under the cost prorate (constant percent) method for individual age x assuming multiple retirement ages (p. 133).

${}^{CP\,*r'}(NC)_x$ = Normal cost under the cost prorate (constant percent) method for individual age x assuming multiple retirement ages with actuarially equivalent early retirement benefits (p. 134).

${}^{PT}(NC)_x$ = Normal cost under the plan termination method for individual age x assumed to retire at age r (p. 95).

$(NR)_t$ = Nominal return in year t (p. 245).

$(NRI)_t$ = Net realized income in year t (p. 265).

P = Rate of productivity reflected in salary increases (p. 38).

$p_x^{(d)}$ = Probability of an employee not terminating from service from age x to age $x+1$, excluding consideration of other decrements (p. 21).

${}_np_x^{(d)}$ = Probability of an employee not terminating from service from age x to age $x+n$, excluding consideration of other decrements (p. 21).

$p_x^{(m)}$ = Probability of a life age x living to age $x+1$ (p. 16).

${}_np_x^{(m)}$ = Probability of a life age x living to age $x+n$ (p. 16).

${}^dp_x^{(m)}$ = Probability of a disabled life age x living to age $x+1$ (p. 21).

$p_x^{(r)}$ = Probability of an employee retiring at the beginning of age x, excluding consideration of other decrements (p. 31).

$p_x^{(t)}$ = Probability of an employee not terminating employment from age x to age $x+1$, excluding consideration of other decrements (p. 18).

${}_np_x^{(t)}$ = Probability of an employee not terminating employment from age x to age $x+n$, excluding consideration of other decrements (p. 18).

$p_x^{(T)}$ = Probability of an employee surviving in service from age x to age $x+1$ (p. 31).

${}_np_x^{(T)}$ = Probability of an employee surviving in service from age x to age $x+n$ (p. 32).

${}_n^r p_x^{(T)}$ = Probability of surviving in employment n years, where retirement decrements are included with mortality, termination, and disability decrements (p. 129).

$(PBO)_x$ = Projected benefit obligation for employee age x (p. 181).

${}^{AB\,r}(PCL)_x$ = Plan continuation liability for accrued benefits for employee age x (p. 70).

$(PSC)_x$ = Prior service cost created at an employee's age x (p. 189).

$(PTL)_x$ = Plan termination liability for employee age x (p. 69).

${}^d(PVFB)_x$ = Present value of future disability benefits for employee age x (p. 117).

${}^{HB}(PVFB)_x$ = Present value of future health care benefits for employee age x (p. 261).

${}^{HB'}(PVFB)_x$ = Present value of future health benefits (without health care inflation) for employee age x (p. 267).

${}^r(PVFB)_x$ = Present value of future benefits for employee age x assumed to retire at age r (p. 72).

${}^r(PVFB)_{x,y}$ = Present value of future benefits at age x for an age-y entrant (p. 84).

${}^{r'}(PVFB)_x$ = Present value of future benefits for employee age x under multiple retirement ages (p. 129).

${}^{*r'}(PVFB)_x$ = Present value of future benefits for employee age x under multiple retirement ages with actuarially equivalent early retirement benefits (p. 130).

${}^{s}(PVFB)_x$ = Present value of future surviving spouse benefits for employee age x (p. 118).

${}^{v}(PVFB)_x$ = Present value of future vested termination benefits for employee age x (p. 115).

${}^{r}(PVFNC)_y$ = Present value of future normal costs for employee age y (p. 80).

$q_x'^{(d)}$ = Disability rate at age x (p. 21).

$q_x^{(d)}$ = Probability of decrementing from active service due to disability during age x (p. 31).

$q_x'^{(m)}$ = Mortality rate at age x (p. 16).

$q_x^{(m)}$ = Probability of decrementing from active service due to death during age x (p. 32).

${}^{d}q_x'^{(m)}$ = Mortality rate for a disabled life at age x (p. 21).

$q_x'^{(r)}$ = Retirement rate at age x (p. 25).

$q_x^{(r)}$ = Probability of retiring at the beginning of age x (p. 31).

$q_x'^{(t)}$ = Termination rate at age x (p. 18).

$q_x^{(t)}$ = Probability of decrementing from active service due to termination during age x (p. 31).

$q_x^{(T)}$ = Probability of decrementing from active service during age x (p. 32).

$(\Delta QAA)_t$ = Addition to a qualified asset account in year t (p. 265).

$(QDC)_t$ = Qualified direct cost in year t (p. 265).

r = Normal retirement age (p. 5).

r' = First age at which an employee becomes eligible for early retirement (p. 5).

r'' = Age by which all employees are assumed to be retired (p. 5).

$(RA)_t$ = Reconciliation account balance (p. 163).

$(RR)_t$ = Real return in year t (p. 245).

${}^{MC}R_x$ = Expected reimbursements from Medicare (p. 256).

${}^{Other}R_x$ = Expected reimbursements from other governmental programs and/or other private insurance programs (p. 256).

s_x = Current dollar salary for a participant age x (p. 38).

$s_{x,y}$ = Salary at age x for an age-y entrant (p. 86).

S_x = Cumulative salary from entry age y up to, but not including, age x (p. 38).

s_y = Entry age dollar salary (p. 38).

$S_{r,y}$ = Cumulative salary from entry age y to retirement age r (p. 86).

$s_{\overline{1}|}^{(4)}$ = Factor for accumulating quarterly contributions with interest to end of year (p. 141).

$(SS)_x$ = Merit salary scale at age x (p. 38).

$(\sum SC)_t$ = Sum of all explicit supplemental costs associated with prior increases or decreases in the unfunded liability that are not yet fully amortized (p. 140).

$(SC)_x$ = Service cost for employee age x (p. 184).

$(SC_n)_j$ = jth supplemental cost for the nth unfunded liability increment (p. 101).

${}^{AB}(SC_n)_x$ = Implicit supplemental cost at age x for nth unfunded liability change under the accrued benefit method (p. 105).

${}^{CD}(SC_n)_x$ = Implicit supplemental cost at age x for nth unfunded liability change under the cost prorate (constant dollar) method (p. 109).

${}^{CE}(SC)_t$ = Supplemental costs associated with contingent events (p. 151).

${}^{CE}_{7\text{-}pay}(SC)_t$ = Supplemental costs associated with amortizing contingent event liabilities over 7 years (p. 155).

${}^{CP}(SC_n)_x$ = Implicit supplemental cost at age x for nth unfunded liability change under the cost prorate (constant percent) method (p. 109).

${}^{I/PA}(SC)_t$ = Sum of all explicit supplemental costs for the initial unfunded actuarial liability and plan amendments that are not yet fully amortized (p. 151).

${}^{New}(SC)_t$ = Supplemental cost associated with the *new* unfunded current liability as of the current year, excluding any remaining *old* unfunded current and any unfunded contingent event liabilities (p. 151).

${}^{Old}(SC)_t$ = Supplemental cost associated with *old* unfunded current liability established in 1988 and amortized over the succeeding 18 years (p. 151).

${}^{AAB}{}^{r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate accrued benefit method with implicit supplemental cost (p. 107).

${}^{ABD\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate benefit prorate (constant dollar) method with implicit supplemental cost (p. 108).

${}^{ABP\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate benefit prorate (constant percent) method with implicit supplemental cost (p. 108).

${}^{ACD\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate cost prorate (constant dollar) method with implicit supplemental cost (p. 111).

${}^{ACP\,r}(TC)_t$ = Total cost (normal plus supplemental cost) under the aggregate cost prorate (constant percent) method with implicit supplemental cost (p. 111).

${}^{CD}(TC)_x$ = Total cost (normal plus supplemental cost) under the cost prorate (constant dollar) method with implicit supplemental cost (p. 109).

${}^{d}(TC)_x$ = Term cost of disability benefits for an employee age x (p. 116).

${}^{s}(TC)_x$ = Term cost of surviving spouse benefits for an employee age x (p. 117).

${}^{v}(TC)_x$ = Term cost of vested termination benefits for an employee age x (p. 115).

$(TP)_t$ = Transition percentage, equal to 20 percent for 1993 and increasing by 10 percent per year to 100 percent by 2001 (p. 155).

Tx = Unrelated business income tax rate (p. 266).

u = Number of years (positive or negative) that, when added to the participant's age, yields an assumed age for the surviving spouse (p. 117).

$(UCL)_t$ = Unfunded current liability during year t (p. 152).

$(UL)_t$ = Unfunded liability for plan at beginning of year t (p. 98).

$(\Delta_n UL)$ = Unfunded liability (positive or negative) developed during year n (p. 98).

$(\Delta_n UL)_x$ = nth unfunded liability developed during a prior year (p. 101).

${}^{CE}(UL)_t$ = Unfunded liability associated with prior contingent events (p. 154).

${}^{Old}(UL)_t$ = Balance of unfunded *old* current liability (p. 153).

$(ULB)_t$ = Unamortized liability balance in year t (p. 162).

$(\Delta_n ULB)_j$ = Unfunded liability balance at the beginning of the jth year for the nth unfunded liability change (p. 101).

v^n = Present value of one dollar due in n years at an annual compound rate of interest equal to i (p. 36).

w = Waiting period before disability benefits commence (p. 116) and serial correlation of successive year's inflation (p. 244).

${}^{FD}(Waiver)_t$ = Funding deficiency waiver (p. 162).

x = Employee's attained age.

y = Employee's entry age.

$(Yrs)_n$ = Amortization years remaining for the nth supplemental cost (p. 167).

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