PENSION MATHEMATICS with Numerical Illustrations

Second Edition

Howard E. Winklevoss, Ph.D., MAAA, EA
President
Winklevoss Consultants, Inc.

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Chapter 7

Supplemental Costs

Theoretically, the accumulation of past normal costs will precisely equal the cost method's actuarial liability determined prospectively (i.e., the present value of future benefits less the present value of future normal costs). By the same token, if contributions are made to the trust fund each year in the amount of the normal cost, then theoretically plan assets will also equal the actuarial liability. There are several reasons, however, why these equalities will not obtain.

- *Experience Variations:* The experience of the plan will differ from the underlying actuarial assumptions, referred to as *actuarial gains and losses*.
- *Assumption Changes:* The actuarial assumptions will be changed from time to time.
- *Benefit Changes:* The plan's benefit formula may be increased or decreased periodically by plan amendment, with the change frequently being retroactive.
- *Past Service Accruals:* The plan sponsor may have granted benefit credits to years prior to the establishment of the plan, sometimes referred to as the *past service liability*.

Additionally, plan assets may not equal the plan's actuarial liability for yet a fifth reason:

- *Contribution Variances:* The plan sponsor may contribute more or less than the normal cost under the cost method in use.

The discrepancy that develops between plan assets and the prospectively determined actuarial liability is called the plan's *unfunded liability*. Although the term "unfunded liability" is used, the discrepancy could be either positive or negative. For example, if the plan's assets exceed the actuarial liability, a surplus (i.e., negative unfunded liability) would exist.

Supplemental costs are designed to amortize the plan's unfunded liability. In this sense, they are like the plan's normal costs which are designed to amortize ${}^r(PVFB)_y$ from age y to r, but which may fail to do so because of one or more of the factors listed above. The failure of normal costs to amortize ${}^r(PVFB)_y$ creates an unfunded liability which, in turn, generates a supplemental cost.

The supplemental cost can be a one-time cost equal in value to the unfunded liability created during a given year, or it can extend over a period of years. Supplemental costs can, at least in theory, take on any pattern over any time period, or the supplemental costs can be geared to the corresponding normal cost pattern of the actuarial cost method in use.

The supplemental cost associated with experience variations and assumption changes are almost always formulated on a group basis. In other words, the unfunded liability is determined in the aggregate for all plan members and amortized thereafter either without reference to individual participants or with reference only to the current group of active participants.

The unfunded liabilities and supplemental costs associated with benefit changes, particularly past service accruals, are dealt with on an individual or group basis, depending on the actuarial cost method being used and/or the preference of the plan sponsor or actuary. The purpose of this chapter is to describe alternative supplemental cost methods. First, however, unfunded liabilities that give rise to such costs are defined.

UNFUNDED ACTUARIAL LIABILITY

Since the plan's unfunded actuarial liability is generally determined on a group basis rather than on a participant by participant basis, the notation in this section is similarly based on the entire membership. The plan's total unfunded liability (UL) at the beginning of year t, sometimes referred to as the supplemental liability, is equal to the difference between the actuarial liability and plan assets:

$$(UL)_t = (AL)_t - (Assets)_t \tag{7.1}$$

where

$(AL)_t$ = actuarial liability at beginning of year t

$(Assets)_t$ = plan assets at beginning of year t.

The unfunded liability created during year n from all sources is equal to the difference between the *actual* unfunded liability at the beginning of the next year less the *expected* unfunded liability at that point:

$$(\Delta_n UL) = (UL)_{n+1} - E[(UL)_{n+1}] \tag{7.2a}$$

where

$(\Delta_n UL)$ = unfunded liability (positive or negative) developed during year n

$(UL)_{n+1}$ = actual unfunded liability at the beginning of year $n+1$

$E[(UL)_{n+1}]$ = expected unfunded liability at year end, or the beginning of year $n + 1$ prior to any contributions.

The expected unfunded liability at year end is calculated as of the beginning of year n, under the assumption that none of the previously listed contingencies occur (i.e., experience variations, assumption changes and so forth), as discussed below.

First, $E[(UL)_{n+1}]$ in (7.2a) is replaced by its component parts, as defined by (7.1):

$$(\Delta_n UL) = (UL)_{n+1} - \{E[(AL)_{n+1}] - E[(Assets)_{n+1}]\}. \tag{7.2b}$$

The expected actuarial liability at $n + 1$ is given by

$$E[(AL)_{n+1}] = [(AL)_n + (NC)_n - B_n](1+i) \tag{7.3}$$

where $(NC)_n$ is the plan's normal cost during year n and B_n represents the benefits paid, both assumed to occur at the beginning of the year (or, equivalently, adjusted with interest to the beginning of the year). In other words, the expected actuarial liability is equal to the prior year's actuarial liability plus the normal cost less the benefits paid, all increased with interest. A liability-based unfunded liability results when the actuarial liability at $n+1$ differs from $E[(AL)_{n+1}]$.

The expected assets at $n+1$ may be written as

$$E[(Assets)_{n+1}] = [(Assets)_n + (Cont)_n - B_n](1+i) \quad (7.4)$$

where $(Cont)_n$ represents the contributions to the plan at the beginning of year n. An asset-based unfunded liability results when assets at $n+1$ differ from $E[(Assets)_{n+1}]$.

Substituting (7.3) and (7.4) into (7.2b), then canceling and rearranging terms, we have

$$(\Delta_n UL) = (UL)_{n+1} - [(UL)_n + (NC)_n - (Cont)_n](1+i). \quad (7.5)$$

In words, (7.5) shows that no unfunded liability will develop during year n if that year's unfunded liability, $(UL)_n$, plus the net change in the unfunded liability, $(NC)_n - (Cont)_n$, all accumulated with interest, are equal to the unfunded liability at the beginning of the next year.

Observe from (7.5) that if $(NC)_n = (Cont)_n$, and if $(\Delta_n UL)$ is zero for the year, then $(UL)_{n+1} = (UL)_n(1+i)$. In this case contributions equal to the normal cost *plus* the present value of the interest on the unfunded liability will keep the unfunded liability constant from year to year, that is,

$$(Cont)_n = (NC)_n + v\,i\,(UL)_n \quad (7.6a)$$

$$= (NC)_n + d\,(UL)_n \quad (7.6b)$$

where $d = vi$, the rate of discount. Generally, however, the plan sponsor will want to reduce the unfunded liability through the payment of supplemental costs.

The incremental unfunded liability created during year n can be categorized into the five factors listed previously. Experience variations can be determined by (7.5), if we assume that none of the other four factors occur. Moreover, it is possible to further allocate experience variations according to each actuarial assumption used in the valuation process.[1] The unfunded liability attributed to assumption changes and/or benefit changes can be determined by simply calculating the actuarial liability at time n with the prior set of assumptions and/or benefits and noting the

[1] Allocating experience variations by source is beyond the scope of this book. It should be noted, however, that the variation attributed to one assumption may affect the variation attributed to another. In other words, the allocation process may not be unique.

difference. Past service accruals, likewise, are generally recorded at the time such accruals are granted. Contribution variances, of course, can be determined by a simple comparison. As we will see, it is necessary to amortize each year's change in the unfunded liability in order to satisfy the FASB accounting requirements and, under most actuarial cost methods, to determine the ERISA minimum required and maximum tax deductible contribution limitations.

Unless steps are taken to amortize the unfunded liabilities that are created each year, they will grow with interest. Thus, the nth unfunded liability change, equal to the unamortized portion of the unfunded liability created during some prior year, has the following value from year t to year $t + 1$:

$$(\Delta_n UL)_{t+1} = (\Delta_n UL)_t (1 + i) . \tag{7.7}$$

Supplemental costs, designed to amortize the year-to-year unfunded liability changes, can be either explicit or implicit and, for the latter, either individual-based or group-based. All such variations are defined in this section.

EXPLICIT SUPPLEMENTAL COSTS

If a given unfunded liability change is amortized by a method that bears no relationship to the actuarial cost method or the plan participants, then it is known as an *explicit supplemental cost method.* Explicit costs can be used with all actuarial cost methods studied in Chapter 6, and they are probably the most widely used of all supplemental cost methods. As noted above, each year's unfunded liability change grows with interest, much like any other debt. Thus, it is necessary to pay interest on the unfunded liability plus a portion of the outstanding balance if the quantity it to be amortized. Since contributions are assumed herein to be paid in advance, the annual interest is equal to d, the rate of discount defined previously, times the unfunded liability balance remaining after the contribution. In theory, any portion of the unfunded liability could be paid in the current year; however, the typical procedure is to amortize the increment over a finite period according to a specified pattern.

The so-called *straight line method* calls for annual payments equal to the interest on the outstanding balance plus $1/m$ of the original debt (assuming an m-year amortization period). Thus, the beginning-of-year jth supplemental cost payment ($1 \leq j \leq m$) associated with the nth unfunded liability is as follows:

$$(SC_n)_j = d\left[(\Delta_n ULB)_j - \frac{1}{m}(\Delta_n UL)\right] + \frac{1}{m}(\Delta_n UL) \qquad (7.8a)$$

where

$(SC_n)_j$ = jth supplemental cost for the nth unfunded liability increment

$(\Delta_n ULB)_j$ = unfunded liability balance at the beginning of the jth year for the nth unfunded liability change

$(\Delta_n UL)$ = nth unfunded liability developed during a prior year.

If the amortization payments occur at the end of the year, as is the case for pension accounting (see Chapter 11), then the first section of (7.8a) would equal interest times the unfunded liability balance (rather than discount times the unfunded liability balance reduced by the current year's payment).

An alternative and widely used method is to amortize the unfunded liability increment with a series of constant dollar payments, with each payment representing both interest and principal. The supplemental cost at the beginning of the jth year during the m-year amortization period under the *constant dollar amortization method* is

$$(SC_n)_j = \frac{(\Delta_n UL)}{\ddot{a}_{\overline{m}|}} \qquad (7.8b)$$

where

$\ddot{a}_{\overline{m}|}$ = m-year period certain annuity.

Another explicit approach, argued by some to be the most appropriate for salary-based pension benefit formulas, is to have a series of increasing supplemental cost payments, with the annual increase equal to the non-merit portion of the salary assumption. The objective, of course, is to produce a cost pattern approximately equal to a constant percent of payroll. The *constant percent amortization method* can be defined as follows:

$$(SC_n)_j = \frac{(\Delta_n UL)}{{}^{s}\ddot{a}_{\overline{m}|}}\,[(1 + I)(1 + P)]^{j-1} \qquad (7.8c)$$

where

$^{s}\ddot{a}_{\overline{m|}}$ = m-year period certain annuity, with payments increasing by the inflation and productivity components of the salary assumption

I = inflation component of salary assumption

P = productivity component of salary assumption

j = jth supplemental cost payment ($1 \leq j \leq m$).

Figures 7–1a and 7–1b show the annual payments, first in nominal dollars and then in dollars that are "deflated" by the non-merit salary components (i.e., 5 percent), under the three explicit supplemental cost methods. These data assume an amortization period of 15 years, interest of 8 percent (the model interest rate assumption), salary increases of 5 percent (the model inflation plus productivity assumption), and an initial unfunded liability of 100 dollars. Figures 7–2a and 7–2b show the remaining unfunded liability as a percent of the original amount, again in nominal dollars and salary-adjusted dollars.[2]

IMPLICIT SUPPLEMENTAL COSTS

Supplemental costs calculated according to the same principles used in determining normal costs are referred to as *implicit methods*. Each cost method studied in Chapter 6 has a supplemental cost counterpart. Sometimes in practice, or in the literature, a so-called "normal cost" is calculated under these methods, which include the normal cost as previously defined plus the implicit supplemental cost yet to be defined in this section. While it may be unnecessary, and undoubtedly inconvenient, to separate this total cost into its underlying components, it is nevertheless imprecise and confusing not to do so. As stated previously,

[2]If the amortization period under the constant percent method is in the range of 20 to 30 years, its supplemental costs are less than the interest due on the unfunded liability for a period of time; hence, the outstanding balance increases. When expressed as a percent of total annual payroll, however, the outstanding balance represents a continually smaller amount.

FIGURE 7–1a

Supplemental Costs per $100 of Unfunded Liability Under Alternative Methods

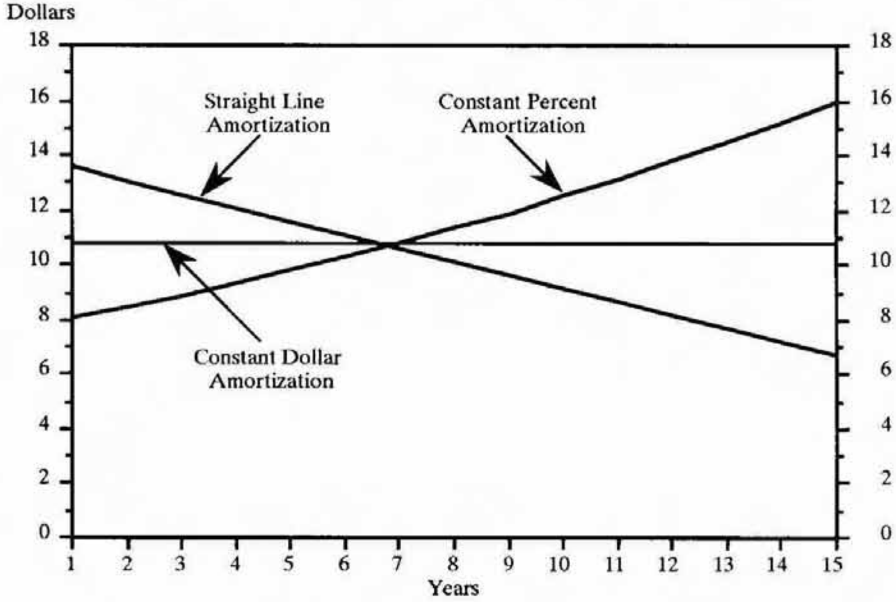


FIGURE 7–1b

Supplemental Costs per $100 of Unfunded Liability Under Alternative Methods
(Dollars "deflated" by 5%, equal to the non-merit salary increase components)

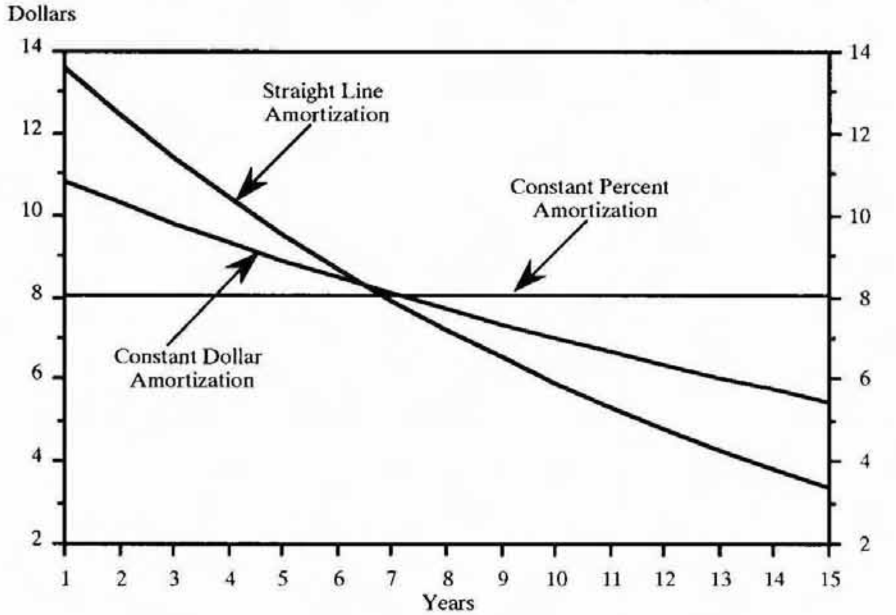


FIGURE 7–2a

Unfunded Liability Balance per $100 of Initial Balance Under Alternative Methods

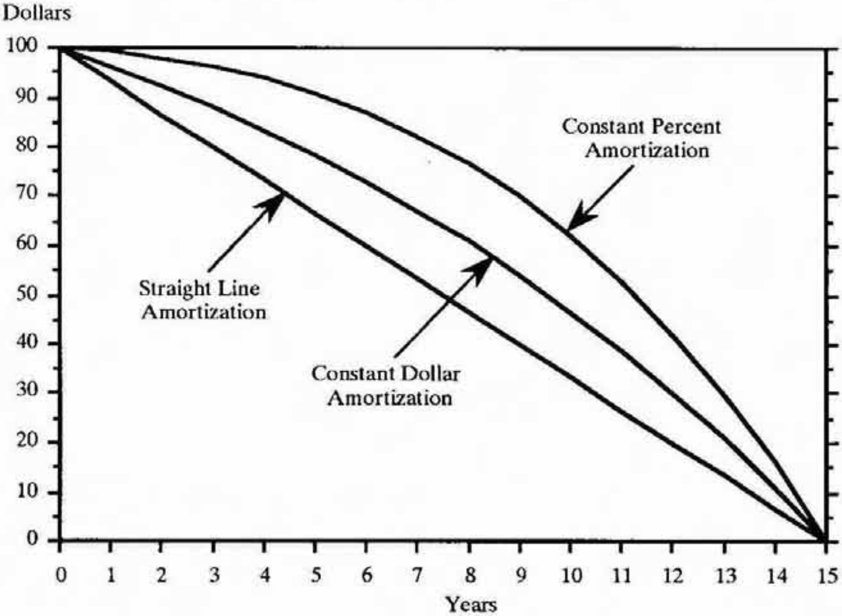
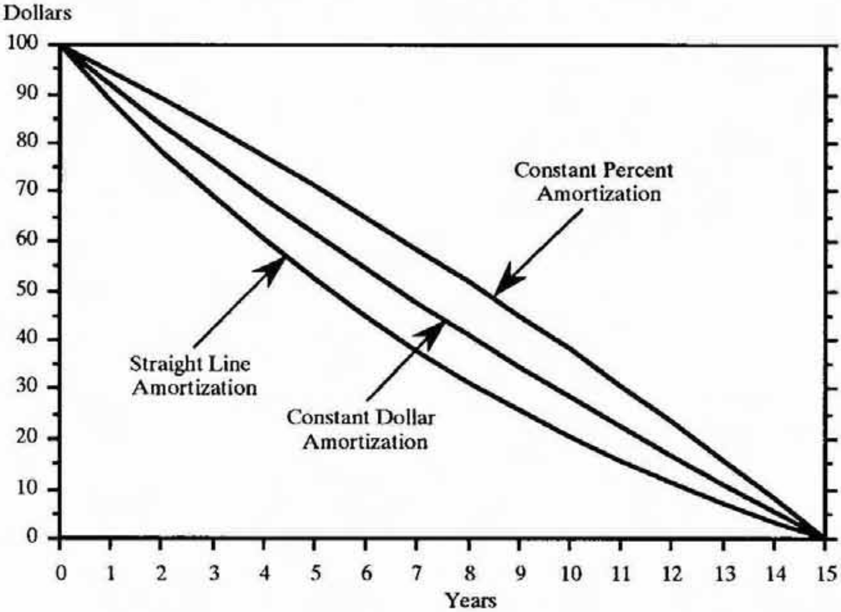


FIGURE 7–2b

Unfunded Liability Balance per $100 of Initial Balance Under Alternative Methods
(Dollars "deflated" by 5%, equal to the non-merit salary increase components)



the term normal cost used in this book never includes any supplemental costs.

Whereas explicit supplemental costs are almost exclusively determined on a group basis, implicit supplemental costs are sometimes determined on a participant-by-participant basis, especially for amortizing past service credits. The other sources of unfunded liability, however, are likewise almost always amortized on a group basis. In fact, a relatively common practice is to use an implicit supplemental cost method for amortizing past service credits and an explicit method to amortize all others sources of unfunded liability. Thus, it is possible to have both types of supplemental cost methods being used on different sources of unfunded liability.

Since implicit supplemental costs are geared to the various actuarial cost methods, the following discussion is organized accordingly.

Accrued Benefit Method

The normal cost under the accrued benefit method, it will be recalled, was found by using the natural benefit accruals for b_x in (6.1). This formulation is repeated here for convenience:

$$^{AB\,r}(NC)_x = b_x \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r . \tag{6.7a}$$

The implicit supplemental cost can be expressed as follows:

$$^{AB}(SC_n)_x = C_n \; b_x \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \tag{7.9}$$

where

$^{AB}(SC_n)_x$ = accrued benefit supplemental cost at age x for nth unfunded liability change

C_n = coefficient to the benefit accrual for the nth unfunded liability change.

During the age that an unfunded liability is created (the nth unfunded liability change in the above formulation), a coefficient, C_n, is determined such that the present value (from attained age x to retirement) of the future designated supplemental costs is equal to the unfunded liability change. Determining the present value of future normal costs, which is an intermediate step in deriving C_n, can be avoided by using the identity given by (6.3a):

$$^{r}(AL)_x = {}^{r}(PVFB)_x - {}^{r}(PVFNC)_x . \tag{6.3a}$$

In other words, the present value of future normal costs (PVFNC) can be expressed as the present value of future benefits less the actuarial liability. Thus, C_n can be determined as follows:

$$C_n = \frac{(\Delta_n UL)_x}{(PVFB)_x - (AL)_x} \tag{7.10}$$

where

$(\Delta_n UL)_x$ = nth unfunded liability created during age x.

If the implicit supplemental cost under the accrued benefit method is relegated to determining only past service credits (the typical usage of this method), or perhaps benefit changes, then the determination of C_n becomes even more straightforward. To illustrate, assume that the plan is initiated at the employee's age z, yet benefit credits are granted from the employee's entry age. The past service benefit would be equal to B_z. The value of C_n is equal to the following:

$$C_n = \frac{B_z}{B_r - B_z}. \tag{7.11a}$$

In words, C_n is equal to the past service benefit divided by the future service benefit. Applying this ratio to the benefit accrual function in (7.9) is logical, since the sum of such "modified accruals" from age z to retirement is B_z, the past service benefit being funded by the supplemental costs. The plan's total cost, excluding any other supplemental costs that might be appropriate, is found by using the following benefit accrual, where the superscript T indicates the *total* benefit accrual (i.e., the normal cost benefit accrual plus the supplemental cost "benefit accrual" for past service credits):

$$b_x^T = (C_n + 1)\, b_x = \frac{B_r}{B_r - B_z}\, b_x. \tag{7.11b}$$

Therefore, the attained age benefit accrual is increased by the amount that the projected benefit, B_r, exceeds the future service benefit, $B_r - B_z$. This increase is a logical consequence of allocating the past service accrual, B_z, over future ages.

The *aggregate* version of the accrued benefit method has a total cost defined as follows, again assuming that only the supplemental cost for past service accruals are considered:

$$^{AAB\,r}(TC)_t = \left(\sum l_{x,y}\, b_{x,y}\right)\left[\frac{\sum l_{x,y}\ {}^r(PVFB)_{x,y}}{\sum l_{x,y}\ (B_{r,y} - B_{z,y})}\right]. \tag{7.11c}$$

If only one employee is considered, the "total accrual" in (7.11c) reduces to (7.11b). As new employees enter the plan in future years, their value for $B_{z,y}$ will be zero; hence, eventually equation (7.11c) will be equal to the aggregate accrued benefit method given by equation (6.7b).

Benefit Prorate Methods

The *constant dollar* and *constant percent* versions of the benefit prorate method have normal costs defined by substituting the benefit accrual functions given by (3.15a) and (3.16a) for b_x in (6.1):

$$^{CD}b_x = \frac{B_r}{(r-y)}, \qquad (y \le x < r) \tag{3.15a}$$

$$^{CP}b_x = \frac{B_r}{S_r} s_x. \qquad (y \le x < r) \tag{3.16a}$$

These benefit accruals would be modified by a C_n factor, analogous to the modification in (7.9), in determining their respective implicit supplemental costs. In other words, the "modified accruals" would be determined such that their present value, from attained age x to retirement, equaled the unfunded liability increment to be amortized.

Restricting consideration to amortizing past service accruals would give the total accrual at age x as follows for the constant dollar version:

$$b_x^T = \frac{B_r}{r-y} + \frac{\frac{B_r}{r-y}(z-y)}{r-z} = \frac{B_r}{r-z}. \tag{7.12a}$$

The term to the right of the first equal sign is the retirement benefit accrual under the benefit prorate method, whereas the second term is the supplemental cost "benefit accrual," the latter equal to the past service allocation (the numerator) divided by future service credits.[3] These two accrual factors reduce to the

[3]Note that the past service "allocation" is not defined to be equal to the true (or legally defined) past service accrual, but rather, to the past service years

projected benefit divided by future service credit, an intuitively appealing result. In other words, the constant dollar normal cost plus past-service-credit supplemental cost is found by simply allocating the projected benefit over future service.[4]

The analogous total benefit factor for the constant percent benefit prorate method is given by

$$b_x^T = \frac{B_r}{S_r - S_z} s_x . \tag{7.12b}$$

Again, the projected benefit is prorated by future service salary from age z to age r instead of from entry age to age r.

Finally, the *aggregate* versions of these two methods have total costs (normal cost plus past-service-accruals supplemental cost) defined by the following two equations:

$$^{ABD\,r}(TC)_t = \left(\sum l_{x,y}\right)\left[\frac{\sum l_{x,y}\ {}^r(PVFB)_{x,y}}{\sum l_{x,y}\ (r-z)}\right]; \tag{7.13a}$$

$$^{ABP\,r}(TC)_t = \left(\sum l_{x,y}\ s_{x,y}\right)\left[\frac{\sum l_{x,y}\ {}^r(PVFB)_{x,y}}{\sum l_{x,y}\ (S_{r,y} - S_{z,y})}\right]. \tag{7.13b}$$

Cost Prorate Methods

Normal costs under the cost prorate methods are equal to a constant dollar amount or a constant percent of salary during each employee's career. By the same token, implicit supplemental costs amortize the unfunded liability in the same manner from the age at which the unfunded liability is created to retirement. If the nth unfunded liability were created at age x, the corresponding supplemental costs from that age to retirement can be represented as follows:

$$^{CD}(SC_n)_x = \frac{(\Delta_n UL)_x}{\ddot{a}^T_{x:\overline{r-x}|}}; \tag{7.14a}$$

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times the constant dollar benefit accruals. This is necessarily the case, since implicit supplemental costs are based on principles identical to those used in determining the cost method's normal costs.

[4]This methodology could also be useful if the first funding age exceeds the employee's plan eligibility age, in which case the methodology used must make up for accruals that occurred before funding commenced.

$$^{CP}(SC_n)_x = \frac{(\Delta_n UL)_x}{s_x \, {}^s\ddot{a}^T_{x:\overline{r-x}|}}. \tag{7.14b}$$

The $^{CD}(SC_n)_x$ supplemental cost is a constant dollar amount applicable to each future age. The dollar value of the constant percent supplemental cost is found by multiplying the salary at each future age by the $^{CP}(SC_n)_x$ fraction.

The *individual* implicit supplemental costs under the cost prorate methods are generally used to amortize the liability associated with the plan's prior service accruals, and are sometimes used to amortize the other sources of unfunded liability. Using the constant dollar version for illustrative purposes, the unfunded liability associated with past service accruals at age z (the assumed starting age of the plan) can be expressed as previously set out in equation (5.8b):

$$^{CD\,r}(AL)_z = \frac{\ddot{a}^T_{y:\overline{z-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} {}^r(PVFB)_z. \tag{7.15a}$$

If this quantity is amortized from age z to age r by $\ddot{a}^T_{z:\overline{r-z}|}$, adding it to the normal cost under this method yields the following total cost:

$$^{CD}(TC)_x = \frac{^r(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}} + \frac{\ddot{a}^T_{y:\overline{z-y}|}\,{}^r(PVFB)_z}{\ddot{a}^T_{y:\overline{r-y}|}\,\ddot{a}^T_{z:\overline{r-z}|}}. \tag{7.15b}$$

Establishing a common denominator, we have

$$^{CD}(TC)_x = \frac{^r(PVFB)_y\,\ddot{a}^T_{z:\overline{r-z}|} + \ddot{a}^T_{y:\overline{z-y}|}\,{}^r(PVFB)_z}{\ddot{a}^T_{y:\overline{r-y}|}\,\ddot{a}^T_{z:\overline{r-z}|}}. \tag{7.15c}$$

Equation (7.15c) can be rewritten in the following manner:

$$^{CD}(TC)_x = \frac{^r(PVFB)_z}{\ddot{a}^T_{z:\overline{r-z}|}}\left[\frac{{}_{z-y}p_y^{(T)}\,v^{z-y}\,\ddot{a}^T_{z:\overline{r-z}|} + \ddot{a}^T_{y:\overline{z-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}}\right]$$

$$= \frac{^r(PVFB)_z}{\ddot{a}^T_{z:\overline{r-z}|}}. \tag{7.15d}$$

The bracketed term is equal to unity, since the numerator is the sum of a deferred temporary employment-based annuity from age

z to r, plus a temporary annuity from age y to z. Thus, we see that the total cost for a participant under the constant dollar cost prorate method, when past service accruals are amortized implicitly, is equal to the present value of future benefits at the start of the plan amortized over future service. The same result holds for the constant percent version of this method.

An approach to determining the annual cost at age x under either version of the cost prorate method, assuming all supplemental liability increments and decrements are to be amortized by the implicit methodology, is illustrated for the constant dollar version. First, solving (6.16) for the normal cost, we have

$$^{CD\,r}(NC)_x = \frac{^r(PVFB)_x - {}^{CD\,r}(AL)_x}{\ddot{a}^T_{x:\overline{r-x}|}}. \qquad (7.16a)$$

In words, the normal cost, which is a constant dollar amount for each attained age, is equal to the portion of the PVFB not yet funded by normal costs (i.e., the PVFB less the actuarial liability) amortized from age x to retirement. If the actuarial liability is replaced by the assets allocated to the employee, then any discrepancy between the actuarial liability and assets (or the unfunded liability balance) will be amortized from age x to retirement (i.e., the implicit amortization method). Thus, the total cost can be expressed as

$$^{CD\,r}(TC)_x = \frac{^r(PVFB)_x - (Assets)_x}{\ddot{a}^T_{x:\overline{r-x}|}}. \qquad (7.16b)$$

If no unfunded liability exists for the employee, (7.16b) simply produces the constant dollar normal cost. If the plan is started at age z, and no other unfunded liability is created, the total cost given by (7.16b) becomes equal to the total cost given by (7.15d), that is, where the past service accruals are amortized over the ages z to r. The important point, however, is that equation (7.16b), and its analogue for the constant percent version, can be used to amortize implicitly the year-to-year (positive or negative) changes in the unfunded liability over the expected future working lifetime of the plan participant.

The total cost under the *aggregate* versions of the cost prorate methods, assuming all unfunded liabilities are amortized implicitly, are given by the following equations:

$$^{ACD\,r}(TC)_t = \left(\sum l_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ ^r(PVFB)_y\right\} - (Assets)_t}{\sum l_{x,y}\ \ddot{a}^T_{y:\overline{r-y}|}}\right]; \quad (7.17a)$$

$$^{ACP\,r}(TC)_t = \left(\sum l_{x,y}\ s_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ ^r(PVFB)_y\right\} - (Assets)_t}{\sum l_{x,y}\ s_y\ ^s\ddot{a}^T_{y:\overline{r-y}|}}\right]. \quad (7.17b)$$

These two versions are popular, in part because of computational convenience, and are often referred to as the *aggregate method.*

EXPLICIT AND IMPLICIT SUPPLEMENTAL COST COMBINATIONS

As noted previously, more than one supplemental cost method can be used with a given actuarial cost method. For example, the individual implicit method might be used for past service accruals under either the accrued benefit or benefit prorate methods, while amortizing all other sources of unfunded liability by an *aggregate* explicit method over m years.

A frequently used method combines the *aggregate* version of the cost prorate method with both *aggregate* implicit and explicit supplemental costs. The appropriate modifications to (7.17a) and (7.17b) are given below, where $\sum SC$ equals the sum of all explicit supplemental costs.

$$^{ACD\,r}(TC)_t = \left(\sum l_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ ^r(PVFB)_y\right\} - (Assets)_t - (ULB)_t}{\sum l_{x,y}\ \ddot{a}^T_{y:\overline{r-y}|}}\right] + \sum SC; \quad (7.18a)$$

$$^{ACP\,r}(TC)_t = \left(\sum l_{x,y}\ s_{x,y}\right)\left[\frac{\left\{\sum l_{x,y}\ ^r(PVFB)_y\right\} - (Assets)_t - (ULB)_t}{\sum l_{x,y}\ s_y\ ^s\ddot{a}^T_{y:\overline{r-y}|}}\right] + \sum SC. \quad (7.18b)$$

The unfunded liability balance (ULB) represents the balance, if any, of the unfunded liability amortized by an explicit method. If the ULB is zero, then (7.18a) and (7.18b) equal (7.17a) and (7.17b). If the ULB is equal to the entire unfunded liability, which requires that the ULB be determined annually using the actuarial liability under the individual cost prorate method, the total cost will be nearly equal to the individual prorate normal cost, the only difference being attributable to the numerical variation caused by the averaging process in (7.18a) and (7.18b).

Another common procedure, sometimes referred to as the *frozen initial liability method* or *frozen entry age method*, is to determine the plan's total unfunded liability at a point in time and amortize it by an explicit method. Similarly, any unfunded liability for future plan or assumption changes is added to this so-called "frozen" liability and amortized explicitly. Experience variations, however, are amortized by the *aggregate* implicit methodology.[5] Under this version, the *ULB* in the above equations would be the current year's balance of the frozen initial unfunded liability (plus any additions thereto).

Still another variation on this theme is to define the unfunded liability at the outset of the plan equal to the present value of accrued benefits (i.e., the actuarial liability as determined under the accrued benefit method). Future benefit and assumption changes are also amortized explicitly, again based on the accrued benefit actuarial liability. Experience variations, however, are amortized by the *aggregate* implicit methodology. This procedure is referred to as the *attained age normal method*.

SUMMARY OF ACTUARIAL COST METHODS

Five normal cost methods were discussed in Chapter 6, as listed in Table 7–1. Three of these methods are benefit-based and two are cost-based. All accomplish the same goal of accounting for or accumulating sufficient funds to meet the benefit obligations under a defined benefit pension plan; however, the incidence of cost during the employee's career is different. This chapter listed five sources of unfunded liabilities and four supplemental cost methods to fund or account for such unfunded liabilities. Again, the goal of each supplemental cost method is the same, namely, to accumulate funds to eliminate the unfunded liability. They differ by the incidence of costs and the period over which the unfunded liability is resolved. It was also noted that more than one supplemental cost method can be used on the various unfunded liability sources. Finally, both the normal cost and supplemental cost methods can be based on either individual

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[5] A variant on this methodology, sometimes called the "frozen, frozen" initial liability method, is to amortize all future changes in the unfunded liability by the implicit methodology.

participants or on the group of participants. Consequently, there are a large number of potential normal cost/supplemental cost combinations, only a few of which are used in practice. Chapter 10 includes a discussion of the more commonly used combinations.

TABLE 7–1
Summary of Actuarial Cost Methods

Normal Cost Methods	**Supplemental Cost Methods**
Accrued Benefit Method	Explicit Methods:
Benefit Prorate Methods:	Straight Line
Constant Dollar	Level Dollar
Constant Percent	Level Percent
Cost Prorate Methods:	Implicit Method
Constant Dollar	
Constant Percent	**Sources of Unfunded Liability**
	Experience Deviations
	Assumption Changes
	Benefit Changes
	Past Service Accruals
	Contribution Variances