PENSION MATHEMATICS with Numerical Illustrations

Second Edition

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Published by

Pension Research Council
Wharton School of the University of Pennsylvania

and

University of Pennsylvania Press
Philadelphia

Library of Congress Cataloging-in-Publication Data
Winklevoss, Howard E.
Pension mathematics with numerical illustrations / Howard E. Winklevoss. –2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-8122-3196-1
1. Pensions–Mathematics. 2. Pensions–Costs–Mathematics. 3. Pension trusts–Accounting. I. Title.
HD7105.W55 1993
331.25'2–dc20 92-44652
CIP

Printed in the United States of America

Chapter 6

Normal Costs

Pension costs can be categorized into two fundamental types: *normal costs* and *supplemental costs*. Normal costs represent the annual cost attributed to the current year of service rendered by active participants, with such costs being defined by one of several actuarial cost methods. In theory, the actuarial accumulation of normal costs from entry age to retirement age will be equal to the liability for the employee's pension benefit at retirement (the retirement-date actuarial liability). The experience of the plan, however, will not precisely match the underlying actuarial assumptions. Moreover, the plan may have granted benefit credits to years prior to its formation (i.e., for periods when normal costs were not calculated), and/or benefit changes or actuarial assumption changes may occur from time to time. Hence, actual normal costs will not accumulate to the retirement-date liability. Supplemental costs are designed to resolve the difference between the theoretical and actual accumulation of normal costs, again according to a specified methodology.

This chapter defines and discusses normal costs, while Chapter 7 deals with supplemental costs. Once both of these cost functions are defined, the minimum and maximum tax deductible contribution limits imposed by federal statutes and the accounting expense required by FASB can be set forth.

Normal costs can be determined on a participant by participant basis, with the plan's overall costs equal to the sum of each individual's costs, or they can be determined by a nearly equivalent calculation involving an aggregation of plan participants. These two methodologies suggest another type of classification for actuarial cost methods, namely, *individual* versus *aggregate*. The term "aggregate" will be reserved herein to refer to this type of aggregation, as opposed to the aggregation of normal and supplemental costs, which is sometimes used in the pension literature and in practice. Moreover, unless the term "aggregate" is used, it is to be understood that the discussion pertains to an "individual" normal cost method.

The discussion begins with a generalized normal cost function, followed by specific definitions. At this point only the normal cost associated with retirement benefits (based on retirement at age r) is considered. The normal cost associated with ancillary benefits is given in Chapter 8.

GENERALIZED NORMAL COST FUNCTION

The retirement-benefit normal cost (NC) for an employee aged x can be represented by the following generalized function:

$$^{r}(NC)_x = b'_x \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r . \quad (y \leq x < r) \qquad (6.1)$$

The prescript on the $^{r}(NC)_x$ notation indicates that only retirement benefits are considered. Any normal cost can be specified by the appropriate definition of b'_x, as described more fully in this chapter.[1] Before defining the normal cost under various actuarial cost methods, we will examine some fundamental normal cost relationships.

In general, normal costs are designed to amortize $^{r}(PVFB)_y$ over the employee's working lifetime, the pattern of amortization payments being governed by the particular actuarial cost method. Thus, the present value of a participant's future normal costs at age y is equal to $^{r}(PVFB)_y$. Notationally, this relationship may be expressed as follows, assuming that normal costs are made at the beginning of each age from the employee's entry age y to one year prior to retirement age r:

$$^{r}(PVFB)_y = {}^{r}(PVFNC)_y = \sum_{t=y}^{r-1} {}^{r}(NC)_t \; {}_{t-y}p_y^{(T)} \; v^{t-y} . \qquad (6.2a)$$

This equation of equality can be demonstrated by writing the left side of (6.2a) in its basic form and substituting (6.1) for $^{r}(NC)_t$:

[1]Normal costs are defined for ages $y \leq x < r$. This age range, however, will not be repeated for each normal cost equation presented.

$$B_r \; {}_{r-y}p_y^{(T)} \; v^{r-y} \; \ddot{a}_r = \sum_{t=y}^{r-1} \left[b_x' \; {}_{r-t}p_t^{(T)} \; v^{r-t} \; \ddot{a}_r \right] {}_{t-y}p_y^{(T)} \; v^{t-y}. \quad (6.2b)$$

The product of ${}_{r-t}p_t^{(T)}$ and ${}_{t-y}p_y^{(T)}$ is ${}_{r-y}p_y^{(T)}$ and the product of v^{r-t} and v^{t-y} is v^{r-y}. Thus, (6.2b) reduces to the defined relationship:

$$B_r = \sum_{t=y}^{r-1} b_x'. \quad (6.2c)$$

This relationship is applicable for the normal costs under all actuarial cost methods, and illustrates that normal costs do indeed amortize ${}^r(PVFB)_y$ over the period from age y to age r.

Continuing with the amortization concept, it also follows that the actuarial liability at age x is equal to the present value of future benefits (PVFB) at that age less the present value of future normal costs (PVFNC) yet to be made (i.e., the portion of ${}^r(PVFB)_x$ not yet amortized):

$${}^r(AL)_x = {}^r(PVFB)_x - {}^r(PVFNC)_x. \quad (6.3a)$$

Demonstrating that this relationship holds, we write ${}^r(PVFB)_x$ and ${}^r(PVFNC)_x$ in their basic form, as follows:

$$\begin{aligned} {}^r(AL)_x &= B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r \\ &\quad - \sum_{t=x}^{r-1} \left[b_x' \; {}_{r-t}p_t^{(T)} \; v^{r-t} \; \ddot{a}_r \right] {}_{t-x}p_x^{(T)} \; v^{t-x}. \end{aligned} \quad (6.3b)$$

The bracketed term in equation (6.3b) represents the normal cost function. The right side of (6.3b) can be written as

$$= B_r \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r - \left(\sum_{t=x}^{r-1} b_x' \right) {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r, \quad (6.3c)$$

and since

$$B_r - \sum_{t=x}^{r-1} b_x' = B_x', \quad (6.3d)$$

equation (6.3b) simplifies to

$${}^r(AL)_x = B_x' \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r. \quad (6.3e)$$

Equation (6.3e) is the actuarial liability definition given by (5.3); hence, the relationship defined by (6.3a) is valid.

Another definition of the actuarial liability in terms of normal costs is the so-called retrospective approach, in contrast to the prospective approach studied above. Under this definition the actuarial liability is equal to the accumulated value of past normal costs (AVPNC):[2]

$$ {}^{r}(AL)_x = {}^{r}(AVPNC)_x . \tag{6.4a} $$

The right side of this equation can be expressed as

$$ {}^{r}(AL)_x = \sum_{t=y}^{x-1} {}^{r}(NC)_t \, (1+i)^{x-t} \frac{1}{{}_{x-t}p_t^{(T)}} \tag{6.4b} $$

and, substituting the normal cost definition given in (6.1), we have

$$ {}^{r}(AL)_x = \sum_{t=y}^{x-1} \left[b_t^{'} \; {}_{r-t}p_t^{(T)} \, v^{r-t} \, \ddot{a}_r \right] (1+i)^{x-t} \frac{1}{{}_{x-t}p_t^{(T)}} \tag{6.4c} $$

which reduces to

$$ {}^{r}(AL)_x = \left(\sum_{t=y}^{x-1} b_t^{'} \right) {}_{r-x}p_x^{(T)} \, v^{r-x} \, \ddot{a}_r \tag{6.4d} $$

$$ = B_x^{'} \; {}_{r-x}p_x^{(T)} \, v^{r-x} \, \ddot{a}_r . \tag{6.4e} $$

Equation (6.4e) is equal to the prospective actuarial liability definition given by (5.3); hence, the actuarial accumulation of past normal costs at age x is equal to the actuarial liability at that age.

The prior normal costs in equation (6.4b) increase to the current age by two factors: the benefit of interest and what is referred to as the "benefit of survivorship." The benefit of interest is a straightforward concept, but the benefit of survivorship may not be at first glance. In order to explore the latter, equation

[2]In actual fact, past normal costs may not accumulate to the value of ${}^{r}(PVFB)_x$ because of the granting of past service credits for years prior to the plan establishment, plan changes, actuarial assumption changes, or experience deviation from actuarial assumptions.

(6.4b) is written with ${}_{r-x}p_x^{(T)}$ replaced by its corresponding service table components:[3]

$$^r(AL)_x = \sum_{t=y}^{x-1} {}^r(NC)_t \, (1 + i)^{x-t} \frac{l_t^{(T)}}{l_x^{(T)}}. \qquad (6.5)$$

In this form we see that a normal cost is generated on behalf of all of the hypothetical employees at each age t, yet the total accumulation at age x is allocated to the lesser number of those who survive in service to this age, hence, the term benefit of survivorship in service. Although this effect is called the "benefit of survivorship," the normal cost is determined with full recognition of this gain from non-survivors.

In theory, normal costs can take on any positive or negative value during an employee's working lifetime. The only theoretical restriction on age-specific normal cost values is that their present value (or accumulated value) satisfy the above relationships. Since an infinite number of normal cost patterns could be determined such that these conditions hold, there exists an infinite number of possible actuarial cost methods, only a few of which are formally recognized and discussed in this book.

A more general retrospective definition of the actuarial liability, appropriate both before and after retirement, is given by

$$^r(AL)_x = \sum_{t=y}^{x-1} [{}^r(NC)_t - B_t] \, (1 + i)^{x-t} \frac{1}{{}_{x-t}p_t^{(T)}}. \qquad (6.6)$$

The annual pension benefit, B_t, in this formulation is zero prior to retirement and equal to B_r after retirement. Conversely, ${}^r(NC)_x$ would be positive prior to retirement and zero after retirement.

NORMAL COST UNDER ACTUARIAL COST METHODS

Each actuarial cost method discussed in Chapter 5 has a corresponding normal cost function. In effect, the normal cost represents the growth in the actuarial liability from one year to the next, reflecting a larger accrued benefit along with interest and

[3] $l_x^{(T)}$ is the number of survivors out of an arbitrary number of employees beginning at some entry age y and who are exposed to the various decrements from y to x. See Chapter 3, Table 3–2, for an example of the $l_x^{(T)}$ function.

survivorship adjustments. As indicated by the general normal cost function given in (6.1), the normal cost represents the present value of the current year's benefit accrual. Normal costs differ among actuarial cost methods by the benefit deemed to accrue at each attained age.

Accrued Benefit Method

The *individual* normal cost under the accrued benefit method is defined by equation (6.1) with b_x determined by the natural accruals that result from applying the plan's benefit formula to the employee's current service and, if applicable, current salary:

$$^{AB\,r}(NC)_x = b_x \; {}_{r-x}p_x^{(T)} \; v^{r-x} \; \ddot{a}_r . \tag{6.7a}$$

The normal cost for the entire plan is equal to the sum of the normal costs for each participant.

The *aggregate* accrued benefit (AAB) normal cost for the entire active plan membership is defined as:

$$^{AAB\,r}(NC) = \left(\sum l_{x,y} \, b_{x,y}\right) \left[\frac{\sum l_{x,y} \; {}^r(PVFB)_{x,y}}{\sum l_{x,y} \; B_{r,y}}\right] \tag{6.7b}$$

where

$\sum$ = summation over all entry age, attained age combinations ($y < r$; $y \le x < r$)

$l_{x,y}$ = number of age-y entrants currently age x

$b_{x,y}$ = benefit accrual at age x for an age-y entrant

$B_{r,y}$ = accrued benefit at age r for an age-y entrant

${}^r(PVFB)_{x,y}$ = present value of future benefits at age x for an age-y entrant.

If there is only one active employee, then (6.7b) simplifies to the normal cost under the individual accrued benefit method given by (6.7a).

The individual normal cost version is used in practice and recognized by federal statutes as a valid method for use with benefit formulas other than the final average salary type. The aggregate version is of only theoretical interest, yet the formulation establishes a methodology for aggregate methods in general, namely, that the numerator and denominator of such methods are weighted by the number of participants at each x,y combination, with the result then multiplied by the total number of plan participants (or compensation). The numerical results of the individual and aggregate methodologies, while not identical, are typically quite similar.

Benefit Prorate Methods

Specifying b_x in (6.1) as previously defined in either (3.15a) or (3.16a) defines the normal cost under two benefit prorate methods. The first definition represents a service proration of B_r which produces a constant dollar benefit accrual, while the second represents a salary proration which produces a benefit accrual equal to a constant percent of salary. The equation for each is given below, with the prescripts indicating the type of cost method and type of proration, respectively:

$$^{BD}\,{}^{r}(NC)_x = \frac{B_r}{r-y}\;{}_{r-x}p_x^{(T)}\; v^{r-x}\; \ddot{a}_r; \tag{6.8}$$

$$^{BP}\,{}^{r}(NC)_x = \frac{B_r}{S_r}\; s_x\;{}_{r-x}p_x^{(T)}\; v^{r-x}\; \ddot{a}_r. \tag{6.9}$$

The actuarial liability associated with these normal costs are given by equations (5.6a) and (5.7a), respectively.

Both of these individual methods have aggregate counterparts which, again, are only of theoretical interest inasmuch as they are seldom, if ever, used in practice. First, equations (6.8) and (6.9) are rewritten in an equivalent form using the ${}^{r}(PVFB)_x$ function:

$$^{BD}\,{}^{r}(NC)_x = \frac{{}^{r}(PVFB)_x}{r-y}; \tag{6.10}$$

$$^{BP}\,{}^{r}(NC)_x = s_x\,\frac{{}^{r}(PVFB)_x}{S_r}. \tag{6.11}$$

The aggregate versions, which involve determining the numerator and denominator for the entire group of active employees, is given by the following two equations:

$$^{ABD}\,{}^{r}(NC) = \left(\sum l_{x,y}\right)\left[\frac{\sum l_{x,y}\;{}^{r}(PVFB)_{x,y}}{\sum l_{x,y}\;(r-y)}\right]; \tag{6.12}$$

$$^{ABP\,r}(NC) = \left(\sum l_{x,y}\, s_{x,y}\right)\left[\frac{\sum l_{x,y}\;^{r}(PVFB)_{x,y}}{\sum l_{x,y}\, S_{r,y}}\right] \qquad (6.13)$$

where

$s_{x,y}$ = salary at age x for an age-y entrant

$S_{r,y}$ = cumulative salary from age y to r.

Cost Prorate Methods

The traditional terminology for this family of normal costs is the *projected benefit cost method* or *entry age cost method*. As we will see, the term "cost prorate" is more precise in describing the methodology used with these methods.

The previously described normal cost methods make the *benefit accrual* associated with the employee the independent variable and the corresponding normal cost the dependent variable. In contrast, the cost prorate methods designate the *normal cost* as the independent variable and the corresponding benefit accrual, which can be derived from the normal cost function, becomes the dependent variable. The switching of the independent and dependent variables has significant financial implications.

There are two forms of this method: one with normal costs equal to a constant dollar amount throughout the employee's working lifetime, and the other with costs equal to a constant percentage of the employee's salary, hence, the term *cost prorate method*. The first form is typically used with plans where benefits are not based on salary, while the second is used with career average and final average benefit formulas.

The "cost prorate, constant dollar" normal cost is defined by first writing the fundamental identity that future normal costs at age y must equal the present value of future benefits at that age, and then solving for the constant dollar normal costs.

$$^{CD\,r}(NC)_y\; \ddot{a}^{T}_{y:\overline{r-y|}} = {}^{r}(PVFB)_y ; \qquad (6.14a)$$

$$^{CD\,r}(NC)_y = \frac{^{r}(PVFB)_y}{\ddot{a}^{T}_{y:\overline{r-y|}}}. \qquad (6.14b)$$

The temporary annuity was defined previously by (3.22). The normal cost at age y, by definition, is applicable to all attained ages under this method. Equation (6.14b) also illustrates, once again, that the present value of future benefits is amortized by normal costs.

The "cost prorate, constant percent" normal cost can be determined first by equating the present value of a portion, K, of the participant's future salary to the present value of future benefits:

$$K\ s_y\ {}^s\ddot{a}^T_{y:\overline{r-y}|} = {}^r(PVFB)_y; \tag{6.15a}$$

$$K = \frac{{}^r(PVFB)_y}{s_y\ {}^s\ddot{a}^T_{y:\overline{r-y}|}}. \tag{6.15b}$$

Then, the normal cost at age x is simply this factor times attained age salary:

$${}^{CP\,r}(NC)_x = K\ s_x. \tag{6.15c}$$

If salary is an increasing function of age, the normal cost under this version represent an ever-increasing dollar amount.

The actuarial liabilities associated with these two methods were given previously by equations (5.8a) and (5.9a), respectively. The more conventional expressions for these actuarial liabilities, however, are the prospective definitions:

$${}^{CD\,r}(AL)_x = {}^r(PVFB)_x - {}^{CD\,r}(NC)_x\ \ddot{a}^T_{x:\overline{r-x}|}; \tag{6.16}$$

$${}^{CP\,r}(AL)_x = {}^r(PVFB)_x - {}^{CP\,r}(NC)_x\ {}^s\ddot{a}^T_{x:\overline{r-x}|}. \tag{6.17}$$

The equality of these expressions with those of (5.8b) and (5.9b) is as follows, using (6.16) to illustrate the identity. First, the normal cost symbol in (6.16) is replaced by (6.14b):

$${}^{CD\,r}(AL)_x = {}^r(PVFB)_x - \frac{{}^r(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}}\ \ddot{a}^T_{x:\overline{r-x}|}. \tag{6.18a}$$

Replacing ${}^r(PVFB)_y$ by ${}_{x-y}p_y^{(T)}\ v^{x-y}\ {}^r(PVFB)_x$ and factoring out ${}^r(PVFB)_x$, one obtains

$${}^{CD\,r}(AL)_x = {}^r(PVFB)_x \left[1 - \frac{{}_{x-y}p_y^{(T)}\ v^{x-y}\ \ddot{a}^T_{x:\overline{r-x}|}}{\ddot{a}^T_{y:\overline{r-y}|}} \right]. \tag{6.18b}$$

With a common denominator, (6.18b) becomes

$$^{CD\,r}(AL)_x = {}^r(PVFB)_x \left[\frac{\ddot{a}^T_{y:\overline{r-y}|} - {}_{x-y}p_y^{(T)}\, v^{x-y}\, \ddot{a}^T_{x:\overline{r-x}|}}{\ddot{a}^T_{y:\overline{r-y}|}} \right]. \tag{6.18c}$$

The numerator represents a temporary employment-based annuity running from age y to age x, i.e., a temporary annuity from y to r minus a deferred temporary annuity payable from x to r. Thus, equation (6.18c) simplifies to (5.8b). The same result can be obtained for the constant-percent-of-salary version.

The above formulations of the cost prorate methods do not produce a benefit accrual factor, b_x, that can be used in the generalized normal cost equation (6.1). Such a factor can be derived, however, by replacing the normal cost notation in (6.1) with the cost prorate normal cost and solving for b_x. This is illustrated using the cost prorate, constant dollar normal cost:

$$\frac{^r(PVFB)_y}{\ddot{a}^T_{y:\overline{r-y}|}} = b_x\ {}_{r-x}p_x^{(T)}\, v^{r-x}\, \ddot{a}_r; \tag{6.19a}$$

$$b_x = \frac{B_r\ {}_{x-y}p_y^{(T)}\, v^{x-y}}{\ddot{a}^T_{y:\overline{r-y}|}}. \tag{6.19b}$$

The second and third factors of the numerator decrease significantly with age, implying that the portion of B_r allocated to each attained age is a sharply decreasing function. This is in contrast to the portion of B_r allocated under the other methods discussed previously, where B_r is increasing or constant. The corresponding benefit accrual for the cost prorate, constant percent method is

$$b_x = \frac{s_x\ B_r\ {}_{x-y}p_y^{(T)}\, v^{x-y}}{s_y\ {}^s\ddot{a}^T_{y:\overline{r-y}|}}. \tag{6.19c}$$

The *aggregate* versions of these two normal cost methods can be written in the following manner:

$$^{ACD\,r}(NC) = \left(\sum l_{x,y}\right) \left[\frac{\sum l_{x,y}\ {}^r(PVFB)_y}{\sum l_{x,y}\ \ddot{a}^T_{y:\overline{r-y}|}} \right]; \tag{6.20a}$$

$$^{ACP\,r}(NC) = \left(\sum l_{x,y}\ s_{x,y}\right) \left[\frac{\sum l_{x,y}\ {}^r(PVFB)_y}{\sum l_{x,y}\ s_y\ {}^s\ddot{a}^T_{y:\overline{r-y}|}} \right]. \tag{6.20b}$$

Summary of Normal Costs Under Actuarial Cost Methods

The *individual* normal costs defined thus far can all be expressed as a fraction of the ${}^{r}(PVFB)_x$ function, much the same way their corresponding actuarial liabilities were expressed:

$$ {}^{r}(NC)_x = k\ {}^{r}(PVFB)_x \tag{6.21} $$

where k is defined as follows:

k	*Actuarial Cost Method*
$\dfrac{b_x}{B_r}$	Accrued Benefit Method
$\dfrac{s_x}{S_r}$	Benefit Prorate Constant Percent Method
$\dfrac{1}{r-y}$	Benefit Prorate Constant Dollar Method
$\dfrac{s_x\ {}_{x-y}p_y^{(T)}\ v^{x-y}}{s_y\ {}^{s}\ddot{a}^{T}_{y:\overline{r-y}\rvert}}$	Cost Prorate Constant Percent Method
$\dfrac{{}_{x-y}p_y^{(T)}\ v^{x-y}}{\ddot{a}^{T}_{y:\overline{r-y}\rvert}}$	Cost Prorate Constant Dollar Method

In theory there need not be any restriction on the value of k in (6.21), provided that the sum of such fractions, from entry age y to one year prior to retirement r, equal unity. Thus, an infinite number of actuarial cost methods exist, since this condition can be met by an infinite number of attained age patterns of k.

It is interesting to consider two extreme cases for the value of k, where k is equal to zero at all ages except one, at which age it takes on the value of unity. If the single age is the employee's entry-age y, the entire projected benefit is accounted for at that point. The normal cost, of course, would equal ${}^{r}(PVFB)_y$ and the actuarial liability at each age thereafter would equal ${}^{r}(PVFB)_x$. This method is known as *initial funding* in the context of pension plan funding.

If the single age for which k is equal to unity is the participant's retirement age, then the normal cost is equal to ${}^{r}(PVFB)_r$ for this single age. The actuarial liability is zero up to this age, and equals ${}^{r}(PVFB)_x$ thereafter (for $x > r$). In the context of pension plan funding, this method is known as *terminal funding.*

Table 6–1 shows the various normal costs for an age-30 entrant under the model actuarial assumptions, all expressed as a percent of attained age salary. These values are plotted in Figure 6–1. The methods produce normal cost values that are relatively dispersed near the employee's entry age, reasonably close midway through the employee's career, and substantially different as the employee approaches retirement. Clearly, those methods with the lowest initial costs have the highest costs near retirement, and vice versa.

The cost patterns under the accrued benefit cost method and the benefit prorate (constant percent) method might appear to be undesirable inasmuch as they increase sharply throughout the employee's career. However, a large plan census with a relatively stable age and service distribution will produce a reasonably constant normal cost percentage for the entire plan under all actuarial cost methods. In this case, the normal cost percentages will differ among the methods depending on the average age and service of plan participants. The benefit prorate methods will produce the lowest costs for a relatively undermature active employee population and vice versa for a relatively overmature population. These dynamics will be displayed at a later point in this book.

The percentage of the age-30 entrant's projected retirement benefit allocated to each age under each actuarial cost method and the model assumptions is given in Table 6–2 and graphed in Figure 6–2. Table 6–3 shows the cumulative percentages given in Table 6–2, with these values being graphed in Figure 6–3. The cost prorate methods have cumulative benefit allocations that are far greater than those of the benefit prorate methods. For example, one-half of the projected benefit is allocated by age 35 under the constant dollar version and by age 40 under the constant percent version. This is in sharp contrast to the accrued benefit method which allocates one-half of the projected benefit by age 57.

Figure 6–4 shows the various normal costs for the model pension plan and the mature population, all expressed as a percent of payroll. These normal cost values include the cost associated with ancillary benefits, a subject yet to be discussed. The normal

TABLE 6–1

Normal Cost Functions as a Percent of Attained Age Salary

Age	Accrued Benefit Method	Benefit Prorate Methods Constant Percent	Benefit Prorate Methods Constant Dollar	Cost Prorate Methods Constant Percent	Cost Prorate Methods Constant Dollar
30	0.12	0.25	1.05	3.24	6.08
32	0.19	0.41	1.46	3.24	5.15
34	0.32	0.59	1.82	3.24	4.38
36	0.50	0.81	2.13	3.24	3.74
38	0.73	1.09	2.44	3.24	3.21
40	1.04	1.43	2.76	3.24	2.76
42	1.45	1.86	3.10	3.24	2.39
44	1.99	2.40	3.46	3.24	2.07
46	2.71	3.08	3.87	3.24	1.81
48	3.65	3.94	4.34	3.24	1.58
50	4.89	5.05	4.88	3.24	1.39
52	6.54	6.48	5.52	3.24	1.22
54	8.70	8.32	6.27	3.24	1.08
56	11.14	10.33	6.91	3.24	0.96
58	13.79	12.42	7.41	3.24	0.86
60	17.12	15.05	8.04	3.24	0.77
62	21.44	18.45	8.86	3.24	0.69
64	27.31	23.10	10.02	3.24	0.62

FIGURE 6–1

Normal Costs as a Percent of Salary Under Various Actuarial Cost Methods

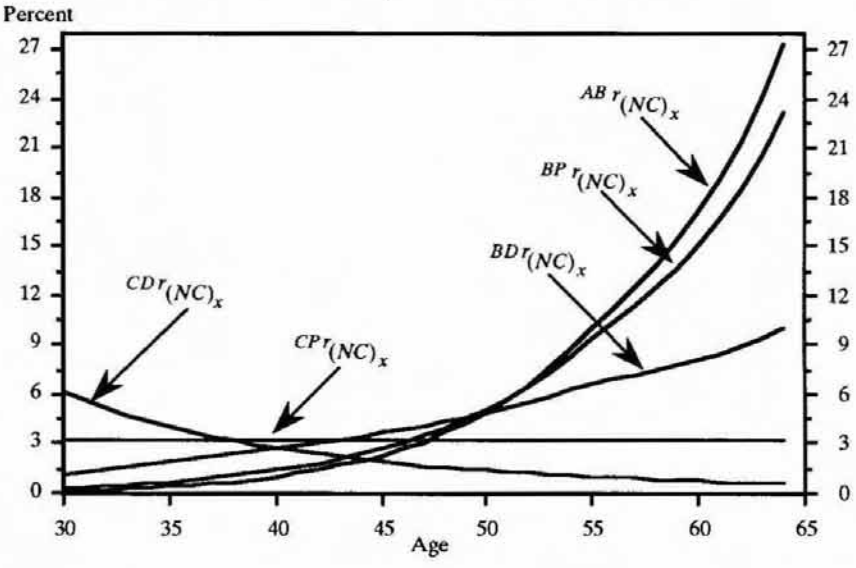


TABLE 6–2

Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

	Accrued	Benefit Prorate Methods		Cost Prorate Methods	
	Benefit	Constant	Constant	Constant	Constant
Age	*Method*	*Percent*	*Dollar*	*Percent*	*Dollar*
30	0.32	0.67	2.86	8.78	16.48
32	0.37	0.79	2.86	6.34	10.09
34	0.50	0.93	2.86	5.09	6.89
36	0.67	1.09	2.86	4.35	5.03
38	0.85	1.28	2.86	3.80	3.76
40	1.08	1.48	2.86	3.36	2.86
42	1.34	1.71	2.86	2.99	2.21
44	1.64	1.98	2.86	2.67	1.71
46	2.00	2.27	2.86	2.39	1.33
48	2.40	2.60	2.86	2.13	1.04
50	2.86	2.96	2.86	1.90	0.81
52	3.38	3.35	2.86	1.68	0.63
54	3.96	3.79	2.86	1.48	0.49
56	4.61	4.27	2.86	1.34	0.40
58	5.31	4.79	2.86	1.25	0.33
60	6.08	5.35	2.86	1.15	0.27
62	6.91	5.95	2.86	1.04	0.22
64	7.79	6.59	2.86	0.92	0.18

FIGURE 6–2

Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

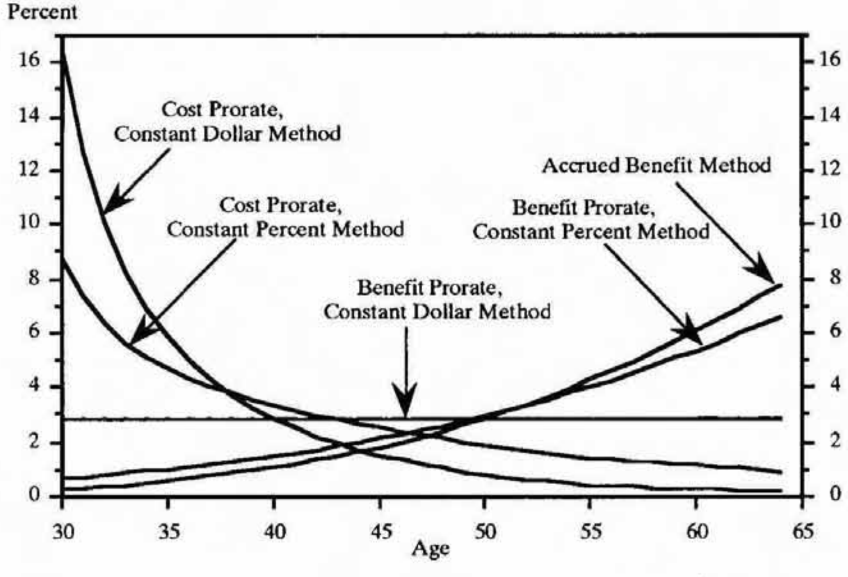


TABLE 6–3

Cumulative Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

Age	Accrued Benefit Method	Benefit Prorate Methods: Constant Percent	Benefit Prorate Methods: Constant Dollar	Cost Prorate Methods: Constant Percent	Cost Prorate Methods: Constant Dollar
30	0.00	0.00	0.00	0.00	0.00
32	0.65	1.41	5.71	16.12	29.16
34	1.45	3.06	11.43	28.08	47.50
36	2.53	5.01	17.14	37.84	60.24
38	3.95	7.28	22.86	46.24	69.60
40	5.77	9.93	28.57	53.60	76.65
42	8.05	13.01	34.29	60.13	82.02
44	10.87	16.57	40.00	65.94	86.17
46	14.33	20.66	45.71	71.14	89.39
48	18.52	25.36	51.43	75.79	91.90
50	23.55	30.73	57.14	79.93	93.85
52	29.53	36.83	62.86	83.61	95.38
54	36.58	43.76	68.57	86.86	96.57
56	44.82	51.57	74.29	89.72	97.50
58	54.38	60.36	80.00	92.35	98.25
60	65.38	70.21	85.71	94.80	98.88
62	77.96	81.20	91.43	97.05	99.40
64	92.21	93.41	97.14	99.08	99.82
65	100.00	100.00	100.00	100.00	100.00

FIGURE 6–3

Cumulative Percentage of Projected Retirement Benefit Allocated to Each Age Under Various Actuarial Cost Methods

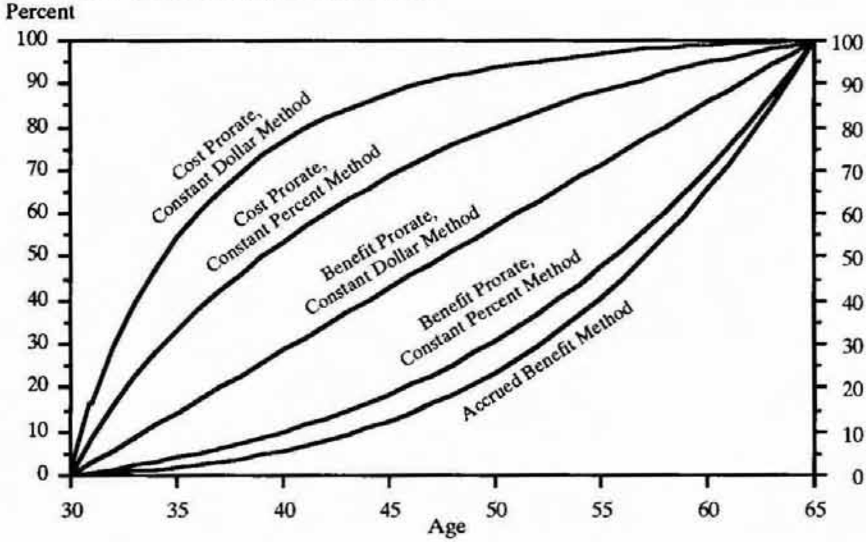
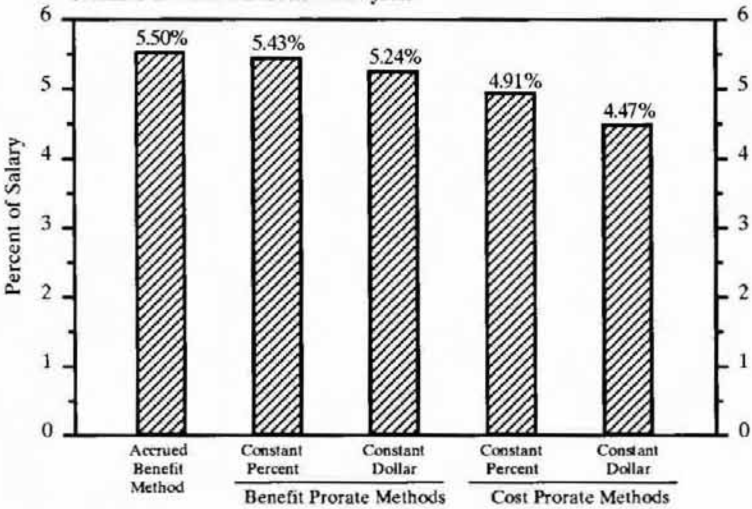


FIGURE 6–4
Normal Costs as a Percent of Payroll



costs for each method are not substantially different; the accrued benefit cost method has a 5.5% normal cost, while at the other extreme, the cost prorate, constant dollar method has a 4.5% normal cost. These differences, however, become more significant when compared for undermature and overmature plans.

PLAN TERMINATION COST METHOD

This chapter has defined the normal cost for each of the actuarial liabilities set forth in Chapter 5. In addition, the point was made that there exists an infinite number of actuarial liabilities and corresponding normal costs. Therefore, it is logical to ask what the so-called "normal cost" would be for the plan continuation and plan termination liabilities presented in Chapter 5. Since the plan continuation liability for accrued benefits is mathematically identical to the actuarial liability under the accrued benefit cost method, it stands to reason that they have identical normal costs.

The normal cost for the plan termination liability can be derived by examining the progression of year-to-year values.[4] The normal cost at age x is equal to the difference between the present value of the liability at age $x + 1$ less the liability at age x:

[4]The normal cost for each of the previously defined actuarial cost methods can also be defined in terms of the annual growth in their actuarial liability.

$$^{PT}(NC)_x = p_x^{(T)} v\,(PTL)_{x+1} - (PTL)_x. \tag{6.22a}$$

Upon substituting the components making up the PTL, equation (6.22a) becomes

$$^{PT}(NC)_x = p_x^{(T)} v\left[B_{x+1}\ {}_{r-x-1}p_{x+1}^{(m)}\ v^{r-x-1}\ \ddot{a}_r\right] - \left[B_x\ {}_{r-x}p_x^{(m)}\, v^{r-x}\, \ddot{a}_r\right]. \tag{6.22b}$$

Equation (6.22b) reduces to

$$^{PT}(NC)_x = \left[B_{x+1}\ p_x^{(T)}\ {}_{r-x-1}p_{x+1}^{(m)} - B_x\ {}_{r-x}p_x^{(m)}\right] v^{r-x}\, \ddot{a}_r. \tag{6.22c}$$

Since $p_x^{(T)} = p_x^{(m)}\, p_x^{(w)}\, p_x^{(d)}\, p_x^{(r)}$, equation (6.22c) may be written as

$$^{PT}(NC)_x = \left[B_{x+1}\, p_x^{(w)}\, p_x^{(d)}\, p_x^{(r)} - B_x\right]\, {}_{r-x}p_x^{(m)}\, v^{r-x}\, \ddot{a}_r. \tag{6.22d}$$

Although B_{x+1} exceeds B_x, it is possible for this excess to be more than offset by the product of the withdrawal, disability, and retirement rates, especially for young employees. If this were to occur, the normal cost as given by (6.22d) could take on negative values at some ages beyond age y.

Figure 6-5 compares the normal cost of the plan termination liability method to that of the accrued benefit cost method for an age-30 entrant.

FIGURE 6–5

Normal Cost Under Plan Termination Method vs. Accrued Benefit Method