PENSION MATHEMATICS
with Numerical Illustrations

Second Edition

Howard E. Winklevoss, Ph.D., MAAA, EA
President
Winklevoss Consultants, Inc.

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Chapter 15

Investment Policy: Asset Allocation

The plan's investment policy includes the asset allocation decision, investment manager selection, and performance measurement. The purpose of this chapter is to discuss only the methodology for establishing the plan's asset allocation policy, since the other facets of the investment policy are beyond the scope of this book.

The plan's asset allocation is one of the most important, if not *the* most important, decision that the plan sponsor makes. While it is true that alternative actuarial assumptions and cost methods can have a significant impact on current year costs and the incidence of costs over time, they have no permanent impact on the economics of the pension plan. The asset allocation, on the other hand, has a direct impact on such economics. The ultimate cost of a pension plan can be represented as follows:

$$Ultimate\ Cost = B_t - d\ (Assets)_t \tag{15.1}$$

where

B_t = total benefits paid in year t

$$d = \frac{i}{1+i}.$$

The larger the level of assets, the lower the ultimate cost. Since the asset allocation decision has a direct bearing on the level of assets, its importance cannot be overemphasized.

Ironically, the asset allocation decision is given far less attention than other aspects of the plan's investment policy. Corporate America is notorious for spending large amounts of effort and money on investment manager selection and performance

measurement, even though less than 10 percent of the plan's total return over an extended period of time is accounted for by this aspect of the investment policy. Conversely, 90 percent of the plan's return is due to the asset allocation decision.

More often than not, the asset allocation decision is improperly analyzed. The focus is often on the near-term volatility of alternative asset classes and, in particular, the possibility that equity returns will be poor for a period of time. In most cases, this is the wrong focus. The critical variables are the volatility of contributions, pension expense and, possibly, the funded ratio of the plan. Because of the various asset smoothing techniques employed in pension funding and accounting, most plans can withstand a significant near-term negative return on assets without having a dramatic increase in pension costs.

For example, if plan assets are valued on a 5-year moving average basis and investment gains and losses are amortized over 5 years, then approximately 5 percent of any year's market loss is recognized for cost purposes in the following year. The SFAS 87 pension cost is similarly insulated from sharp downturns in asset values. The point is that most pension plans can withstand significant near-term losses in investments without having a dramatic affect on costs; hence, the equity exposure can be higher than what management may believe to be prudent by only considering their risk/return characteristics.

If one believes that equities will outperform fixed income investments over the long term, and if the plan's costs and pension expense are sufficiently smoothed, then it is clear that a high percentage of equities (perhaps even 100 percent) is the logical asset allocation. This conclusion might not be appropriate, however, if the plan sponsor viewed the disclosure of the plan's funded status on a market value basis to be particularly important to its overall financial welfare. For example, if the plan were underfunded in the first instance and/or if the corporation's capital structure were highly leveraged, then it may be prudent to "leave some money on the table" with respect to the long-term return from equities as compared to fixed income investment, and to reduce the equity exposure, whether or not contributions and expenses are well insulated from gyrations in the equity market. Similarly, a corporation not paying taxes (or subject to the alternative minimum tax), may want to avoid an increase in near-term contributions by selecting a high percentage of short-term, fixed

income investments even though the plan's long-term costs are likely to be higher as a result of this asset allocation decision.

Although it is important to consider the corporation's current circumstances in making the asset allocation decision, in most cases the end result will contain a far greater proportion of equities than currently exists.

CAPITAL MARKET SIMULATION

The first step in performing an asset allocation study is to simulate the returns of various asset classes. The conventional method is to establish, for each asset class, a real rate of return, the volatility of returns (standard deviation), and the covariability (or correlation) of returns among the classes. In addition, since inflation affects asset returns as well as liability values, this variable should also be included.

An equation for simulating inflation is given by

$$I_t = w\, I_{t-1} + (1 - w)\, I_\infty + {}^{Inf}e_t \qquad (15.2)$$

where

I_t = inflation in year t

w = serial correlation of successive year's inflation

I_∞ = long-term rate of inflation

${}^{Inf}e_t$ = error term for unexpected inflation in year t, drawn from a log normal distribution with mean zero and specified standard deviation.

The value of w is typically between .60 and .75, with 2/3 being a reasonable choice. In effect, this coefficient says that 2/3 of the time last year's inflation rate is a good predictor of next year's, whereas 1/3 of the time next year's rate is significantly different from the previous year's. The simulation process is performed by drawing random variables from the error term distribution and adding these values to the weighted average of the previous year's simulated rate and long-run inflation.

The nominal return for a given asset class can be written as follows:

$$(NR)_t = (RR)_t + E(I_t) + {}^{AC}e_t \qquad (15.3)$$

where

$(NR)_t$ = nominal return in year t

$(RR)_t$ = real return in year t

$E(I_t)$ = expected inflation in year t

$E(I_t)$ = $w\, I_{t-1} + (1 - w)\, I_\infty$ from equation (15.2)

${}^{AC}e_t$ = error term for asset class (AC) in year t, drawn from a log normal distribution with mean zero and specified standard deviation.

The error terms for each asset class and inflation are linked to each other by a specified covariance (or correlation).

There are a number of variations that one could make in the above equations, such as allowing the real returns, error terms, and/or correlations to follow a dynamic pattern over time rather than being stationary. The following numerical illustrations, however, are based on the previous equations without such embellishments.

Capital Market Statistics

Table 15–1 shows statistics on inflation and returns on five asset classes over the 20-year period 1971–1990.[1] Inflation is based on the Consumer Price Index for All Urban Consumers. The statistic shown under "nominal return" for inflation is the geometric average of the CPI over this period, whereas the standard deviation and correlations for inflation are based on an unexpected inflation variable.[2] T-Bills are based on 30-day bonds. Long-term bonds (L-Bonds) are based on Solomon Brothers Long-Term, High-Grade Corporate Bond Index, which includes nearly all bonds rated Aaa and Aa. The S&P 500 index is used for domestic stock returns. Small stocks (S-Stocks) are defined as the fifth (smallest) quintile on the NYSE, plus stocks on the AMEX and OTC with the same capitalization upper and lower bounds as the NYSE fifth quintile. World stocks (W-Stocks),

[1]The source of these data is Ibbotson Associates' Benchmark PLUS (Chicago, Illinois, 1991).

[2]There are a number of ways to define unexpected inflation from an inflation index. The method used in the above statistic was to determine the delta from a linear regression over the period from 1926 to 1990.

TABLE 15–1

Capital Market Statistics, 1971–1990
(Means and Standard Deviations in Percent)

	Nominal Return	*Real Return*	*Std. Dev.*	*Real Return Correlations*					
				*Inf**	*T-Bills*	*L-Bonds*	*S&P*	*S-Stocks*	*W-Stocks*
Inf*	6.26		2.64	1.00					
T-Bills	7.66	1.40	3.07	–.71	1.00				
L-Bonds	9.01	2.75	13.97	–.79	.68	1.00			
S&P	11.15	4.89	17.15	–.66	.41	.60	1.00		
S-Stocks	13.35	7.09	23.20	–.46	.10	.28	.71	1.00	
W-Stocks	15.79	9.53	27.13	–.30	.03	.30	.64	.41	1.00

*Standard deviations and correlations are based on unexpected inflation.

which exclude U.S. stocks, is based on the FT-Actuaries World Indices which includes data from 23 countries. The latter data are available for the past 10 years only.

The capital market simulation is parameterized by using the geometric mean rather than the arithmetic mean; hence, this statistic is shown for each index. The geometric mean is less than the arithmetic mean, the degree to which depends on the dispersion of yearly returns. As an example, the arithmetic mean of the S&P 500 nominal return is 12.49 percent over the 20-year period versus the geometric mean of 11.15 percent.

The real returns for all asset classes are negatively correlated with unexpected inflation, whereas the various asset classes are positively correlated to each other over this time period.

Illustrative Capital Market Parameters

Based on the data presented in Table 15–1, and to emphasize the point that the parameterization of a capital market simulation should be based, in part, on historical data and, in part, on judgment, the capital market parameters used in the simulations presented in this chapter are shown in Table 15–2. The long term inflation rate is set at 4 percent, with the incremental real returns being approximately equal to those shown in Table 15–1, but reduced for S-Stocks and W-Stocks. The standard deviations and correlations were selected to be "round numbers" reasonably consistent with historical data.

TABLE 15–2

Capital Market Assumptions
(Means and Standard Deviations in Percent)

	Nominal Return	*Real Return*	*Std. Dev.*	*Real Return Correlations*					
				*Inf**	*T-Bills*	*L-Bonds*	*S&P*	*S-Stocks*	*W-Stocks*
Inf*	4.0		2.0	1.00					
T-Bills	5.0	1.0	2.0	–.70	1.00				
L-Bonds	7.0	3.0	10.0	–.75	.60	1.00			
S&P	9.0	5.0	18.0	–.55	.40	.50	1.00		
S-Stocks	10.0	6.0	25.0	–.35	.10	.20	.70	1.00	
W-Stocks	11.0	7.0	27.0	–.30	.00	.20	.60	.35	1.00

*Standard deviations and correlations are based on unexpected inflation.

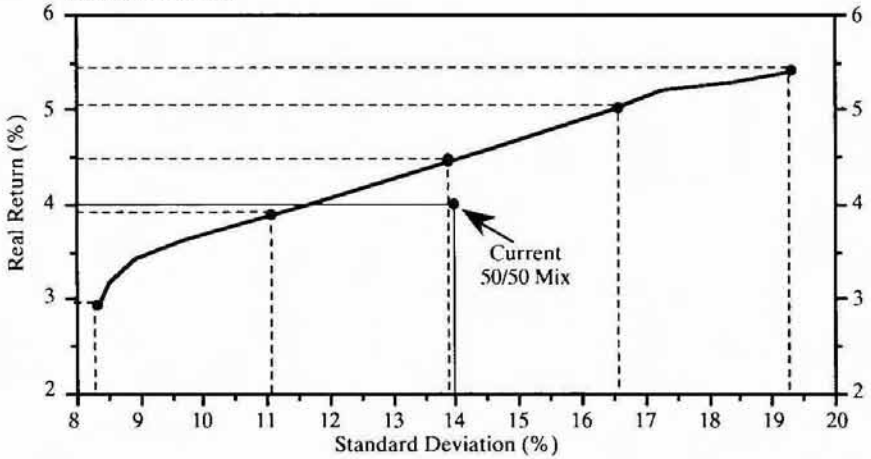
Efficient Frontier

There are an infinite number of asset mixes that one could derive from the five asset classes specified in the illustrative capital market simulation. The *efficient frontier* represents all combinations of these asset classes that have the highest return for a given risk or, conversely, the lowest risk for a given return. Figure 15–1 shows the efficient frontier for the illustrative assumptions, along with the five mixes to be studied in the asset allocation analysis. In deriving the efficient frontier, T-Bills were limited to a minimum of 5 percent and a maximum of 15 percent, and both S-Stocks (small stocks) and W-Stocks (world stocks) were limited to a maximum of 20 percent each. Under the assumptions and constraints used in this illustration, the efficient frontier is linear from Mix 2 to Mix 4 because the only change over this interval is the mix between the S&P and L-Bonds asset classes.

The current 50-50 stock-bond mix is also plotted in Figure 15–1. This mix is inefficient since has the same standard deviation as Mix 3, while having the same real return as Mix 2.

The five mixes selected from the efficient frontier range from the least risky to the most risky, with the intermediate mixes having a uniform difference in standard deviations. The graph shows that equal increases in the risk (standard deviation) results in proportionally smaller increases in real returns. The real returns

FIGURE 15–1
Efficient Frontier



	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
T-Bills	.15%	.05%	.05%	.05%	.05%
L-Bonds	.79	.65	.37	.08	.00
S&P	.00	.10	.38	.67	.55
S-Stocks	.00	.00	.00	.00	.20
W-Stocks	.06	.20	.20	.20	.20
	1.00	1.00	1.00	1.00	1.00
Real Return	2.9	3.9	4.5	5.0	5.4
Std. Dev.	8.4	11.1	13.9	16.6	19.3

and standard deviations for the various mixes are shown at the bottom of Figure 15–1. Since the long-term inflation rate is 4 percent, the nominal returns are equal to the indicated real returns plus 4 percent.

Because of the input assumptions for S&P and W-Stocks, both are preferred in the efficient frontier to the S-Stock asset class. Whether the inputs used in this illustration are reasonable will depend on the plan sponsor's analysis of these asset classes. The purpose of this chapter is simply to illustrate the asset allocation methodology, not to promote one or more specific asset classes over others.

Real Return Simulations

The 1-year, 5-year and 10-year simulated real returns for each of the five mixes are shown in Figures 15–2a through 15–2c, respectively. In each case, percentiles ranging from the

5th to the 95th are indicated. Note that the *y*-axis scale is different for each graph. Mix 5, for example, has simulated 1-year returns ranging from –21 to 40 percent, while the 5-year returns range from –4 to 19 percent and the 10-year returns range from –6 to 20 percent.

The attractiveness of alternative mixes depends on the time period under analysis. As the planning period is extended, the compression of compound real returns (known as time diversification), coupled with the higher expected returns, favors a higher exposure to equities. Since most pension plans have the luxury of a long planning horizon, a high equity exposure is often an appropriate asset allocation. Moreover, because of the asset smoothing that takes place in determining contributions and expense, high equity exposures make even more sense, as shown in the next section.

STOCHASTIC PENSION SIMULATIONS

A summary of the asset allocation results is presented in this section. Figure 15–3a shows the distribution of contributions for the 5th year of the stochastic projection under each of the five mixes, while Figure 15–3b shows the SFAS 87 expense and Figure 15–3c shows the economic liability funded ratio. Figures 15–4a, through 15–4c show these same results for the 10th year of the projection.

The distributions for employer contributions do not lend themselves to "floating bar charts" because they are often not continuous, due to the fact that (1) contributions often can be locked into the plan sponsor's target cost, provided such costs are within the ERISA minimum and maximum limits, and (2) because contributions cannot be negative, hence, a large probability of zero contributions can occur. Therefore, every 5th percentile of the contribution distribution is shown in Figure 15–3a. All of the mixes have a fairly substantial probability of allowing the 7.35 percent target cost to be contributed in the 5th year. This contribution for Mix 1 applies from the 35th to the 75th percentile, whereas for Mix 5 it applies from the 50th to the 75th percentile. There is very little chance that contributions will be zero in the 5th year under Mix 1, versus a 25 percent chance under Mix 5. On the other hand, the "worst case" (or 95th per-

FIGURE 15–2a
One-Year Real Returns

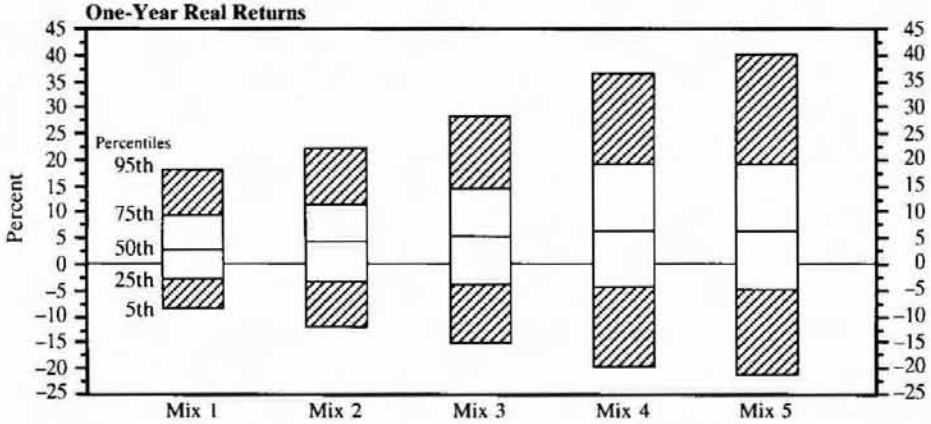


FIGURE 15–2b
Five-Year Compound Real Returns

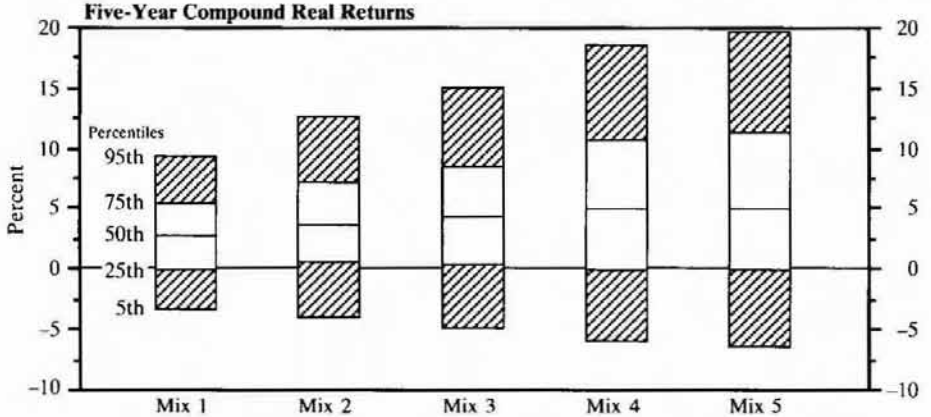


FIGURE 15–2c
Ten-Year Compound Real Returns

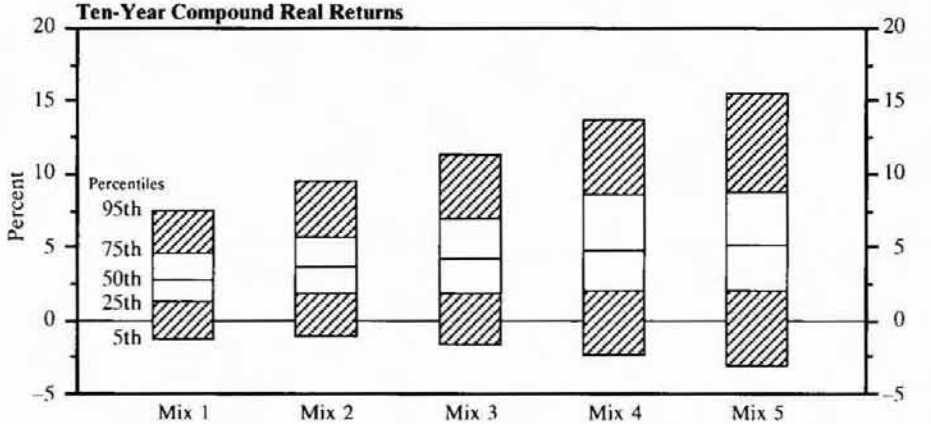


FIGURE 15–3a
Contributions in Year 5 Under Alternative Asset Mixes

Percentile	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
95th	7.90	8.23	8.62	9.78	9.53
85th	7.35	7.35	7.60	8.16	8.26
75th	7.35	7.35	7.35	7.35	7.35
65th	7.35	7.35	7.35	7.35	7.35
55th	7.35	7.35	7.35	7.35	7.35
50th	7.35	7.35	7.35	7.35	7.35
45th	7.35	7.35	7.35	7.24	7.09
35th	7.35	7.28	6.99	6.37	5.61
25th	7.23	6.64	6.39	2.09	0.00
15th	6.78	3.10	0.00	0.00	0.00
5th	0.85	0.00	0.00	0.00	0.00

FIGURE 15–3b
Pension Expense in Year 5 Under Alternative Asset Mixes

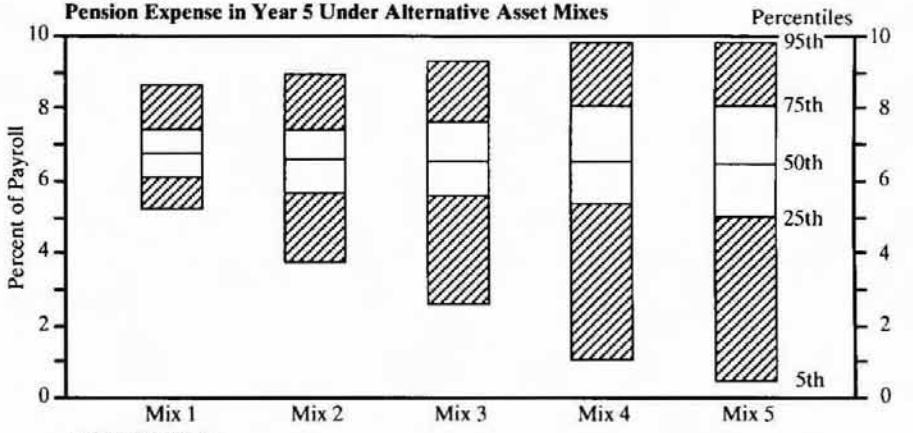


FIGURE 15–3c
Economic Liability Funded Ratio in Year 10 Under Alternative Asset Mixes

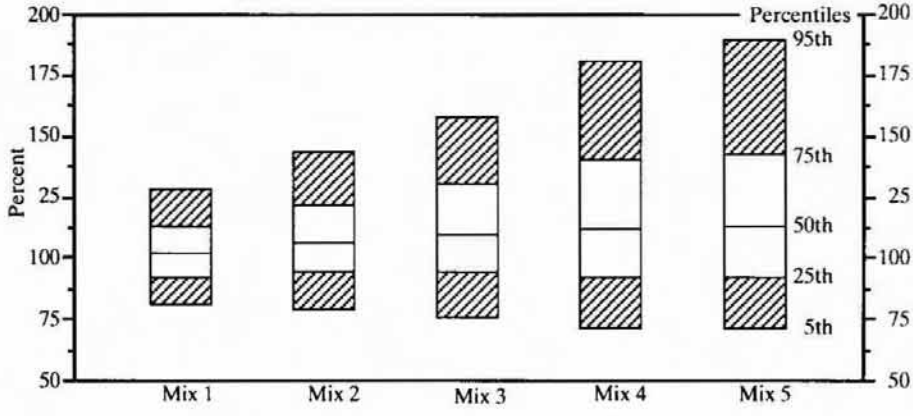


FIGURE 15–4a
Contributions in Year 10 Under Alternative Asset Mixes

Percentile					
95th	7.88	7.94	8.32	9.49	9.50
85th	7.35	7.35	7.35	7.35	7.35
75th	7.35	7.35	7.35	7.35	7.35
65th	7.35	7.35	7.35	7.35	6.90
55th	7.35	6.73	6.24	5.91	5.63
50th	6.97	6.20	5.66	5.17	4.97
45th	6.58	5.69	5.24	4.72	0.00
35th	5.63	4.78	4.12	0.00	0.00
25th	4.93	2.71	0.00	0.00	0.00
15th	4.09	0.00	0.00	0.00	0.00
5th	0.00	0.00	0.00	0.00	0.00
	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5

FIGURE 15–4b
Pension Expense in Year 10 Under Alternative Asset Mixes

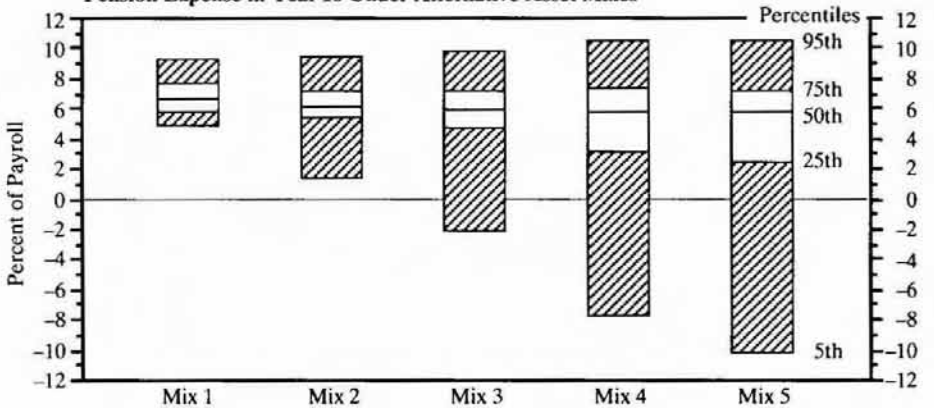
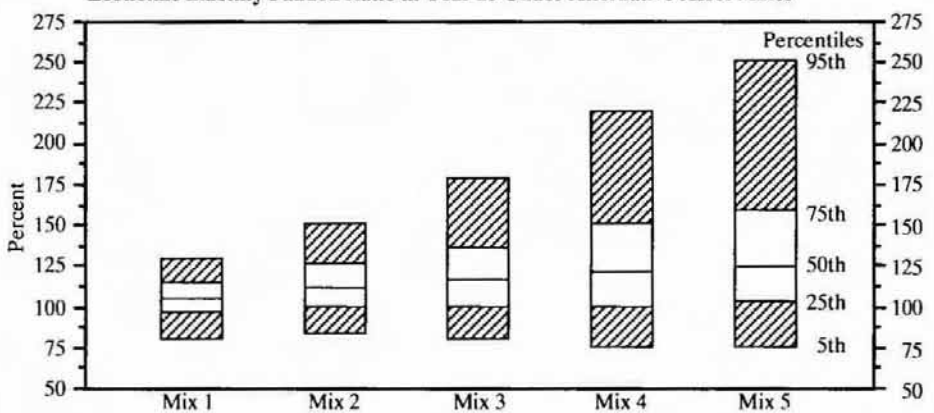


FIGURE 15–4c
Economic Liability Funded Ratio in Year 10 Under Alternative Asset Mixes



centile) for Mix 1 is 7.9 percent versus 9.53 for Mix 5 (note that Mix 4 has a higher "worst case" contribution in the 5th year). The "worst case" profile in the 10th year (see Figure 15–4a) is nearly identical to the "worst case" for the 5th year; however, the higher equity mixes have considerably lower expected contributions. Generally speaking, Mix 5 appears to be the superior mix except, perhaps, for those firms that place an extremely high priority on contribution predictability and stability.

There is little question that the SFAS 87 and economic funded ratio "floating bar charts" support Mix 5: the "worst case" is not any worse, yet the "best case" is considerably better for this mix than the others. This is true for both the 5th and 10th year of the analysis. Consequently, unless the "worst case" contributions under Mix 5 are of concern, which is unlikely for most plan sponsors, Mix 5 would be the preferred mix. It will be recalled that this mix is essentially an all-equity mix, having 55 percent of the S&P asset class, 20 percent each of S-Stocks and W-Stocks, and 5 percent of T-Bills (the latter representing the minimum required percent for this asset class).