PENSION MATHEMATICS
with Numerical Illustrations

Second Edition

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Chapter 12

Alternative Actuarial Assumptions

The purpose of this chapter is to investigate the sensitivity of pension costs (1) to changes in valuation assumptions and (2) to changes in the experience of the plan. This analysis will provide insight into the relative importance of the various assumptions used with pension plans.

The valuation sensitivities show the relative impact of each actuarial assumption change on the normal cost and actuarial liability of three actuarial cost methods: the accrued benefit method, the constant dollar benefit prorate method, and the constant percent cost prorate method. The impact on minimum required contributions, of course, would depend on the funded status of the plan.

The experience sensitivities are based on maintaining the valuation assumptions while varying the projected experience of the plan over time. These analyses are based on the constant dollar benefit prorate method, with contributions being made at the minimum required level.

MORTALITY RATES

The impact of mortality is shown in Table 12–1, where the mortality rate multiple indicates the change made to the model assumption. For example, a multiple of .50 indicates that the age-specific rates are reduced to one half the standard rates (except for the rate at the assumed end of the life span, which retains a value of unity), while a multiple of 1.50 indicates a 50 percent increase in the age-specific rates (unless a value greater than unity results, in which case unity is used). As noted in Chapter 2,

changes in mortality rates affect the cost of surviving spouse benefits in the opposite direction of the cost of retirement, vesting, and disability benefits.

The impact of variations in the mortality assumption is relatively uniform across funding methods and for both the normal cost and actuarial liability. Variations of up to 25 percent in such rates affect costs and liabilities by 10 to 15 percent, a relatively minor impact given the fact that a 25 percent increase or decrease in mortality rates is a substantial change. The last row of Table 12–1 shows the impact of changing the mortality assumption from the GAM–71 to the GAM–83 table, indicating that long-run costs can be expected to increase by about 7 percent.[1]

TABLE 12–1

Effect of Alternative Mortality Rates

Mortality Rate Multiple	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
.50	120.3	120.7	121.4	121.1	122.3	121.0
.75	108.9	109.1	109.4	109.2	109.8	109.2
1.00	100.0	100.0	100.0	100.0	100.0	100.0
1.25	86.6	86.4	85.8	86.1	85.2	86.1
1.50	76.7	76.5	75.3	76.0	74.3	75.9
GAM 83	107.2	106.9	107.7	107.2	108.1	107.2

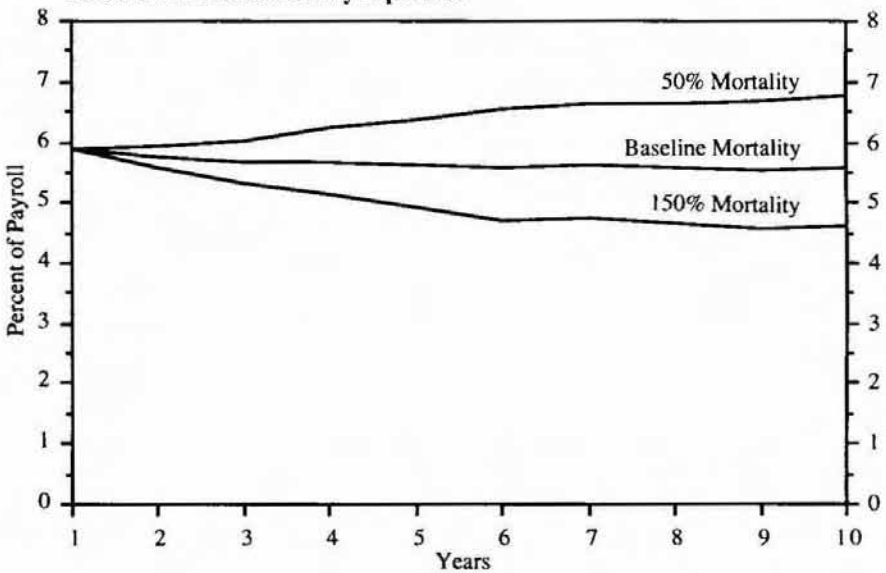
Figure 12–1 shows the financial implications if the valuation assumptions are held constant at the GAM–71 table while the experience of the plan is either 50 percent greater or less than this assumption. By the end of 10 years, costs are affected by about one percent of payroll, or approximately 20 percent of the baseline costs. These results assume that actuarial gains and losses generated from the mortality experience are amortized over 5 years, consistent with the minimum required contribution requirements.

After 10 years of the mortality experience differing from the valuation assumption by 50 percent, the cost impact is about the same as changing the mortality valuation assumption in the initial year, as indicated in Table 12–1. On the other hand, if a plan

[1]Short-term costs could be greater or less than 7 percent, depending on the plan's funded status.

were to experience this mortality and then change the valuation assumption, the net effect on costs would be a combination of the results in Table 12–1 and Figure 12–1, or a cost impact of approximately 40 percent for this example.

FIGURE 12–1
Effect of Alternative Mortality Experience



While valuation mortality rates will occasionally need to be changed, and while experience will fluctuate from the underlying assumption from time to time, variations in the mortality assumption are not likely to have a substantial impact on costs. As Figure 12–1 indicates, even extreme deviations in mortality over an extended period of time have a comparatively minor impact on costs.

TERMINATION RATES

Termination rates for active employees are not only greater in magnitude than mortality rates, they are also subject to considerably more variation, both among plans and for a given plan over time. Although a 50 percent variation in the mortality assumption for a large group of plan members is unlikely, this is not the case for termination rates. Table 12–2 indicates the impact of 25

and 50 percent changes in termination rates for valuation purposes.

Results under the accrued benefit method are hardly affected by these changes, nor is the actuarial liability under the other cost methods. This is the case since the dominant portion of the normal cost under the accrued benefit method, as well as the actuarial liabilities under all methods, are for participants who are at an age for which the withdrawal rates are small or zero. The benefit prorate method shifts the incidence of normal costs to younger ages as compared to the accrued benefit method, and the cost prorate method has an even greater shift; hence, the normal costs under these methods are affected more by a change in termination rates. Since the actuarial liabilities are affected minimally, the plan's supplemental cost will likewise be affected minimally; therefore, the effect on total costs will be less than the effect on normal costs.

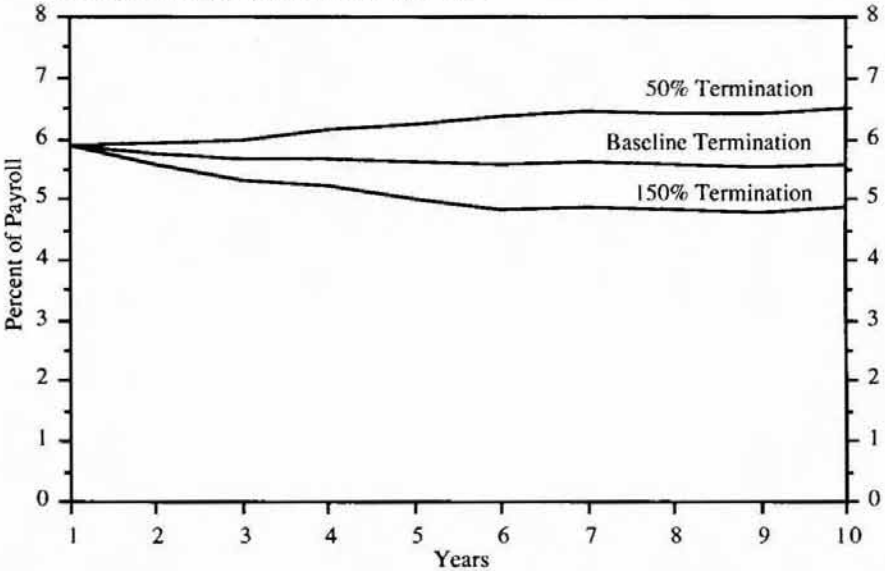
TABLE 12–2

Effect of Alternative Termination Rates

Termination Rate Multiple	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
.50	102.2	100.4	115.2	104.0	127.4	100.1
.75	101.0	100.2	106.6	101.8	113.3	100.2
1.00	100.0	100.0	100.0	100.0	100.0	100.0
1.25	98.2	99.7	90.4	97.1	77.3	98.9
1.50	96.6	99.4	83.6	94.8	60.2	97.2

Figure 12–2 shows the results of experiencing 10 years of termination rates running 50 percent higher and 50 percent lower than the underlying actuarial assumption. Pension costs are affected by less than 20 percent by the end of the 10-year projection, a result similar to the impact of a 50 percent deviation in mortality rates over this period. As noted previously, however, there is a much greater likelihood of this differing experience occurring with termination rates than with mortality rates. Additionally, as was the case with mortality sensitivities, the results in Table 12–2 would be applicable any time termination rates are changed during the projection in Figure 12–2. Consequently, a plan could experience the combined financial impact of both sensitivity analyses.

FIGURE 12–2
Effect of Alternative Termination Experience



DISABILITY RATES

Table 12–3 displays the financial implications of changing the valuation disability rates by 25 and 50 percent. These changes have virtually no impact on either costs or liabilities. This occurs for two reasons: (1) the disability rates are relatively small and even large proportionate changes in such rates do not have a significant impact, and (2) the change in disability-based costs is largely offset by the change in retirement-based costs. For example, if disability rates are increased, the cost of disability increases but the reduction in retirement-related costs virtually offsets this increase. As indicated in Figure 12–3, there is virtually no perceptible difference in costs over a 10 year period if disability rates deviate by 50 percent from the underlying actuarial assumption.

It will be recalled that the disability provision under the model plan provides the accrued benefit, payable immediately for life, upon disability, provided the employee is age 40 and has 10 years of service. Other disability provisions may be affected

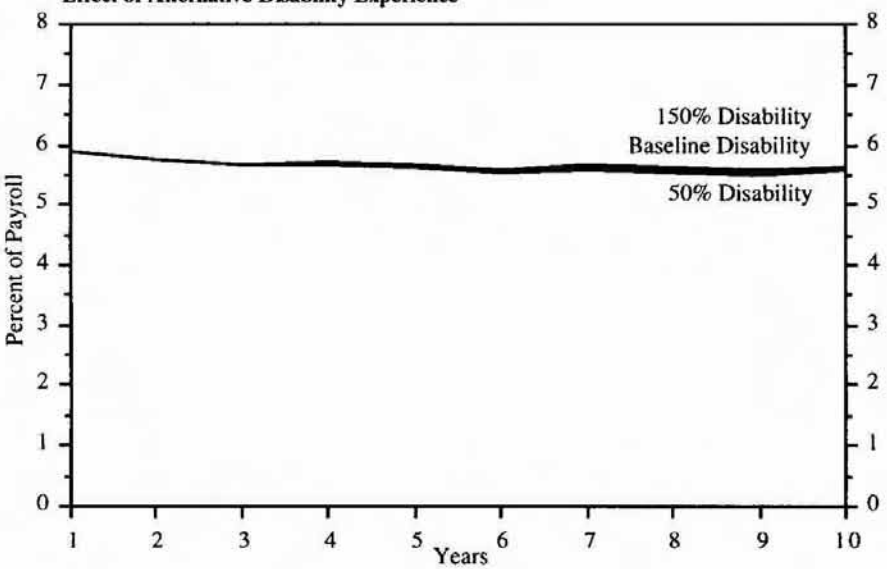
TABLE 12–3

Effect of Alternative Disability Rates

Disability Rate Multiple	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
.50	98.9	99.4	99.4	99.5	99.7	99.7
.75	99.5	99.7	99.7	99.8	99.8	99.8
1.00	100.0	100.0	100.0	100.0	100.0	100.0
1.25	101.1	100.6	100.6	100.5	100.3	100.3
1.50	102.1	101.2	101.1	100.9	100.6	100.7

FIGURE 12–3

Effect of Alternative Disability Experience



differently than the results presented here; however, it is unlikely that disability rates or disability experience will have a major impact on the financial results of the pension plan.[2]

[2]Public sector pension plans, and particularly police and firefighter plans, are notorious for abusing the disability provisions of the plan; hence, both the disability valuation assumption and experience may be very important to the financial condition of such plans.

RETIREMENT RATES

The costs of providing *actuarially reduced* early retirement benefits for retirements both earlier and later than the standard assumption are shown in Table 12–4. The normal cost and actuarial liability under the accrued benefit method are only minimally affected by changes in the incidence of retirement. The actuarial liabilities of the benefit prorate and cost prorate methods are only moderately affected, whereas the normal costs of these methods are affected up to 25 percent (i.e., early retirements reduce costs while delayed retirements increase costs).[3]

TABLE 12–4

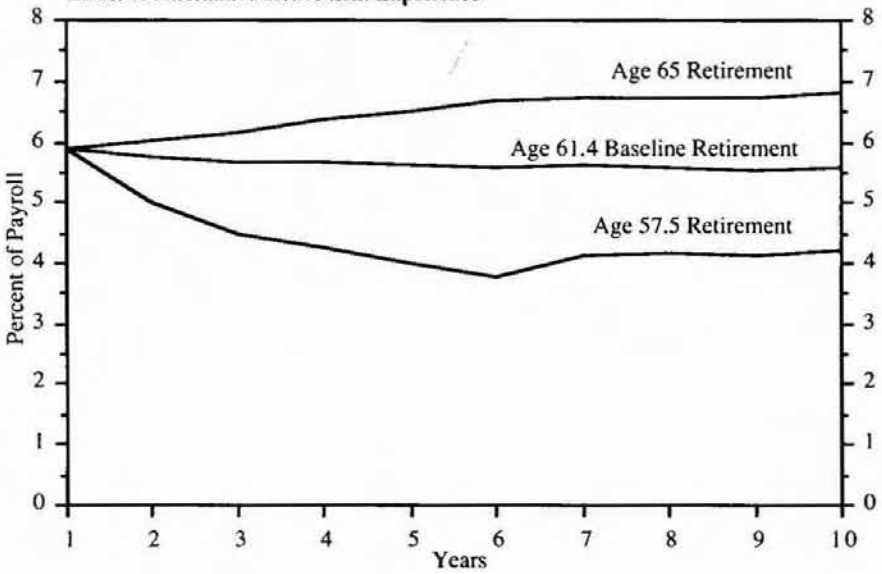
Effect of Alternative Retirement Rates

Average Retirement Age	Accrued Benefit		Constant Dollar Benefit Prorate		Constant Percent Cost Prorate	
	NC	AL	NC	AL	NC	AL
65.0	106.6	100.9	117.2	109.2	120.7	113.8
63.4	103.8	100.6	109.8	105.2	111.9	107.6
61.4	100.0	100.0	100.0	100.0	100.0	100.0
59.4	97.1	99.4	90.7	95.1	87.5	93.0
57.5	89.6	98.6	78.8	89.6	75.1	85.0

If retirements are assumed for valuation purposes to conform to the distribution of retirements given by Table 2–9, while actual retirements conform either to a distribution of rates averaging 57.5 or to 100 percent of retirements at age 65, then costs will be affected by actuarial gains and losses. As indicated in Figure 12–4, the financial effects of retirement-age deviations have a fairly significant effect on pension costs after a few years, with such costs being increased by about 25 percent after 5 years when retirements are older than expected, and decreased by 25 percent or more after only 3 or 4 years when retirements are earlier than expected. The cost of providing non-reduced accrued benefits at early retirement, as opposed to actuarially reduced benefits, is considered in the following chapter.

[3]If the plan provides an early retirement reduction that only approximates a true actuarial reduction, such as 6 percent per year below the plan's normal retirement age, the sensitivity of alternative retirement ages may be greater than indicated in Table 12–4.

FIGURE 12–4
Effect of Alternative Retirement Experience



SALARY RATES

Pension costs are directly proportional to the level of benefits provided under the plan. Consequently, for plans with a salary-based benefit formula, the assumed rate of growth in salaries has an important bearing on costs. The underlying theory for future salary increases, as set out in Chapter 2, is that merit, productivity, and inflation represent the components of the increases. The model merit scale, although conforming to a concave function, has about a 2 percent compounding effect for an age-30 entrant, the productivity factor is assumed to be 1 percent, and the inflation factor 4 percent. The first two components, merit and productivity, are relatively stable as compared to the inflation component. The purpose of this section is to analyze the impact of introducing each salary increase component into the salary assumption, and to study the effects of various levels of inflation, both in the valuation assumptions and the experience of the plan.

Table 12–5 shows the results of alternative valuation salary assumptions. The normal cost under the accrued benefit method changes by about 10 percent for each 2 percentage points change in the salary rate. This is in sharp contrast to its actuarial liabil-

ity, which not only is minimally affected by different salary assumptions, but is affected in the opposite direction. As the salary rate is decreased, the actuarial liability increases and vice versa. The reason for this is that a change in the salary assumption affects each participant's *assumed* accrued benefits under this method. In particular, the flatter the salary assumption, the *larger* the assumed salaries of plan participants for prior years; this in turn increases the assumed accrued benefits. Thus, for a given set of current salaries, the flatter the salary, the larger will be the actuarial liability under the accrued benefit method.

TABLE 12–5

Effect of Alternative Salary Rates

Salary Rate	Accrued Benefit		Constant Dollar Benefit Prorate		Constant Percent Cost Prorate	
	NC	AL	NC	AL	NC	AL
0	56.0	109.1	64.7	85.1	43.0	84.8
M	65.8	107.2	70.0	87.3	53.1	87.1
M+P	73.3	105.7	74.4	89.2	60.1	89.2
M+P+2%	87.3	102.7	85.3	93.8	77.3	94.2
M+P+4%	100.0	100.0	100.0	100.0	100.0	100.0
M+P+6%	111.6	97.5	120.4	108.2	129.4	106.6
M+P+8%	122.2	95.2	149.0	119.1	166.5	113.8
M+P+10%	131.9	93.1	190.1	133.9	211.8	121.1

M = Merit Scale; P = Productivity Assumption of 1%

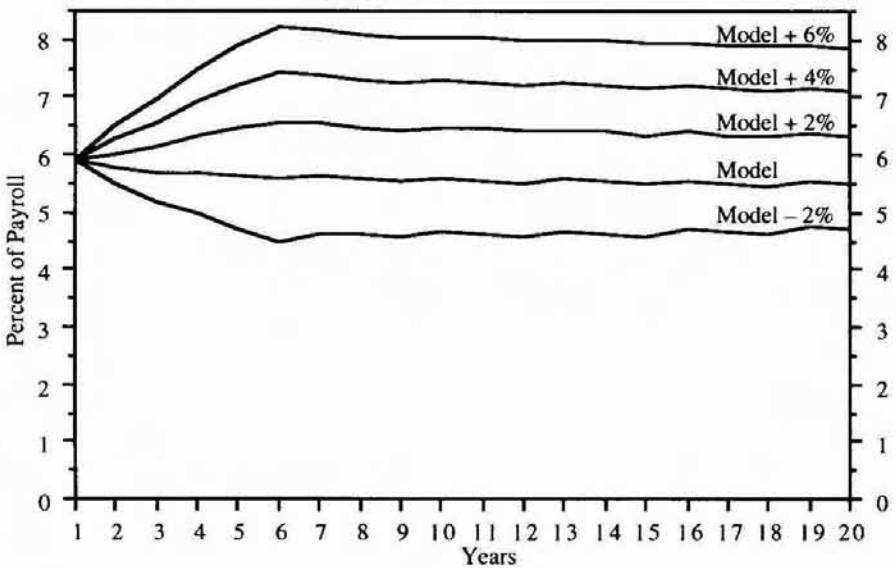
The normal cost under the benefit prorate method is affected by about 10 percent for each 1 percentage point change in the salary assumption, or about double the impact for the accrued benefit method. The actuarial liability is affected by about half of this amount. Finally, the cost prorate method is affected even more, with a 2 percentage point change bringing about a 30 percent change in costs. The actuarial liability, however, is affected to a lesser degree than that of the benefit prorate method.

In theory, at least, it is inappropriate to alter the inflation component of the salary assumption without a commensurate change in the assumed investment return. The effects of simultaneous changes in these two assumptions is studied at a later point in this chapter.

The effects of experiencing salary increases greater or less than the valuation assumptions is provided in Figure 12–5. After 5 years, costs are affected by about 20 percent for each 2 percent-

age point deviation in the experience from the underlying valuation salary assumption. This impact, however, does not continue to increase, with the relative differential after 20 years being approximately the same as after 5 years.[4] Since salary deviations of 2 to 4 percent for several years can easily occur, this actuarial assumption merits closer scrutiny than the decrement assumptions, for which substantial deviations are less likely to occur.

FIGURE 12–5
Effect of Alternative Salary Experience



INTEREST RATES

The interest rate assumption, like the salary assumption, has associated with it an underlying theory that was discussed in Chapter 2. This theory states that the interest assumption consists of three components: one to account for the risk-free rate of return, one to account for the risk inherent in the portfolio of assets held, and one to account for inflation. These components, it will be remembered, are 1 percent, 3 percent, and 4 percent, respectively, for the model assumptions.

[4]This projection, as well as the following two, are for 20 years instead of 10 years. The time period was extended to illustrate that pension costs reach an ultimate level rather than continuing to increase or decrease for a given experience deviation.

Table 12–6 shows the results of introducing the various interest rate components and varying the inflation rate around the 4 percent model assumption. A brief inspection of this table shows that costs are more sensitive to the interest rate assumption than they are to any parameter thus far studied. For example, assuming zero interest causes costs to increase by 400 to 900 percent, depending on the cost measure. A more meaningful analysis is the impact of a change in the inflation component of the interest assumption. The cost prorate method is the most sensitive of the various measures to such changes. A zero inflation rate component (or a 5 percent interest rate) has the effect of increasing the normal cost nearly threefold, while a 10 percent inflation component (or a 14 percent interest rate) reduces costs to one fourth of the cost under the model assumption. The actuarial liability is only about half as sensitive to changes in the interest rate as the normal cost.

TABLE 12–6

Effect of Alternative Interest Rates

Interest Rate	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
0	580.5	387.7	895.0	494.4	880.4	365.2
RF	440.4	310.9	638.4	381.8	665.2	304.8
RF+RP	213.9	176.9	260.3	196.2	289.4	182.4
RF+RP+2%	142.6	130.1	156.4	136.4	168.6	133.3
RF+RP+4%	100.0	100.0	100.0	100.0	100.0	100.0
RF+RP+6%	73.1	79.7	67.6	76.7	60.7	77.5
RF+RP+8%	55.4	65.5	47.8	61.0	37.8	61.2
RF+RP+10%	43.2	55.1	35.3	50.0	24.2	49.8

RF = Risk Free Rate of 1%; RP = Risk Premium of 3%

The normal costs under the other two cost methods are less sensitive to interest rate changes than the cost prorate normal cost, although their actuarial liability values are affected by about the same amount.

The rule-of-thumb that pension costs are altered by 6 to 7 percent for each 1/4 of one percent change in the interest rate is well known and used often in connection with pension plans. If we take the midpoint of this range, or 6.5 percent, the rule tells us that a 1 percentage point increase in the interest rate will reduce costs by 22 percent [100 $(1–1.065^{-4})$], while a 1 percentage point

reduction will increase costs by 29 percent [100 (1.065^4–1)]. The results of a 2 percentage point increase or decrease reduces costs by 60 percent or increases costs by 160 percent, respectively, while a 4 percentage point change reduces costs by 37 percent or increases costs by 274 percent. The normal cost under the benefit and cost prorate methods conform to this rule reasonably well, but their actuarial liabilities as well as the normal cost under the accrued benefit method appear to follow a 4 percent rule rather than the 6 to 7 percent rule.

The inflation component of the interest rate assumption was altered in this section without a simultaneous change in the inflation component of the salary assumption. While the results are both interesting and important, it is believed that the sensitivity of pension costs to the inflation parameter as analyzed in the next section, is somewhat more meaningful and valuable.

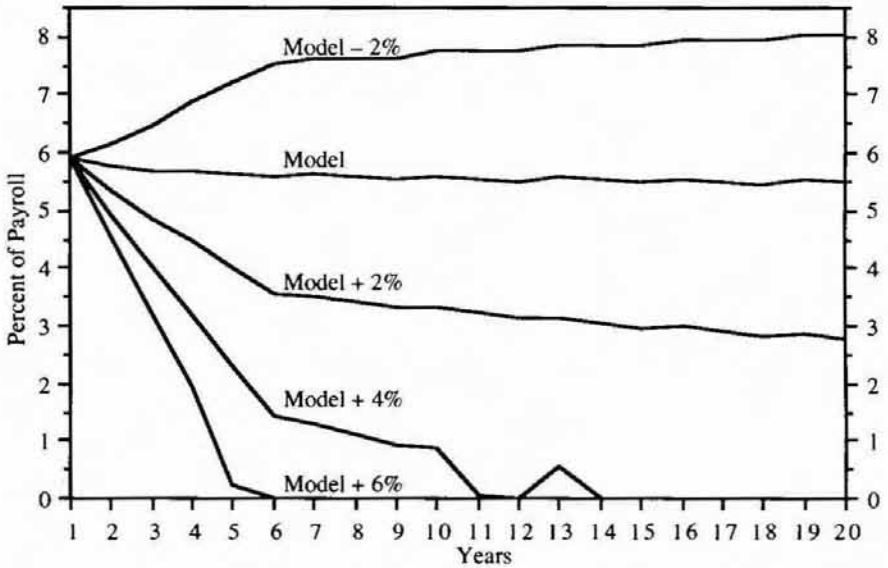
Figure 12–6 shows the implications of experiencing asset returns different from the 8 percent valuation interest rate. The impact on costs continues to grow throughout the 10 year period, with a 2 percentage point deviation causing costs to be affected by over 30 percent (or about 2 percent of payroll) by the end of the 10 year projection.

It is ironic that there is almost a perfect positive correlation between the importance of a given assumption, whether for valuation purposes or for experience purposes, and the assumption's stability and degree to which it can be predicted. The interest rate is by far the most important assumption and by far the most difficult to establish accurately. On the other hand, mortality rates, for which reasonable variations are not particularly crucial to pension costs, are highly predictable.

INFLATION RATES

The effect of changing the assumed rate of inflation, a component of both the salary rate and interest rate assumptions, is analyzed in this section. Some plan sponsors naively believe that equal changes in the interest rate and salary rate (in this case being brought about by a change in the inflation component of each) will tend to cancel out, since these two assumptions have counterbalancing effects on pension costs. This is not the case,

FIGURE 12–6
Effect of Alternative Investment Experience



however, since the salary scale operates up to the participant's retirement age, while the interest discount factor extends to the end of the assumed life span.[5] Thus, a change in the inflation component of the interest assumption will have a *greater* impact on pension costs than its counterpart in the salary assumption.

The results of assuming an inflation component of zero up through 10 percent are given in Table 12–7. Pension costs, as expected, are inversely related to changes in the inflation rate: the higher the rate of inflation the lower the dollar cost of the plan *for the current year.* Although the *dollar* cost of the plan experiencing high inflation will eventually be greater at some future point in time than if lower inflation were to be experienced, the cost as a *percentage of payroll* will be less. The impact among cost methods and between the normal cost and actuarial liability is reasonably constant.

Theoretically, if a sponsoring firm's earnings were to be perfectly insulated from the effects of inflation, then greater rates of inflation might be viewed as a cost reducing factor in a relative

[5]This would not be true if the plan has a cost-of-living clause, in which case the inflation component of the salary assumption would extend beyond retirement.

sense. The more typical case, however, is where inflation impairs the earnings potential of the firm and the corresponding increase in pension dollar costs simply adds to other problems created by inflation. The effects of inflation are even more serious if this component in the salary assumption becomes applicable at ages beyond retirement due to cost-of-living escalators, as will be seen in the following chapter.

TABLE 12–7

Effect of Alternative Inflation Rates

Inflation Rate	*Accrued Benefit*		*Constant Dollar Benefit Prorate*		*Constant Percent Cost Prorate*	
	NC	*AL*	*NC*	*AL*	*NC*	*AL*
0%	161.9	187.6	175.6	164.0	178.9	161.1
2%	125.2	133.7	130.0	126.0	131.0	125.1
4%	100.0	100.0	100.0	100.0	100.0	100.0
6%	81.9	77.8	79.3	81.6	78.8	81.9
8%	68.4	62.6	64.4	68.1	63.8	68.5
10%	58.2	51.7	53.3	57.9	52.7	58.2

Figure 12–7 shows the financial results if inflation is different from the base case projection. It will be recalled that the pension plan being projected provides an *ad hoc* COLA every 3 years equal to 25 percent of cumulative inflation. The higher the level of inflation, the lower is the cost of the pension plan, even with the *ad hoc* COLAs being given. After 10 years of inflation deviating from the valuation assumption, the ultimate level of costs is obtained, with costs remaining at that level throughout the remainder of the projection.

FIGURE 12–7
Effect of Alternative Inflation Experience