PENSION MATHEMATICS
with Numerical Illustrations

Second Edition

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Chapter 11

Pension Accounting

In 1985 the Financial Accounting Standards Board (FASB) promulgated Statement 87 (SFAS 87), *Employers' Accounting for Pensions*, effective for fiscal years beginning after December 15, 1986. The primary objective of this statement is to achieve consistency, uniformity, and comparability with respect to pension plan accounting among plan sponsors. SFAS 87 specifies the methodology used to determine pension expense, termed the *net periodic pension cost*, the liability, if any, to be reported on the employer's balance sheet, and the various disclosure items to be reported in the sponsor's financial statements.[1]

This chapter describes the essential components of pension accounting as set forth in SFAS 87 using previously defined equations. The discussion begins with two liability values that are an integral part of the SFAS 87.

LIABILITY VALUES

SFAS 87 defines the *Accumulated Benefit Obligation* (ABO) and the *Projected Benefit Obligation* (PBO), both of which must be disclosed in the employer's financial statements. In addition, the ABO is used to determine whether a liability must be reported on the employer's balance sheet, while the PBO plays an integral part in determining annual pension costs.

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[1]SFAS 87 applies both to qualified pension plans (the subject of this book) and non-qualified plans. Because of the effect of the benefit limits (IRC §415) and the maximum limit on compensation [IRC §401(a)(17)] the SFAS 87 costs of supplemental non-qualified plans have become more significant.

Accumulated Benefit Obligation (ABO)

The ABO is equal to the present value of accrued benefits, taking into account all future decrements. The ABO for an active employee age x is given by (11.1a), shown on the following page.[2]

The summation in (11.1a) runs from attained age x to the oldest assumed retirement age r''. Since this period involves ages for which some of the benefits are not applicable, zero values for the grading function and/or decrement rates at these ages eliminate the present value calculation. For example, both the retirement grading function and the retirement decrement are zero prior to age r', while the grading functions associated with the various ancillary benefits are defined to be zero at those ages for which benefits are not applicable. The ABO for non-active members is found by multiplying their accrued benefit by the appropriate annuity factor.

The ABO mathematics are identical to (1) the plan continuation liability, (2) the actuarial liability under the accrued benefit cost method, and (3) the statutory current liability. However, each of these may be based on different actuarial assumptions.[3] Each actuarial assumption used in determining the ABO must represent the *employer's* best estimate of the anticipated experience under the plan. While this still leaves the employer some latitude in the selection process, prior accounting promulgations were silent in this regard. With respect to the discount assumption, which is one of the most significant assumptions, SFAS 87 requires the use of a so-called settlement rate, i.e., the interest rate for which the pension obligation could be settled through the purchase of annuities. Rather than explicitly determining such rates each year, two proxies are suggested: (1) the current

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[2]Although each term in (11.1a) has been defined previously, these definitions are provided again for convenience.

[3]The plan continuation liability for retirement benefits at a single retirement age was given by (5.2a), and the actuarial liability under the accrued benefit method by (5.5a). The actuarial liability under the accrued benefit method with ancillary benefits is equal to (8.7), with the benefit accrual being replaced by the accrued benefit. This liability for multiple retirement ages was given by (9.5), again with the benefit accrual being replaced by the accrued benefit (and the summation going to age r''). The statutory current liability is given by (10.4a) through (10.4c).

$$(ABO)_x = B_x \sum_{k=x}^{r''} {}_{k-x}p_x^{(T)}\, v^{k-x} \cdot \left(q_k^{(t)}\, {}^{v}F_k + q_k^{(d)}\, {}^{d}F_k + q_k^{(m)}\, {}^{s}F_k + q_k^{(r)}\, {}^{r}F_k\right) \qquad (11.1a)$$

where

B_x = accrued benefit based on service, salary and the plan's benefit accrual rate determined at age x[4]

${}_{k-x}p_x^{(T)}$ = probability that an employee age x will survive in employment to age k

v^{k-x} = discount, at rate i, from age x to age k

$q_k^{(t)}$ = probability of terminating employment at age k

${}^{v}F_k$ = value of termination benefit at age k (for model plan, termination grading function times mortality-based life annuity deferred to age r)

$q_k^{(d)}$ = probability of becoming disabled at age k

${}^{d}F_k$ = value of disability benefit at age k (for model plan, disability grading function times disabled-mortality-based life annuity deferred to end of waiting period)

$q_k^{(m)}$ = probability of dying at age k

${}^{s}F_k$ = value of death benefit at age k (for model plan, survivor grading function times probability of having a surviving spouse times life annuity, reflecting age of spouse)

$q_k^{(r)}$ = probability of retiring at age k

${}^{r}F_k$ = value of retirement benefit at age k (for model plan, retirement grading function times life annuity).

[4]Equation (11.1a) shows the accrued benefit as a constant for each future decrement age; however, if the accrued benefit for a given employee at age x is constrained by maximum benefit or maximum salary limits for qualified plans under federal statutes, the indexed value of these limits should be used at each future decrement age. In this case, the accrued benefit would increase until the indexed limit no longer has an effect.

spot yield on high-yield, long-term corporate bonds (e.g., corporate bonds with a maturity of 20 or 30 years), and (2) the most recent interest rates promulgated by the Pension Benefit Guaranty Corporation (PBGC), or some reasonable approximation of such rate. These actuarial assumption requirements apply also to the PBO and the calculation of net periodic pension cost.

The ABO for SFAS 87 purposes must be a fiscal year-end value (or no more than three months prior to the date of the sponsor's financial statements). In many cases, the valuation will have been prepared six to twelve months earlier; hence, the ABO must be projected to the measurement date. There are two approaches that may be used. The first is to perform a detailed projection of the plan census, plus an estimate of new entrants, to the measurement date. This projection must reflect salary and service increases for active members, decrements for all plan members, and, of course, age increases for all members. Any material plan amendments must also be taken into account.

The second approach is to project the valuation results, with appropriate approximations.[5] Equation (11.1b) is appropriate for a full year's projection: [6]

$$(ABO)_{t+1} \approx [(ABO)_t + K(ABO)_t]\ (1+i) - E(B)_t\,(1+\tfrac{1}{2}i) \tag{11.1b}$$

where

$(ABO)_t$ = ABO for all plan members at time t

i = discount rate used with ABO

$E(B)_t$ = expected benefit payments during year t

K = fraction of ABO to account for service and (if appropriate) salary increases.

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[5]While it is permissible to project the ABO (as well as the PBO) to the measurement date, the actuarial assumptions and the benefits valued must be appropriate at the measurement date. If these differ from the original valuation, then a new valuation must be run with appropriate changes before the projection set forth in (11.1b) is used.

[6]The fractional coefficient in the last term is intended to represent a weighted average of benefit payments throughout the year. If benefits are paid at the beginning of the month, and are expected to be uniform during the year, then 13/24 would be the correct coefficient; however, if benefits are expected to increase (or if lump-sum payments are expected on a non-uniform basis), then the coefficient should reflect this non-uniform expectation. For simplicity, the 1/2 coefficient is used in this equation and others presented in this chapter.

The K coefficient to $(ABO)_t$ is determined precisely as

$$K = \left[\sum^{all\,x} \frac{B_{x+1} - B_x}{B_x} (ABO)_x\right], \qquad (11.2)$$

where the summation is over all plan members and B_x is the accrued benefit at age x.[7] Various approximations to the fractional growth in the accrued benefit can be made, depending on the type of benefit formula.

Vested Benefit Obligation (VBO)

SFAS 87 requires the disclosure of the vested benefit obligation, again a fiscal year-end value. This obligation is simply equal to the vested percent, if any, of each plan member's ABO. Some experts, however, believe that ancillary benefits, such as disability and pre-retirement death benefits, should not be included in this liability because employees do not become vested in such benefits. In other words, a vested benefit is one to which the employee is entitled after termination, with disability and death benefits not falling in this category.

Since minimum vesting requirements are five years under federal statute, and most non-vested employees are relatively young, the VBO is very close in value to the ABO for many pension plans. In fact, these values are often so close that the employer's financial statements simply indicate that they are not materially different, rather than showing the VBO separately.

Projected Benefit Obligation (PBO)

The PBO is equal to the present value of prorated retirement benefits, where the proration is based on service. The mathematics are the same as the actuarial liability under the constant dollar version of the benefit prorate actuarial cost method. For an active employee age x, the PBO is given by

[7] Equation (11.2) represents the normal cost under the accrued benefit cost method, i.e., the present value of the increase in the accrued benefit which is equal to the benefit accrual.

$$(PBO)_x = \sum_{k=x}^{r'} {}^{CD}B_k \; {}_{k-x}p_x^{(T)} \; v^{k-x}$$

$$\cdot \left(q_k^{(t)} \, {}^{v}F_k + q_k^{(d)} \, {}^{d}F_k + q_k^{(m)} \, {}^{s}F_k + q_k^{(r)} \, {}^{r}F_k \right) \quad (11.3a)$$

where

$${}^{CD}B_k = \frac{B_k}{(k-y)} \, (x-y)$$

= accrued benefit projected to age k, prorated by the ratio of current service to projected service at age k.

Each F_k function in (11.3a) represents of the value of the benefit payable at each decrement [see explicit definitions in connection with equation (8.7)].

The prorated accrued benefit in (11.3a) is assumed to be appropriate to each type of decrement, with the grading functions modifying this benefit for any reduction (e.g., the model plan has a survivor benefit equal to 50 percent of the accrued benefit and an early retirement benefit that is actuarially reduced). There are plan designs, however, that require a prorated accrued benefit to be defined separately for one or more decrements. The fundamental principle underlying SFAS 87 is that the allocated accrued benefit (or attributed benefit) follow the plan's formula accruals, and there are certain other requirements that can lead to a unique definition of the attributed benefit for various decrements, as discussed below.

If the retirement benefit, for example, has a years-of-service maximum, then the proration runs over the shorter of actual service or the maximum service. If the maximum for a given plan member occurs at age m, for example, then the proration factor in the above defined attributed benefit would be $(m - y)$ for $k > m$ instead of $(k - y)$. In this case, the attributed benefit increases for ages beyond age m only because of assumed salary increases. If the maximum applies equally to each ancillary benefit, then the attributed benefit can apply to all decrements; otherwise, a different attributed benefit must be defined.

If the benefit formula accrual rate is non-linear (for example, x percent of salary for the first 15 years, and y percent thereafter), then the attributed benefit must be defined as the current year's benefit accrual percentage times the employee's projected salary at age k. This allocation, unlike the one in (11.3a), can produce

an ill-behaved benefit allocation, exacerbated if combined with a maximum benefit, for example. SFAS 87 also makes a distinction for non-service related benefits that are subject to vesting, in which case the attribution period is over the non-vested ages, with the entire benefit being allocated at the point of full vesting. On the other hand, non-service related benefits that are not subject to vesting are to be prorated in the manner set forth in (11.3a).[8]

In addition to the best-estimate actuarial assumptions discussed above, if the plan sponsor has recurring plan changes for active and/or retired employees (e.g., systematic increases in a career average benefit formula), then such benefit increases are to be anticipated in the calculation of the PBO. As with the ABO, limitations imposed by federal statute for determining any maximum tax deductible contributions (e.g., non-indexed benefit and salary limits) must reflect expected increases.[9]

For disclosure purposes, the PBO must be projected to the sponsor's fiscal year end (unless a new valuation has been completed within 3 months prior to this date). Again, a valuation can be performed on a projected census or, as was the case for the ABO, the valuation results can be projected. The following approximation can be used (assuming a full year projection):

$$(PBO)_{t+1} \approx [(PBO)_t + (SC)_t](1+i) - E(B)_t(1+\tfrac{1}{2}i) \qquad (11.3b)$$

where $(SC)_t$ is the normal cost under the constant dollar benefit prorate method (termed "service cost" under SFAS 87, as discussed in the following section). Since the assumptions and benefits valued must be appropriate for the measurement date, not the annual valuation date, a new valuation may be required before the results can be projected by (11.3b).

NET PERIODIC PENSION COST

Table 11–1 outlines the components of the net periodic pension cost, each of which is discussed in this section. For this dis

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[8]The FASB may not have had enough exposure to the variety of pension plan designs in formulating the attribution methodology for SFAS 87. The attribution methodology given in (11.3a) is preferable, on both theoretical and practical grounds, to the allocation methodology set forth in SFAS 87.

[9]These items must also be included in the plan's service cost, yet to be discussed.

TABLE 11-1

Net Periodic Pension Cost Components

Service Cost:	Normal cost under constant dollar benefit prorate method, adjusted with interest to end of year
+	
Interest Cost:	PBO (adjusted for expected distributions during year) times discount rate
−	
Expected Return on Assets:	Market-related value of assets (adjusted for expected distributions and contributions during year) times expected rate of return on assets
+	
Amortization Costs:	Amortization Methodology:
Transition Obligation (Asset)	Straight line over average future service of plan participants expected to receive benefits or, optionally, 15 years, if greater
+	
Prior Service Cost	Fixed schedule over the future service of plan participants expected to receive benefits
+	
Net Loss (Gain)	Rolling schedule over future service of plan participants expected to receive benefits

cussion, it is assumed that the valuation is on the first day of the plan sponsor's fiscal year.[10]

Service Cost

The service cost (SC) is defined as the normal cost under the constant dollar version of the benefit prorate method, increased by a full year's interest. The benefit accrual under this method is

[10]The service cost, if evaluated prior to this time, can be projected to the beginning of the fiscal year with appropriate approximations and adjustments. The assumptions are those used in the prior fiscal year's measurement date, while the benefits are as of the beginning of the fiscal year, generally the same unless a plan change has been adopted. However, the PBO used in this calculation, and also projected to the beginning of the measurement date, will generally not be the PBO disclosed at the end of the year, since another valuation will undoubtedly have been made by that time.

equal to the accrued benefit at each future age prorated by years of service to that age.[11]

The symbol $(SC)_x$ is used to denote the service cost under SFAS 87 for an employee age x. Equation (11.4) defines the service cost, which is identical to (11.3a) except that the constant dollar accrued benefit is replaced by the constant dollar benefit accrual (equal to the change in the accrued benefit from age x to age $x + 1$), and one year's interest is included in the SFAS 87 service cost: [12]

$$(SC)_x = \left[\sum_{k=x}^{r'} {}^{CD}b_{k\,k-x}\,p_x^{(T)}\,v^{k-x}\left(q_k^{(t)v}F_k + q_k^{(d)d}F_k + q_k^{(m)s}F_k + q_k^{(r)r}F_k\right)\right](1+i)\,. \tag{11.4}$$

The service cost for the entire plan during year t, $(SC)_t$, is equal to the sum of (11.4) for each active plan member. There are no other choices available under SFAS 87 for determining this component of the net periodic pension cost. Hence, the FASB has eliminated a potential source of cost variation among plans and, for that matter, the same plan over time.

Some would argue that the methodology of prorating benefits by service is an odd conclusion for the FASB to have reached. Two of the fundamental propositions set forth in SFAS 87 are that pension benefits represent deferred wages and that the projected benefit should be earned, and accounted for, proportionately, as opposed to simply accounting for the legally earned benefit accrual. Logic, then, would lead one to prorate the projected benefit by salary. In other words, the normal cost under the constant percent version of the benefit prorate method would be appropriate. The service cost under this method generally would be somewhat higher; however, the amortization costs (yet to be discussed) would be lower since the actuarial liability under the constant percent version is less than that for the constant dollar version of this method.

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[11] The previous discussion regarding the proration of benefits for the PBO, including maximum service limits and non-linear benefit accruals, applies also to the service cost equation.

[12] The interest cost is included since the service cost, while calculated at the beginning of the year, is reported at the end of the year. This interest cost could have been included in the SFAS 87 "interest cost" component of net periodic pension cost.

Whereas the service cost is the normal cost of a specific actuarial cost method (increased for one year's interest), the second and third pension cost components (i.e., SFAS 87 interest cost and amortization cost) are, in effect, what has been termed heretofore as the plan's supplemental cost.

Interest Cost

The second component of the net periodic pension cost is interest on the projected benefit obligation (PBO), with the latter reduced for expected benefit payments throughout the year, $E(B)_t$. Thus, the interest cost (IC) is as follows, where i is the discount rate used in the PBO calculation and the time-weighted expected benefit payments is assumed to be 50 percent of such payments for illustrative purposes:

$$(IC)_t = i\left[(PBO)_t - \tfrac{1}{2}E(B)_t\right]. \qquad (11.5a)$$

As noted previously, the discount rate used in (11.5a) must represent a current settlement rate; hence, it will change whenever there is a material change in such rates. Generally, as the discount rate increases, the interest cost component of the net periodic pension cost decreases (and vice versa), since an increase in i generally produces a greater porportionate decrease in $(PBO)_t$.

Expected Return on Assets

The third component is the expected return on the market-related value of assets (MRA), with the latter adjusted for expected benefit payments, $E(B)_t$, and expected contributions, $E(C)_t$, during the year (the time-weighting factor is again assumed to be 50 percent; however, this would not be appropriate if either of these items was not uniform throughout the year):

$$(EROA)_t = i'\left[(MRA)_t - \tfrac{1}{2}E(B)_t + \tfrac{1}{2}E(C)_t\right] \qquad (11.5b)$$

where i' is equal to the plan sponsor's best-estimate of the expected long-term return on assets. This interest rate, unlike the rate used with the PBO, is not intended to fluctuate substantially in the short term. Moreover, there is considerably more latitude in selecting this rate than the discount rate used with the PBO. If plan assets are substantial, there is an opportunity for the plan sponsor to exert some management over this element of net peri-

odic pension cost, undoubtedly a result not intended by the FASB.

The market-related value of assets used in (11.5b) may represent the actual market value or, optionally, a smoothed market value (such as 3 or 5 year average), provided that the smoothing process does not extend beyond 5 years.[13]

The interest cost less the expected return on assets represents interest on the unfunded obligation. This can be seen more easily if the discount rate and expected return on assets is assumed to be identical and equal to i. Equation (11.5a) less (11.5b) produces

$$\begin{matrix} Net \\ Interest \\ Cost \end{matrix} = i\left[(PBO)_t - (MRA)_t - \tfrac{1}{2}E(C)_t\right]. \qquad (11.5c)$$

In the context of pension funding, (11.5c) would be the annual interest charge on the unfunded liability, with the excess supplemental cost payment being applied toward the amortization of the unfunded liability. For SFAS 87, the interest cost is determined separately from the principal amortization. Again, denoting i as the interest cost and i' as the expected return on assets, (11.5a) less (11.5b) can be written as follows:

$$\begin{matrix} Net \\ Interest \\ Cost \end{matrix} = i\left[(PBO)_t - (MRA)_t - \tfrac{1}{2}E(C)_t\right] + (i-i')\left[(MRA)_t - \tfrac{1}{2}E(B)_t + \tfrac{1}{2}E(C)_t\right] \qquad (11.5d)$$

In other words, SFAS 87 includes a funding-type interest cost plus the difference in interest rates, $(i - i')$, times the market-related value of assets (adjusted for expected cash flow). The latter can have a substantial affect on the net periodic pension cost, depending on the difference in interest rates and the size of plan assets. Moreover, since it is impossible for the plan's experience to satisfy two interest rates simultaneously, the dual interest assumption under SFAS 87 guarantees that interest-based gains and losses will occur each year, unless $i = i'$.

[13]Chapter 10 included a discussion of alternative asset valuation methods, some of which would qualify for determining the market-related value of assets under SFAS 87.

Amortization Cost

Three different amortization methods are required under SFAS 87, depending on the source of the unrecognized obligation (or asset). Since the interest cost is determined separately, as discussed above, all of the methods amortize only the principal portion of the obligation (asset).

Future Service of Employees Expected to Receive Benefits. The three amortization methods yet to be discussed involve *the future service of employees expected to receive benefits.* The future service (FS) for an employee age x is as follows:

$$(FS)_x = \sum_{k=x}^{r^*-1} {}_{k-x}p_x^{(T)}. \tag{11.6a}$$

Equation (11.6a), however, does not necessarily reflect the future service of a plan participant *expected to receive benefits.* This can be seen by transforming (11.6a) into service table notation:

$$(FS)_x = \sum_{k=x}^{r^*-1} \frac{l_k^{(T)}}{l_x^{(T)}} \tag{11.6b}$$

$$= \sum_{k=x}^{r^*-1} \frac{\sum_{t=k}^{r^*-1} \left[d_t^{(t)} + d_t^{(d)} + d_t^{(m)} + d_t^{(r)}\right]}{l_x^{(T)}}. \tag{11.6c}$$

The terms in the numerator of (11.6c) indicate "when and how" the individuals at each future age will leave the plan; however, there is no indication of whether they will be eligible for a benefit. With $E_t^{(v)}$ defined as unity if the individual is eligible for a benefit (for this symbol, a vested benefit) at age t and as zero if not eligible, equation (11.6d) properly accounts for those who are expected to receive a benefit at the time of decrement:

$$(FS)_x = \sum_{k=x}^{r^*-1} \frac{\sum_{t=k}^{r^*-1} \left[E_t^{(v)}\, d_t^{(t)} + E_t^{(d)}\, d_t^{(d)} + E_t^{(s)}\, d_t^{(m)} + E_t^{(r)}\, d_t^{(r)}\right]}{l_x^{(T)}}. \tag{11.6d}$$

The *average* future service (AFS) is found by summing (11.6d) for all plan members and dividing by the number of participants *expected to receive benefits.* The "expected to receive benefits" (ERB) function for one individual is defined by

$$(ERB)_x = \sum_{k=x}^{r^*-1} {}_{k-x}p_x^{(T)}\left[E_k^{(v)}q_k^{(i)} + E_k^{(d)}q_k^{(d)} + E_k^{(s)}q_k^{(m)} + E_k^{(r)}q_k^{(r)}\right]. \tag{11.7}$$

The *average* future service of plan participants expected to receive benefits is given by

$$(AFS)_t = \frac{(FS)_t}{(ERB)_t} \tag{11.8}$$

where the t subscript indicates that these functions are for all plan participants in year t. Average future service, as defined in (11.8) typically will fall in the range of 8 to 15 years.

Prepaid (Accrued) Expense. Another item involved in determining the SFAS 87 amortization cost is the so-called prepaid (accrued) expense. If cumulative contributions to the pension plan from its inception exceed the cumulative accounting expense, the difference is referred to as a *prepaid expense*. In this case, pension funding has "run ahead" of pension accounting, creating an accounting asset that must be accounted for (or amortized) in the future. On the other hand, if the cumulative pension expense exceeds cumulative contributions, the difference is an *accrued expense*. In this case, pension accounting has "run ahead" of pension funding, creating an accounting obligation that must be amortized in the future.

Denoting $(CC)_t$ as cumulative contributions from plan inception to year t and $(CE)_t$ as cumulative expense, a positive difference in (11.9) is a prepaid expense, while a negative difference is an accrued expense (even though it is frequently written without the negative sign):[14]

$$\textit{Prepaid (Accrued) Expense} = \sum (CC)_t - \sum (CE)_t. \tag{11.9}$$

Transition Obligation (Asset). This item, defined as the beginning of the first year to which SFAS 87 applies, is equal to (1) the PBO less (2) the market value of assets (MA) plus any prepaid (or less any accrued) pension expense:

$$\begin{matrix}\textit{Transition}\\ \textit{Obligation}\\ \textit{(Asset)}\end{matrix} = (PBO)_t - (MA)_t + \left[\sum (CC)_t - \sum (CE)_t\right]. \tag{11.10}$$

[14]Cumulative contributions and expense are easily determined by adding the current year's values to a running balance.

If the bracketed term in (11.10) is negative, then an accrued expense is deducted from the otherwise determined obligation, whereas a prepaid expense is added if the term is positive. This is logical because an accrued expense implies that a portion of the obligation has already been accounted for (or expensed) and should be excluded from the amortization. Similarly, a prepaid expense implies that a portion of the obligation has not yet been accounted for (or expensed); hence, it should be included in the amortization.

The transition obligation (asset) is amortized on a straight line basis (i.e., equal installments of principal) over the *average future service of plan participants expected to receive benefits*, equation (11.8) determined at the transition date, or, optionally, 15 years, if greater.[15] This schedule is fixed at the date of transition and is not affected by future changes in benefits or assumptions. The annual amortization cost will be positive if the value of (11.10) is positive (indicating a transition liability) and negative if the transition item is an asset.

Prior Service Cost. The prior service cost is defined as the change in the PBO due to a plan amendment. Its value is zero in the first year of SFAS 87 application, and takes on positive (negative) values as plan benefits are increased (decreased) after this time. The prior service cost is amortized over the future service of plan participants expected to receive benefits as of the date of the plan amendment (i.e., a closed group amortization). This is in contrast to the transition obligation which uses the *average* future service of plan participants expected to receive benefits. Each year's minimum amortization is based on the proportion of the originally determined future service that is expected to be worked in the upcoming year. Thus, the amortization cost (AC) during age k for prior service cost (PSC) created at the employee's age x is given by

$$(AC)_k = \frac{{}_{k-x}(FS)_x}{(FS)_x} (PSC)_x \quad (k \geq x) \tag{11.11}$$

where ${}_{k-x}(FS)_x$ represents the future service from age k to $k+1$. The amortization schedule is determined once and for all at the date of plan amendment by recording the subtotals for each value

[15] If all, or almost all, of the plan's participants are inactive, their average life expectancy can be used as the amortization period.

of k in (11.6d). The amortization cost for the entire plan is found by aggregating the numerator and denominator of (11.11) and applying this fraction to the plan's past service cost for the plan amendment in question.

The prior service cost amortization represents a continually decreasing portion of the initial prior service cost, and the amortization schedule is not complete until the last employee at the time of the plan amendment leaves employment. Since this amortization approach is numerically complex, the plan sponsor can elect to amortize the prior service cost more rapidly, for example, over the *average* future service period as defined by (11.8), or any consistently applied more rapid amortization (even immediate recognition). The annual prior service cost amortization payments will be positive if the plan amendment increases the PBO (the usual case) and negative if the PBO decreases as a result of the plan change.

Net Loss (Gain). The cumulative unrecognized net loss (gain) (i.e., the cumulative amount from the date SFAS 87 is applied less any prior amortizations) is determined in the following manner:

$$\begin{matrix}\text{Loss}\\(\text{Gain})\end{matrix} = \text{PBO} - \left[\begin{matrix}\text{Market-Related}\\\text{Value of Assets}\end{matrix} + \begin{matrix}\text{Unrecognized}\\\text{Transition}\\\text{Obligation}\end{matrix} + \begin{matrix}\text{Unrecognized}\\\text{Prior Service}\\\text{Cost}\end{matrix} - \begin{matrix}\text{Prepaid}\\(\text{Accrued})\\\text{Pension}\\\text{Expense}\end{matrix}\right]. \tag{11.12}$$

The portion of this loss (gain) that must be amortized is the value produced by (11.13), provided that the value is positive; otherwise there is no minimum amortization:

$$\begin{matrix}\text{Minimum}\\\text{Amount to be}\\\text{Amortized}\end{matrix} = \left|\begin{matrix}\text{Loss}\\(\text{Gain})\end{matrix}\right| - .10 \bullet \text{Max}\left[\begin{matrix}\text{PBO}\\\text{Market-Related}\\\text{Value of Assets}\end{matrix}\right]. \tag{11.13}$$

In words, the cumulative gain (or loss) in excess of 10% of the PBO (or market-related value of assets, if larger) must be amortized.

The amortization is based on a "rolling" schedule, as opposed to the two types of fixed schedules used with the transition obligation and prior service cost. Each year the value of (11.8) is

evaluated for the entire group of active participants, with its reciprocal being used to determine the proportion of the minimum loss (gain) amortized in the current year.[16] In the following year a new minimum amount, if any, is determined and amortized according to a newly determined reciprocal of (11.8) and so forth.[17] The amortization payment for the year in question will be positive for a net loss and negative for a net gain.

A corridor smaller than 10%, or even a zero corridor, can be used in (11.13) if done consistently. Since each year's amortization is determined independently of the prior year's, it is possible to have loss (gain) amortization in one year and not the next.

The net loss (gain) reflects both the experience deviations during the year as well as actuarial assumption changes. This is unique to pension accounting, since for pension funding purposes, actuarial assumption changes are treated as a separate item for amortization purposes.

Disclosure of Net Periodic Pension Cost

The components of the net periodic pension cost must be disclosed in the employer's financial statements. SFAS 87 requires a disclosure of the *actual return on assets* along with the other components discussed above. Since these statements are prepared after the close of the firm's fiscal year, the actual return on plan assets is known.

Table 11–2 shows the disclosure items. Note that the asset-based loss (gain) is subtracted from the expected return on assets (line 3 in Table 11–2) and then added into the amortization items (line 4). While these offsetting entries do not affect the value of the net periodic pension cost, the disclosure shows the actual return on assets.

The asset-based loss (gain) can be determined by

$$\begin{matrix}\textit{Asset-Based}\\ \textit{Loss (Gain)}\end{matrix} = (EROA)_t - [(MA)_{t+1} + B_t - C_t - (MA)_t] \quad (11.14)$$

[16]For plans primarily covering inactive participants, the average life expectancy can be used.

[17]It should be noted that this "rolling" schedule would never amortize a given dollar amount, since amortizing $1/n$ th of each year's remaining balance will never fully extinguish the original balance. This is not necessarily a deficiency in the amortization methodology, however, since actuarial gains and losses should be offsetting over time, especially if the underlying assumptions are best-estimates, as required under SFAS 87.

where

$(EROA)_t$ = expected return on assets, equation (11.5b)

B_t = actual distribution (benefit payments)

C_t = actual contributions

$(MA)_t$ = market assets at beginning of year

$(MA)_{t+1}$ = market assets at end of year t.

TABLE 11–2
Disclosure of Net Periodic Pension Cost

1. Service Cost	Equation (11.4)
2. Interest Cost	Equation (11.5a)
3. Actual Return on Assets	Expected Return on Assets – Asset-Based Loss (Gain)
4. Net Amortization and Deferrals	Transition Obligation + Prior Service Cost + Loss (Gain) + Asset-Based Loss (Gain)
5. Net Periodic Pension Cost	(1) + (2) – (3) + (4)

Numerical Illustration of Net Periodic Pension Cost

Table 11–3 shows a 20-year projection of the net periodic pension cost and its various components for the model pension plan. The valuation and experience assumptions are identical to those listed in Tables 10–8 and 10–9 of Chapter 10 used to project minimum and maximum statutory contributions. The expected return on assets is an additional assumption required for the SFAS 87 projection; 8 percent is assumed in the projection. The "expected return on assets" and the "asset-based loss (gain)" are shown in Table 11–3 for clarity even though they are not disclosure items *per se*.

TABLE 11–3

Projection of SFAS 87 Net Periodic Cost Disclosure Items
(End-of-Year Values as Percent of Payroll)

Year	(1) *Service Cost*	(2) *Interest Cost*	(3) *Expected Return*	(4) *Asset Loss (Gain)*	(5) *Actual Return* (3 – 4)	(6) *Transition Obligation Amortization*	(7) *Prior Service Cost Amortization*	(8) *Loss (Gain) Amortization*	(9) *Net Amortization* (6 + 7 + 8 – 4)	(10) *Net Periodic Pension Cost*
1	4.83	8.10	7.36	–1.66	9.02	1.01	0.00	0.00	2.67	6.57
2	4.75	8.43	7.65	–2.04	9.69	0.97	0.00	0.00	3.01	6.49
3	4.71	8.68	7.96	–2.37	10.32	0.92	0.00	0.00	3.29	6.36
4	4.70	8.98	8.29	–2.63	10.92	0.88	0.09	0.00	3.60	6.37
5	4.71	9.15	8.64	–2.83	11.47	0.84	0.09	0.00	3.75	6.15
6	4.72	9.31	9.01	3.02	5.99	0.80	0.09	0.00	–2.14	5.91
7	4.73	9.56	9.25	3.32	5.92	0.76	0.19	0.00	–2.38	5.98
8	4.74	9.69	9.38	3.52	5.85	0.72	0.18	0.00	–2.62	5.95
9	4.75	9.82	9.44	3.65	5.79	0.68	0.17	0.00	–2.79	5.99
10	4.75	10.05	9.47	3.71	5.76	0.65	0.28	0.00	–2.78	6.27
11	4.76	10.16	9.50	–2.04	11.54	0.62	0.27	0.00	2.93	6.31
12	4.77	10.26	9.65	–2.51	12.16	0.59	0.26	0.00	3.36	6.23
13	4.77	10.47	9.90	–2.90	12.79	0.56	0.37	0.00	3.83	6.28
14	4.77	10.55	10.20	–3.21	13.41	0.53	0.35	0.00	4.10	6.02
15	4.77	10.63	10.52	–3.43	13.95	0.51	0.32	0.00	4.26	5.72
16	4.78	10.82	10.83	3.64	7.19	0.00	0.41	0.00	–3.24	5.18
17	4.78	10.88	10.98	3.95	7.03	0.00	0.39	0.00	–3.56	5.06
18	4.78	10.93	11.01	4.15	6.86	0.00	0.35	0.00	–3.80	5.05
19	4.78	11.10	10.96	4.25	6.70	0.00	0.44	0.00	–3.82	5.36
20	4.78	11.13	10.86	4.28	6.59	0.00	0.42	0.00	–3.86	5.46

The service cost, like the normal cost projection in Chapter 10, is relatively constant throughout the projection period.[18] The interest cost increases continuously, from about 8 percent of payroll to 11 percent, an increase resulting from the PBO growing faster than payroll due to the aging of the population. The expected return on assets parallels the interest cost, since assets are approximately equal to the PBO (see Table 11–5 yet to be discussed). The asset loss (gain) alternates between a loss and a gain every 5 years as a result of the actual return on assets assumed in the experimental design. Actual returns, of course, are equal to the expected return less the asset loss (or gain).

The amortization of the transition obligation is a continually decreasing percent of payroll over 15 years, starting at 1.01 percent and reducing to .51 percent by the 15th year. The *ad hoc* COLA's, amounting to 25 percent of cumulative inflation every 3 years, have only a minor amortization cost in the early years of the projection, reaching an ultimate level of about .4 percent of payroll by the end of 20 years.

The amortization of the loss (gain) from investment experience alternating from 5 percent higher to 5 percent lower than the 10 percent expected return on assets has no impact on costs. This illustrates that a 10 percent corridor of the PBO (or market-related value of assets, if greater) provides a substantial cushion for smoothing the actual return on assets.

The net periodic pension expense, representing the interaction of these components, fluctuates in the range of 5.5 to 6.5 percent of payroll throughout the projection period.

BALANCE SHEET

The SFAS 87 accounting requirements nearly always cause contributions and pension expense to be different. This contrasts with past accounting rules and practice, under which contributions and expenses were nearly always equal. Thus, there are annual balance sheet entries recording the difference between these

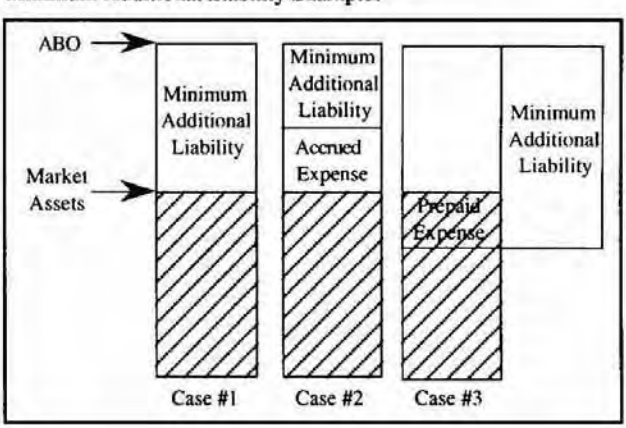
[18]Since the projection of contributions in Chapter 10 is based on the benefit prorate cost method, its normal cost is the same as the service cost for SFAS 87 purposes under the specified assumptions (and assuming there are no applicable maximum benefit and salary limitations). The service cost shown in Table 11–3 is 8 percent larger than the normal cost shown in Table 10–8 because it includes one year's interest.

two quantities, with a prepaid expense (accounting asset) being generated if cumulative contributions exceed cumulative expenses, and an accrued expense (accounting liability) recorded when the reverse it true.

Minimum Additional Liability

SFAS 87 defines a *minimum additional liability* that must be entered on the sponsor's balance sheet. This liability is equal to the difference between the ABO and the market value of assets (reduced for any accrued expense or increased for any prepaid expense). Figure 11–1 illustrates three cases for which a minimum additional liability is required. In case #1 the minimum additional liability is simply the difference between the ABO and market assets, since there is no accrued or prepaid expense. Case #2 has a smaller minimum additional liability because of the existence of an accrued expense (i.e., expensing has "run ahead" of funding). Since an accrued expense is an accounting liability, it is deducted in determining the minimum additional liability recorded on the sponsor's balance sheet. Case #3 has a larger minimum additional liability because of a prepaid expense (i.e., funding has "run ahead" of expensing). Since the prepaid expense is an accounting asset, it must be added in determining the minimum additional liability.

FIGURE 11–1
Minimum Additional Liability Examples



Intangible Asset

If an additional liability is recorded on the balance sheet, an intangible asset can be recorded to avoid a reduction in stockholders' equity. However, the *maximum* intangible asset is limited to the sum of (1) the unrecognized transition obligation and (2) the unrecognized prior service cost. If the minimum additional liability exceeds the maximum intangible asset, stockholders' equity is reduced by the difference.

Reconciliation of Funded Status

The employer's financial statements must include a reconciliation of funded status. The items disclosed in this reconciliation are as given in Table 11–4. It should be noted that the ABO and PBO disclosure items might be based on a valuation different from the valuation used to determine the net periodic pension cost. Figure 11–2 has been constructed to illustrate this situation for a plan having a 1/1 to 12/31 valuation year and a 7/1 to 6/30 fiscal year.

Numerical Illustration of Financial Statement Disclosure Items

A 20-year projection of the SFAS 87 disclosure items are shown in Table 11–5, based on the same assumptions as the projection of the net periodic pension cost given previously. The VBO is within 2 percentage points of the ABO, a result that would be expected given the maturity of the population and a 5-year vesting assumption. The ABO is 78 percent of the PBO at the outset of the projection and climbs steadily to 83 percent by the end of 20 years. Market assets were equal to 85 percent of the PBO in the first year and, because of the initially favorable investment experience, climb to 100 percent after 5 years. This relationship fluctuates between about 90 and 105 percent, as the investment experience fluctuates from 5 percent to 15 percent during the 20-year projection.

The unrecognized transition obligation equals a rapidly declining percentage of the PBO. The unrecognized prior service cost is quite small, never exceeding 2.5 percent of the PBO. The unrecognized loss (gain) from the alternating investment experi-

TABLE 11–4
Reconciliation of Funded Status

1. PBO	Year-end value {or projected year-end value from equation (11.3b) }
2. Market Assets	Year-end value
3. Funded Status	(1) – (2)
4. Unrecognized Transition Obligation	Beginning of year value less current year's amortization amount
5. Unrecognized Prior Service Cost	Beginning of year value less current year's amortization amount
6. Unrecognized Net Loss (Gain)	(3) – (4) – (5) + {Prepaid (Accrued) Expense at B.O.Y. – Net Periodic Pension Cost + Contributions}
7. Prepaid (Accrued) Pension Cost	(3) + (4) + (5) + (6)
8. Additional Liability	Based on year-end value of ABO {or projected year-end value from equation (11.1b) }
9. (Pension Liability) Prepaid Pension Cost	(7) + (8)

FIGURE 11–2
Valuation Year vs. Fiscal Year

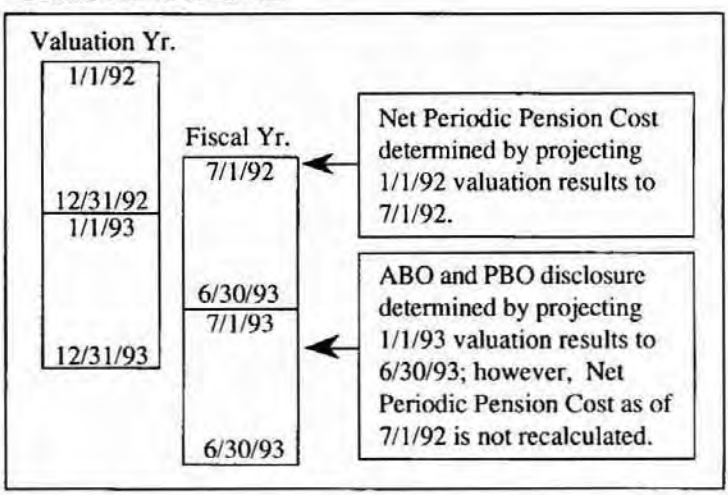


TABLE 11–5

Projection of SFAS 87 Financial Statement Disclosure Items
(Values Expressed as Percent of PBO)

Year	*VBO*	*ABO*	*PBO*	*Market Assets*	*Funded Status*	*Unrecognized Transition Obligation*	*Unrecognized Prior Service Cost*	*Unrecognized Loss (Gain)*	*Additional Liability*	*Prepaid (Accrued) Cost*
1	76.83	77.99	100.00	85.33	14.67	14.67	0.00	0.00	0.00	0.00
2	77.58	78.84	100.00	88.42	11.58	12.58	0.00	-1.60	0.00	-0.60
3	78.03	79.37	100.00	91.76	8.24	10.81	0.00	-3.41	0.00	-0.84
4	78.51	79.92	100.00	94.25	5.75	9.19	0.96	-5.29	0.00	-0.89
5	78.70	80.18	100.00	97.55	2.45	7.86	0.82	-7.21	0.00	-0.99
6	78.84	80.41	100.00	100.64	-0.64	6.68	0.70	-9.17	0.00	-1.16
7	79.31	80.82	100.00	97.63	2.37	5.56	1.61	-6.32	0.00	-1.51
8	79.53	81.01	100.00	95.26	4.74	4.64	1.37	-3.53	0.00	-2.27
9	79.75	81.19	100.00	92.87	7.13	3.81	1.15	-0.83	0.00	-3.01
10	80.14	81.57	100.00	89.78	10.22	3.03	2.01	1.74	0.00	-3.43
11	80.32	81.73	100.00	88.28	11.72	2.38	1.69	4.19	0.00	-3.46
12	80.49	81.89	100.00	91.76	8.24	1.79	1.40	2.29	0.00	-2.77
13	80.85	82.24	100.00	94.72	5.28	1.25	2.23	0.16	0.00	-1.64
14	80.99	82.37	100.00	98.86	1.14	0.79	1.84	-2.05	0.00	-0.55
15	81.12	82.50	100.00	102.82	-2.82	0.37	1.50	-4.36	0.00	0.33
16	81.44	82.81	100.00	105.05	-5.05	0.00	2.33	-6.62	0.00	0.76
17	81.53	82.91	100.00	102.80	-2.80	0.00	1.93	-3.96	0.00	0.77
18	81.61	82.99	100.00	100.07	-0.07	0.00	1.56	-1.26	0.00	0.37
19	81.89	83.27	100.00	96.11	3.89	0.00	2.42	1.38	0.00	-0.09
20	81.94	83.32	100.00	93.65	6.35	0.00	2.01	3.96	0.00	-0.38

ence never exceeds 10 percent of the PBO. Since this is below the minimum amortization corridor, the loss (gain) amortization payments are zero, as previously observed in Table 11–3.

The projection did not require an additional liability to be recorded on the sponsor's balance sheet because assets at all times exceeded the ABO. The last column in Table 11–5 shows that contributions were approximately equal to pension costs, since only a minimum prepaid (accrued) cost occurs during the projection period.

Table 11–6 shows a reconciliation of SFAS 87 values from the end of one year to the end of the next year. This format, with illustrative numbers included, may be useful in better understanding the interaction of the various SFAS 87 components.

CRITIQUE OF SFAS 87

In many respects the SFAS 87 promulgation achieves its objectives of consistency, uniformity, and comparability of pension accounting among plan sponsors. There are several aspects of the promulgation, however, that, if changed, would result in an improved set of accounting procedures. These are briefly discussed in the order of their importance.

Salary vs. Benefit Proration

The service proration methodology used for the service cost and PBO should be changed to a proration based on salary. As noted earlier, the salary proration is consistent with the view that pensions are deferred wages. For example, by the time an employee earns 50 percent of his (or her) career compensation, 50 percent of the career pension benefit should be deemed to have been earned. A byproduct of this recommendation is that the salary assumption becomes less critical, since it is both in the projection of the benefit and in the proration.[19] This would further standardize pension cost accounting among plan sponsors.

[19]The salary assumption in the proration should be the same as in the benefit projection. Thus, a flat benefit formula (e.g., $150 per year of service) would not be affected by the salary proration methodology.

TABLE 11–6

Reconciliation of SFAS 87 Values

		Plan Assets	PBO	Transition Obligation	Loss (Gain)	Prior Service Cost	Periodic Pension Cost	Accrued (Prepaid) Pension Cost
	End of Prior Year	100	(100)	(50)	(10)	40		20
	Service Cost		(10)				10	
	Interest Cost		(15)				15	
	Expected Return on Assets	10					(10)	
Amortization Amounts for:	Transition Obligation			5			(5)	
	Loss (Gain)				2		(2)	
	Past Service Cost					(3)	3	
	Periodic Pension Cost						11	11
	Benefit Payments	(2)	2					
	Contributions	4						(4)
Gain (Loss)	Asset-Based	3			(3)			
	Liability-Based		5		(5)			
	End of Current Year	115	(118)	(45)	(16)	37		(27)

Linear vs. Formula Proration

The projected benefit should be uniformly prorated from entry age to each future decrement age. This is in contrast to the SFAS 87 procedure of allocating benefits according to the plan's benefit formula which, due to various maximums, front or back loading formulas, step-up or step-down formulas, offsets and so forth, can produce anomalous prorations. This recommendation

should be implemented regardless of whether the service or salary proration scheme is used.

Interest Cost

Basing the interest cost on a so-called settlement rate, reflective of the cost of annuities, overemphasizes a "wind up" or termination contingency. Annuity rates include insurance company risk, expense, and profit charges which should not be part of the interest rate determination. On the other hand, if the FASB desires the discount rate to reflect market conditions, then the rate should be keyed to the spot rate on investment-grade, long-term corporate bonds as of the measurement date. Changes should be made only when the index causes the prior year's rate to be changed by more than a meaningful amount, such as 50 basis points. In any case, the author favors allowing the long-term return on assets to be selected with an eye towards management's best estimate of such return, taking into account the asset allocation policy of the plan.

Gain (Loss) Amount

It would be useful to show the effect of discount rate changes, as well as other actuarial assumption changes, separate from the effects of experience differing from the underlying assumptions. This is particularly important in light of the continually changing discount rate. This should be done whether the amortization of these two items are treated the same or differently.

Gain (Loss) Amortization

The 10 percent corridor around the larger of the PBO or market-related assets can produce some strange results. For example, (1) a plan that has assets equal to or larger than the PBO has a 10 percent asset-based cushion, (2) a plan that has assets equal to 50 percent of the PBO has a 20 percent asset-based cushion, and (3) a plan that has assets equal to 20 percent of the PBO has a 50 percent asset-based cushion. If anything, one might be inclined to be harsher on lower funded plans rather than more lenient in the degree of asset volatility that enters the accounting expense. If the *lower* of these two values were used, then the benefit to lower

funded plans would be removed; however, plans with funded ratios in excess of 100 percent would still be penalized (e.g., a plan with a 200 percent funded plan would have only a 5 percent cushion). This problem could be resolved if one simply based the corridor on assets alone, but then PBO-based gains and losses would be affected differently for different funded ratios.

The shortcoming in the gain (loss) corridor is that it focuses on the wrong item. Since the ultimate objective is to adjust *costs* if gains and losses become too significant, on the one hand, without being overly sensitive, on the other, the corridor might be based on the service cost. For example, the amortization amount associated with the full unrecognized gain (loss) might be determined and, then, the portion of the amortization payment outside a 10 percent corridor of the service cost, for example, might be recognized.

Amortization Periods

The future service of employees, instead of the future service of employees *expected to receive benefits*, should be used. While the SFAS 87 approach is conceptually correct, it is computationally complex and simply not worth the effort, in spite of the availability of high speed computers.

VBO Disclosure

The separate disclosure of the VBO adds little value and should be eliminated as a required disclosure item.

Terminology

Past service cost should be "past service obligation (asset)." Similarly, the transition obligation should be "transition obligation (asset)." Loss (gain) should be "experience loss (gain), and changes in assumptions should be labeled "assumption change obligation (asset)." These cosmetic changes would facilitate understanding, which is one of the SFAS 87 objectives.