PENSION MATHEMATICS
with Numerical Illustrations

Second Edition

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Chapter 1

Pension Plan Benefits

The primary function of a pension plan is to provide income to employees in their retirement. Pension plans are not limited to providing retirement income, however, and all plans provide at least some of the following types of benefits: (1) vested termination benefits, (2) disability benefits, and (3) death benefits, the latter consisting of either a lump sum benefit or an annuity to a surviving spouse or other dependents. The eligibility requirements and benefit formulas typically associated with each of these benefits are discussed in this chapter. In addition, the benefits associated with a model pension plan used to illustrate pension costs throughout this book are given.[1]

There are two basic types of pension plans: *defined contribution* (DC) and *defined benefit* (DB). A defined contribution plan, as the name implies, has a defined amount of employer and/or employee contributions set aside each year, often as a specified percentage of salary.[2] The employee's retirement benefit is determined by the size of the accumulation at retirement. For example, if the accumulation totals $100,000 and is applied to provide an annuity payable for life, the annual benefit might fall in the range of $10,000 to $15,000 per year, depending on the em-

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[1]For comprehensive treatments of pension plan design see: Everett T. Allen, Jr., Joseph J. Melone, Jerry S. Rosenbloom and Jack L. VanDerhei, *Pension Planning*, 7th ed. (Homewood, Ill.: Richard D. Irwin, 1992); Jerry S. Rosenbloom, *The Handbook of Employee Benefits: Design, Funding and Administration*, 3rd ed. (Homewood, Ill.: Richard D. Irwin, 1992); and Dan M. McGill and Donald S. Grubbs, Jr., *Fundamentals of Private Pensions*, 6th ed. (Homewood, Ill.: Richard D. Irwin, 1989).

[2]Profit sharing plans, which are a type of "defined contribution" plan, often define the method of allocating contributions among employees rather than defining contributions themselves.

ployee's age at retirement, the interest rate assumed during retirement, the form of the benefit payment (e.g., whether or not payments continue to a surviving spouse after the death of the annuitant), and so forth. The important point is that for a DC plan, yearly contributions to an accumulation fund are *defined*, whereas the benefit ultimately paid to the employee is not known for certain until retirement.

Table 1–1 illustrates the benefits payable, as a percentage of retirement-age salary, for an 8-percent-of-salary DC plan under various assumptions as to employment periods and interest returns on the accumulating funds. The other assumptions used in the illustration are (1) salary increases of 6 percent per year, (2) retirement at age 65, (3) 8 percent interest return in retirement, (4) mortality based on 1971 Group Annuity Mortality (GAM) Table, and (5) benefits payable during the life of the retiree only (i.e., a so-called straight life annuity). The results illustrate a substantial difference in benefits for employees with differing periods of service and/or interest returns on the accumulating funds. The benefit percentages are directly proportional to the contribution rate; hence, a 4 percent contribution formula would produce benefits equal to 50 percent of those shown in Table 1–1.

TABLE 1–1

Retirement Benefits as Percent of Salary at Retirement for 8-Percent-of-Salary DC Plan

Interest Return	*Entry Ages* 25	35	45
6%	39%	29%	20%
8%	60	40	24
10%	92	55	30

A defined benefit plan is one under which the retirement benefit (as opposed to the employer's contribution) is the defined quantity, generally expressed in terms of the employee's salary and length of service. For example, the plan might provide a benefit of 1.5 percent of the employee's average salary during the final 5 years of employment for each year of service rendered. An employee with 20 years of service would receive a benefit equal to 30 percent of final average salary. Since this example is based on the employee's salary, the precise benefit will not be known until retirement, as was the case under a defined contri-

bution plan; nevertheless, the *benefit formula*, as opposed to the *contribution formula*, is defined, hence, the term defined benefit plan.

Table 1–2 illustrates the benefits payable, as a percentage of retirement-age salary, for alternative DB formulas that encompass the range typically found among large corporate pension plans. The benefit formulas are based on the employee's final 5-year average salary. The other assumptions used in the illustration are (1) salary increases of 6 percent per year, (2) retirement at age 65, and (3) benefits payable during the life of the retiree only. The 1.5 percent DB formula is roughly equivalent to the 8 percent DC formula assuming the fund earns 8 percent per year. This might lead one to conclude that the costs associated with a 1.5 percent DB plan would equal 8 percent of salary. While this would be true if neither plan offered vesting prior to retirement, it is not the case if such plans provide the minimum vesting standards required by law. The DB plan is generally less costly because the cost of providing vested accrued benefits to terminating employees is typically less than the value of employer contributions taken by employees terminating under the DC plan prior to retirement.

TABLE 1–2

Retirement Benefits as Percent of Salary at Retirement for Alternative DB Plan Formulas

Benefit Percent	*Entry Ages* 25	35	45
1.0%	35%	26%	17%
1.5%	52	39	26
2.0%	69	52	35

This book presents the mathematics applicable to defined benefit pension plans. In particular, the various methods of determining employer contributions required to fund such benefits are given. The last chapter of this book considers a related topic; namely, the mathematics associated with retiree health benefit plans.

RETIREMENT BENEFITS

The eligibility requirements and benefit formulas typically associated with DB pension plans are discussed in this section, along with the provisions assumed for the model pension plan.[3] The discussion also points out the minimum plan design requirements and other pertinent rules established under the Employee Retirement Income Security Act (ERISA), as well as subsequent legislation.

Eligibility Requirements

There are two categories of eligibility requirements related to retirement benefits, one setting the requirements for plan membership and the other specifying the requirements for retirement under the plan.

The Employee Retirement Income Security Act of 1974 (ERISA), as amended, places a limit on the age and service requirements for plan membership. An employee must be eligible to join the plan after reaching age 21 and rendering one year of service.[4] Naturally, a plan can provide more liberal eligibility requirements than those specified under ERISA, and some plans have no waiting period whatsoever. Unless the plan is contributory, in which case the employee may elect not to participate, benefits frequently begin to accrue at the time these membership eligibility requirements are met. Alternatively, benefit accruals might be granted retroactively to the employee's date of hire, but the law does not require that pre-eligibility service be included for benefit accrual purposes. The pension cost data presented in this book are based on the assumption that plan membership commences at the date of hire.

At the other end of the employment cycle are the eligibility requirements for retirement. The normal retirement age of the plan is stated in the plan document and frequently specified as age 65. The traditional definition of the normal retirement age is the first age at which retirement can occur without any reduction in the benefits calculated according to the plan's benefit formula. This definition does not apply to many large corporate pension

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[3]Table 1–3 on page 11 summarizes all of the model pension plan benefits.

[4]If the plan provides full and immediate vesting, the eligibility requirements for plan participants can be extended to two years of service.

plans, however, since non-reduced benefits are often provided at ages below the plan's normal retirement, sometimes as early as age 60 for a plan having 65 as the normal retirement age. The relevance of the normal retirement age for such plans is that it is frequently used as the commencement date for deferred pension benefits to vested terminated employees and, in some cases, to disabled employees.

The symbol r is used to denote the normal retirement age in the pension mathematics presented in later chapters, with r' denoting the first age at which an employee becomes eligible for early retirement, with either full benefit accruals or reduced benefits, and r" denoting the age by which all employees are assumed to be retired. Unless otherwise stated, r is age 65 in the numerical illustrations and r' is assumed to be based on the employee's age and service.[5] The early retirement eligibility provision assumed for the model pension plan is one permitting retirement, on an actuarially reduced basis, upon the attainment of age 55 and the completion of 10 years of credited service. Thus, for most employees, r' will be age 55.

Benefit Amount

The most common type of benefit formula used in pension plans is the so-called *unit benefit formula*, which provides a unit of benefit for each year of credited service. There are three such formulas associated with defined benefit plans: (1) flat dollar, (2) career average, and (3) final average. The flat dollar benefit formula is the simplest of the three, providing a dollar amount, such as $20, per month for each year of service rendered by the employee. The flat dollar amount is generally increased at periodic intervals by plan amendment, either to keep pace with the inflationary trends in the economy and/or in response to union negotiations. Since it is not permissible under existing IRS regulations to anticipate future benefit increases for funding purposes, the intermittent jumps in the unit benefit can cause erratic contribution patterns and may prevent such plans from reaching full funding.[6]

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[5]The oldest retirement age, r'', is assumed to be equal to r unless otherwise indicated.

[6]The funding characteristics associated with this type of benefit formula are illustrated in Chapter 13.

The career average benefit formula provides a benefit defined in terms of some stipulated percentage of the employee's career average salary. For example, a plan might define an employee's benefit accrual as 2 percent of each year's salary.[7] It is permissible under current law to use a larger percentage for salary in excess of the average Social Security wage base than the percentage applied to salary under this level, an integration formula known as the *step rate method.* The difference in the percentages, which is generally limited to a value of .75 percent (or, if lower, the base percentage itself), is allowed so that an employer's plan can compensate for the fact that Social Security benefits are not based on total salary.[8] Thus, if a plan were to use the maximum difference under the step rate benefit formula, in theory at least, total benefits (pension plus the portion of the Social Security benefit attributable to employer contributions) would constitute roughly the same percentage of salary for all employees. A plan using such a formula is said to be *integrated* with Social Security, and is deemed under current law not to be discriminatory in favor of highly paid employees. Although the employer's plan itself does indeed discriminate in favor of higher paid employees, such discrimination is designed to offset the reverse discrimination inherent in Social Security benefits which favor lower paid employees.

The final average benefit formula is one providing a given percentage of the employee's final average (or highest average) salary per year of service. Since the benefits derived from this type of formula are based on the employee's salary near retirement, the percentage need not be as high as the career average formula percentage in order to provide equivalent benefits.

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[7]Career average benefit formulas are often updated by basing past service benefits on the employee's current salary. Thus, the employee's retirement benefit may approximate the benefits of a final average formula. As under a flat dollar benefit formula, the updating process can cause erratic contribution patterns and may prevent such plans from reaching full funding, as illustrated in Chapter 13.

[8]The .75 percent factor must be reduced if the plan permits normal retirement prior to the employee's normal retirement age under Social Security (which is now dependent on the employee's birth date) and/or if the benefit reduction for early retirement is lower than the reduction specified under law. As a practical matter, the factor is .65 for plans that want to use the same benefit formula for all participants, since age 67 is the applicable retirement age under Social Security for younger participants and most plans have age 65 as the normal retirement age.

This formula can also be integrated with Social Security according to the step rate procedure. For example, a plan could provide 1 percent of an employee's final 5-year average salary up to the applicable Social Security wage base and 1.75 percent on the excess salary (a smaller differential may be required, depending on the plan's normal and early retirement provisions).

Another procedure for integrating benefits with Social Security is known as the *offset method*. Under this procedure, which is often used with final average benefit formulas, the retirement benefit of the plan is determined without regard to the Social Security wage base, but the benefit so determined is offset (reduced) by up to .75 percent of the retiring employee's final 3-year average compensation (up to the covered compensation under Social Security) per year of service. In no event can the reduced benefit be less than 50 percent of the unreduced benefit.[9]

The formula used for the model pension plan is 1.5 percent of the participant's final 5-year average salary for each year of credited service. Social Security integration is not used in order to simplify the various analyses; however, many pension plans are in fact integrated with Social Security. The normal annuity form is a straight life annuity, with early retirement benefits actuarially reduced (i.e., a reduction that offsets the otherwise higher cost of benefits beginning sooner and expected to be paid longer to participants retiring early).

VESTED BENEFITS

An employee has a vested benefit if its payment at retirement is no longer contingent upon remaining in the service of the employer. When an employee terminates employment with a vested benefit, the benefit amount generally becomes payable at the plan's normal retirement age; however, some plans permit pay-

[9]Another offset methodology, which was widely used prior to the Tax Reform Act of 1986, was to reduce the final average formula benefit by a percentage of the employee's Primary Insurance Amount (PIA) payable under Social Security, with the offset generally limited to 50 percent, but possibly as high as 83 1/3 percent. While this method is still used, the benefit payable (when combined with Social Security benefits) cannot represent a higher percentage of salary for higher paid employees than for lower paid employees.

ments to begin at an early retirement age, usually with an actuarial reduction for early payment.[10]

Eligibility Requirements

The Tax Reform Act of 1986 (TRA '86) requires that pension benefits vest according to one of two schedules: (1) full vesting upon the completion of 5 years of service or (2) graded vesting, with 20 percent of the accrued benefit vested after completing 3 years of service and an additional 20 percent per year thereafter, reaching full vesting after 7 years of service.[11] Vesting after 5 years of service is used for the model pension plan.

Benefit Amount

In most cases, the benefit accruals used to determine vested benefits are those defined by application of the retirement benefit formula. The cost of providing benefits to vested terminating employees could be minimized by adopting a formula with disproportionate benefits provided at older ages and/or longer periods of service; however, the law has established guidelines to prevent such *backloading.*

In all cases, benefits attributable to employee contributions are fully and immediately vested. At the time of termination, employees may be entitled to a return of their contributions, usually with interest, instead of leaving them in the plan and receiving a deferred retirement benefit. Prior to the passage of ERISA, the employee's election to take back contributions often had the effect of forfeiting all of the benefits associated with employer contributions.

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[10]Most plans pay small vested benefits as lump sums to terminated employees and many plans allow lump sum payments for larger amounts as well.

[11]ERISA initially mandated three alternative vesting schedules: (1) full vesting after 10 years, (2) graded vesting, with 25 percent vesting after 5 years, increasing by 5 percent per year for the next 5 years and then 10 percent per year for the next five years, and (3) Rule-of-45 vesting, with 50 percent vesting when the participant's age and years of service total 45, and an additional 10 percent for each of the 5 subsequent years.

DISABILITY BENEFITS

Two types of disability benefits are found in pension plans, one providing a deferred pension to disabled employees beginning at the plan's normal retirement age and continuing for life, and the second providing benefits that commence after a specified waiting period, such as 6 or 9 months, and continuing for life. The former type is generally found in combination with a long-term disability (LTD) benefit program operating outside the pension plan, with the LTD plan providing the desired level of benefits from the time of disability to the plan's normal retirement age.

Eligibility Requirements

The eligibility provisions for disability benefits vary widely among plans, but a minimum age or service requirement, or both, usually exists. The disability entitlement might coincide with eligibility for early retirement, for example, age 55 and 10 years of service, or with eligibility for vesting. The model pension plan provides employees with their accrued benefits, payable immediately, if disability occurs after age 40 with 10 years of credited service.

Benefit Amount

The most common method used to define disability benefits is simply to apply the retirement benefit formula to the employee's salary and years of service at the time of disability. Some plans use the total potential service of the employee up to the plan's normal retirement instead of actual service at the date of disability. This is almost invariably the case if the disability pension is the deferred type used in conjunction with an LTD program. In some instances, the amount of the disability benefit is a flat amount per month, irrespective of the employee's service or accrued benefit. The benefit assumed for the model pension plan is the unreduced accrued benefit commencing at the time of disability and payable for life.

DEATH BENEFITS

Death benefits may consist of a lump sum distribution, such as a flat dollar amount or, for active employees, some multiple of salary. Alternatively, the death benefit may take the form of an annuity payable to a surviving spouse.

Eligibility Requirements

Death benefit eligibility provisions are often related to the plan's eligibility for vesting. In fact, ERISA requires that a surviving spouse option be made available to plan members at the time they become vested. The benefit must be payable by the age the employee would have been eligible for early retirement, or it can be made available earlier. In either case, an actuarial reduction can be applied if the benefit commences prior to the employee's normal retirement age.

The model pension plan provides a surviving spouse benefit upon satisfying the requirements for vesting (i.e., 5 years of service).

Benefit Amount

If a surviving spouse benefit is payable, the amount is generally some percentage of the deceased participant's accrued benefit. A common formula is 50 percent of the participant's attained age accruals, possibly reduced for early retirement or for the actuarial cost of the surviving spouse benefit itself. The benefit payable under the model pension plan is 50 percent of the employee's attained age accrued benefit without reduction.

A summary of the benefits provided under the model pension plan is given in Table 1–3. While some plans require employee contributions, the model plan, as indicated in Table 1–3, does not require such contributions.

TABLE 1–3

Summary of Model Plan Benefits

I. Retirement Benefit	
A. Eligibility	
1. Normal Retirement	Age 65
2. Early Retirement	Age 55 and 10 years of service
B. Benefit	1.5 percent of final 5-year average salary per year of service, payable for life, actuarially reduced for early retirement
II. Vested Benefit	
A. Eligibility	Full vesting after 5 years of service
B. Benefit	Accrued benefit (based on the retirement benefit formula applied to final average salary and service at termination), payable at age 65 for life
III. Disability Benefit	
A. Eligibility	Age 40 and 10 years of service
B. Benefit	Accrued unreduced benefit, payable immediately for life
IV. Death Benefit	
A. Eligibility	5 years of service
B. Benefit	50 percent of accrued benefit, payable for life of surviving spouse, commencing when employee would have been eligible for early retirement
V. Employee Contributions	None