The Pension Challenge

Risk Transfers and Retirement Income Security

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Part II

Global Developments in Retirement Risk Transfer

Chapter 8

Understanding Individual Account Guarantees

Marie-Eve Lachance and Olivia S. Mitchell

Demographic aging is prompting workers everywhere to realize that they are vulnerable to the inherent uncertainty that arises from unfunded social security systems. This realization has prompted a global wave of social security reforms, resulting in over twenty countries setting up individual account (IA) plans. Interest in this movement has gained strength in the United States with the release of the President's Commission to Strengthen Social Security (CSSS) Final Report, in which voluntary IAs are proposed as a component of a reformed system.[1]

Key strengths of IAs are that participants gain ownership in their accounts and may diversify their pension investments. But in view of the recent demise of Enron, some have argued that access to capital market investments might impose new risk on IA participants.[2] Concern over capital market volatility has consequently prompted some policymakers to propose "guarantees" for defined contribution pension accumulations.[3] Abroad, such guarantees have already been adopted in several Latin American countries undergoing reform,[4] and more recently, in Japan and Germany.[5]

The purpose of this chapter is to illustrate how one might evaluate pension guarantees in the context of an IA component of a social security reform.[6] Plan designers and budget analysts should recognize guarantee costs and identify how they can be financed. Sensible public policy that proposes new guarantees must identify who will pay for them and why. In what follows, the first section, "An Overview of pension Guarantees," surveys the major guarantee designs adopted or suggested in a social security context. The section entitled "Models for costing pension Guarantees," provides the background necessary to analyze guarantee costs. The third section on Illustrating Guarantee Costs, provides five examples of guarantee designs and

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their cost estimates, using the methodology and assumptions developed in the Appendix. A fourth section on Financing pension Accumulation Guarantees, discusses alternative financing options for the pension guarantee, and a final section concludes.

An Overview of Pension Guarantees

While many alternative pension guarantee mechanisms could be envisaged, they may be classified into two general categories: minimum *rate of return* guarantees, and minimum *benefit* guarantees.

Under a minimum *rate of return* guarantee, plan participants would be entitled to receive payments at least equal to their lifetime contributions to the system plus some rate of return. One variant on this theme is a "principal guarantee," which is equivalent to guaranteeing a nominal rate of return of zero percent. This approach has been adopted in Germany and Japan, under which participants must receive at least their plan contributions at retirement (but not before). A more generous design proposed by Feldstein and Samwick (2001) involves a "real principal guarantee," under which participants would be guaranteed their lifetime contributions adjusted according to an inflation index. Still a more generous guarantee might promise participants their contributions plus some minimum rate of return. For example, participants could be told that they would always receive their contributions plus the return on a government bond (e.g. the 10-year Treasury bond).[7]

Irrespective of the particular guaranteed rate of return adopted, cost will depend in part on how often the guarantee threshold is tested. In many designs, as in the German and Japanese cases, the guarantee is evaluated only once, at the end of the plan participant's worklife. In other instances, the minimum rate of return is imposed annually. In Uruguay, for instance, the investment-based system provides pension participants a minimum *annual* real rate of return of two percent. In Chile, pension funds must pay an annual real rate of return that is a function of the average annual real rate of return earned by the entire set of pension funds and in Colombia, the guaranteed rate of return is evaluated over 3-year periods (Fischer, 1999; Pennachi, 1999).

A prominent alternative to a minimum *rate of return* guarantee is a minimum *benefit* guarantee. In this second approach, plan participants are promised that the benefits they will receive from social security at retirement will be at least as high as a minimum annuity, irrespective of their account's actual investment performance. For instance, the Chilean reform provides a "minimum annuity" to defined contribution participants, financed by a pay-as-you-go program (Zarita, 1994; Pennachi, 1999). Some social security systems have adopted a multi-pillar structure of benefits in which the participant receives the combination of a defined benefit annuity (first-pillar) and an IA (second-pillar). Under this design, evaluating possible program costs must take into account the sum of these benefits, and compare them to the minimum annuity. In the United States context, Feldstein and Samwick (2001) describe a "mixed" system where part of the participant's social security tax could be contributed to an IA while the remainder is used to finance the pay-as-you-go program. Their suggested model also includes a guarantee that participants would receive benefits at least as great as under the present law benefit formula.[8]

The Moral Hazard Issue

In addition to the guarantee formulas described above, another factor influencing the cost of the guarantee is the level of investment risk taken by the IA participant. Participants may boost the cost of the guarantee, if they elect to hold riskier investments in their portfolios. Naturally this can give rise to a moral hazard problem, as recognized by Bodie and Merton (1993) and Smetters (2002), among others.

Several tools are available to address the moral hazard problem. One would be to specify a *standard investment portfolio* and provide the guarantee only to those participants who elected that standard portfolio. Another approach would let participants invest in the portfolio of their choosing, but then guarantee payments would be computed using the standard portfolio as a benchmark, rather than the participant's actual investment returns. This second approach leaves participants with more investment flexibility, though it would not protect them against investment risk greater than experienced by the standard portfolio.

Models for Costing Pension Guarantees

This section models guarantee outcomes under the two approaches outlined above, and it further illustrates likely guarantee costs using financial techniques for determining the economic cost of guaranteed pension payments.

Guaranteeing Retirement Income

It is useful to develop a simple notation for costing both the minimum *rate of return* and minimum *benefit* guarantee approaches. Denote by T the number of years over which a plan participant contributes to his IA. For a young worker (i.e. a new system participant), the period T corresponds to the length of the full worklife. By contrast, when the system is first introduced, a more senior worker would have a much shorter window during which he could contribute to his IA. Further, let IA_T and G_T denote, respectively, the value of the IA and of a given guarantee formula at retirement.

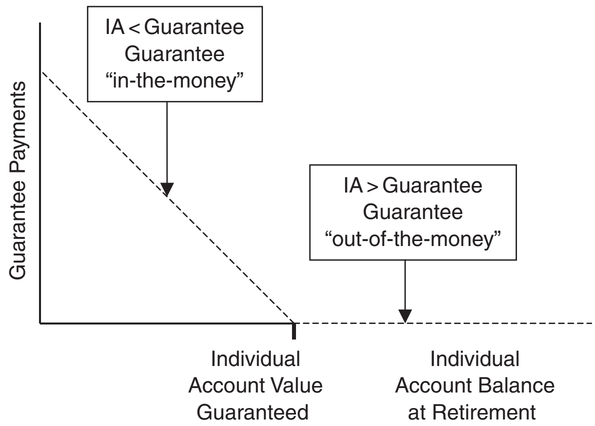

Figure 8-1. Guarantee payments as a function of the IA Value. (*Source*: Authors' calculations.)

The guarantee payments can then be specified depending on the account's investment result. No guarantee is paid at retirement if, at that time, the IA accumulation exceeds the value of the guarantee: $\mathrm{IA}_T > G_T$. But if the value of the IA is below the guaranteed minimum, then the guarantee payment must cover the difference (i.e. $G_T - \mathrm{IA}_T$). The guarantee payoffs, illustrated in Figure 8-1, may be represented as follows:

$$f_T = \max[0, G_T - \mathrm{IA}_T]. \tag{8.1}$$

It must be noted that equation (1) is applicable in the case of a newly created IA system, with no legacy commitment from a prior system. More generally, IA models sometimes develop after a partial or full conversion from a prior pay-as-you-go program. Under a full conversion, the participant would receive the sum of his IA and (possibly) an additional benefit reflecting his participation under the legacy system. Under a partial conversion (or "mixed" system), the participant would receive a combination of his IA and also a defined benefit component as specified under the old plan, perhaps subject to adjustment. Thus under a minimum *benefit* guarantee, it is necessary to adjust equation (1) by adding to the IA value any additional benefits.

To illustrate this point, we examine how one might adjust equation (8.1) for a "mixed" reform. Under the United States social security system, for instance, workers are promised a retirement annuity with present value, SS_T. If voluntary IAs were to be permitted, participant would likely be allowed to divert a portion (but not all) of their social security contributions to a funded defined contribution pension account. To compensate the Trust Fund for the loss in contributions, the promised social security annuity would have to be reduced by an offset amount. The President's Commission (2001) proposed to calculate such an offset by asking, in effect, how much IA contributions would be expected to accumulate under a given rate of return. Letting Offset_T and SS_T^{RED} represent, respectively, the offset and the reduced annuity, then a minimum *benefit* guarantee for a "mixed" social security system can be represented by:

$$f_T = \max\left[0, G_T - (\text{IA}_T + \text{SS}_T^{\text{RED}})\right], \tag{8.2}$$

or equivalently,

$$f_T = \max\left[0, G_T - (\text{IA}_T + (\text{SS}_T - \text{Offset}_T))\right]. \tag{8.3}$$

Finally, we note that in (8.2) and (8.3), social security benefits are assumed to be paid with certainty. Potential costs associated with the funding of social security benefits should be handled separately; that is, legacy system costs are properly attributed to the old system, and not to the guarantee.[9]

Costing Guarantee Payoffs with Option-Pricing Techniques

The discussion above shows that a pension guarantee can provide investors with a floor of protection against the chance of a capital market loss. In turn, the guarantee represents a liability to the sponsor, be it a private sector group—a plan sponsor, an insurer, a financial services firm—or a government entity. Over the last decade, as a result of experience with the Savings and Loan crisis as well as other government guarantee programs, the Congressional Budget Office (CBO) and the General Accounting Office (GAO) have increasingly taken the position that government guarantees should be evaluated and costed as to their budgetary impact. If a pension guarantee were to be included in an IA plan proposal, it would be necessary to estimate and recognize the financial cost of such a promise. That is, irrespective of whether guarantees are provided by a government entity or private sector firms, it is essential to account properly for their costs since real economic resources are required to finance them.

In practice, there is much confusion regarding how to compute the economic value of such guarantee payments. One reason is that the economic cost of providing the pension guarantee may not necessarily equal the recipient's valuation of the guarantee.[10] In this chapter, we focus only on the economic value of the pension guarantee for the provider,[11] referred to as "guarantee costs" below. Another reason is that more than one approach has been suggested to evaluate pension guarantee costs. The simplest approach is to project what pension guarantee payments might be according to a set of stochastic assumptions and take their expectation (cf. Feldstein and Samwick, 2001). This expectation approach has the merit of being easy to apply and explain; it is particularly useful when more sophisticated techniques cannot be adopted. On the other hand, this expectation approach does not incorporate an adjustment for the economic value of risk, so it would tend to underestimate guarantee costs.

An approach that does adjust guarantee costs for risk recognizes that the shape of the guarantee payments in Figure 8-1 conforms to a "put option." Indeed, the pension literature has long recognized that option-pricing techniques[12] can be used to value options related to pension obligations (e.g. Merton, Bodie, and Marcus, 1987). To apply this methodology, one must first detail the stochastic processes for the guarantee formula and the investment portfolio.[13] Then, risk-neutral valuation[14] is used to obtain the guarantee costs from this model. In the special case where the guaranteed portfolio consists of a single contribution that grows with investment returns over time and where the returns follow a lognormal distribution, the risk-neutral valuation technique corresponds to the well-known Black–Scholes formula. The obvious advantage of this approach, as illustrated by Bodie (2001) and Smetters (2002), is that it provides a closed-form solution for the guarantee costs.

More realism in the pension plan design can be introduced by permitting the pension investments to be deposited as a series of periodic contributions, rather than as a one-time investment. In this latter case, a closed-form solution for the guarantee costs is more difficult to find, but Monte Carlo simulations and the risk-neutral valuation technique can be used to model a wide variety of guarantee formulas and portfolio structures. Analysts who have used risk-neutral valuation techniques to value guarantees in this more complex pension framework include Pennachi (1999, 2000) who examined guarantees in Uruguay and Chile, Zarita (1994) who modeled guarantees in Chile, Fischer (1999) who evaluated Colombia's pension guarantee, and Feldstein and Ranguelova (2000) who explored the feasibility of pension collars for the United States.

In the present chapter we also adopt this technique to evaluate the types of pension guarantees that might be suggested in the context of a possible United States social security reform, one that combines a new defined contribution individual account component with a more traditional defined benefit structure. While specific model details are provided in the appendix for interested readers, it is useful to provide a short description of our application of this process. As a first step, it is necessary to risk-adjust the probability distributions of the underlying securities held in the pension portfolio. This probability adjustment is made such that risk-adjusted return processes are expected to yield the risk-free rate. Expectations taken with these risk-adjusted probabilities are represented by the operator $\hat{E}$. Second, the pension guarantee payments can be projected to time T and discounted back at the risk-free rate using the appropriate formulas. Third, the value of the pension guarantee is obtained by taking the risk-adjusted expected value of the discounted guarantee payments. The process may be summarized analytically in equation (8.4) below. Letting $\bar{r}$ represent the average risk-free rate over the period, the no-arbitrage value f of a derivative that pays f_T at time T is given by:[15]

$$f = \hat{E}[\exp(-\bar{r}T)f_T]. \tag{8.4}$$

Nature of the Downside Risk

As was shown above, having a pension guarantee is potentially valuable because of the "downside risk" inherent in IA investments. It is interesting that popular belief regarding the nature of this downside risk tends to downplay the cost of such guarantees. For instance, it is often recommended that investors with long investment horizons hold a larger proportion of stocks in their portfolios. This view is grounded in the argument that stocks are less risky in the long run or, putting it another way, that investors have more time to recoup their losses with longer investment horizons. Historically, stocks have outperformed bonds over long investment horizons,[16] so the belief is that this trend will repeat in the future, resulting in costless guarantees.

Empirical evidence on this point is provided in Figure 8-2, which graphs historical annual nominal returns payable to short-term investors in US stock and bond indexes over the period 1942–2000. The figure

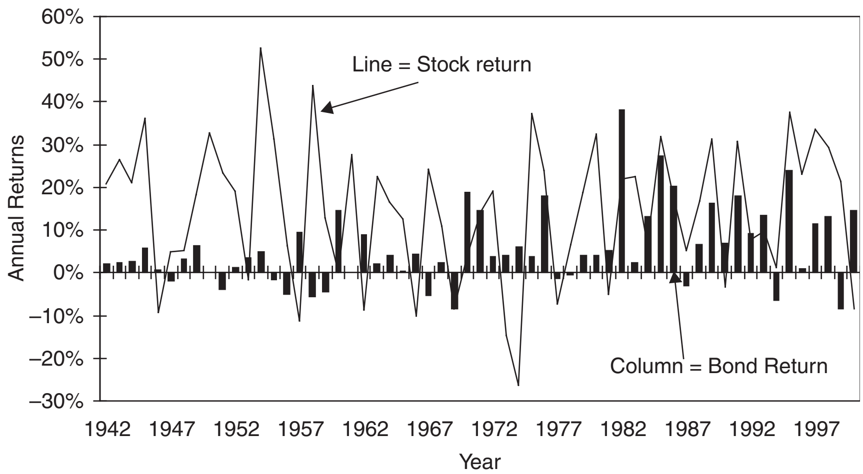

Figure 8-2. Annual returns for US Stock and Bond Markets, 1942–2000. (*Source*: Author's computations, data from CRSP; historical annual stock returns from S&P 500 index including dividends; bond returns from an index of 10-year Treasuries.)

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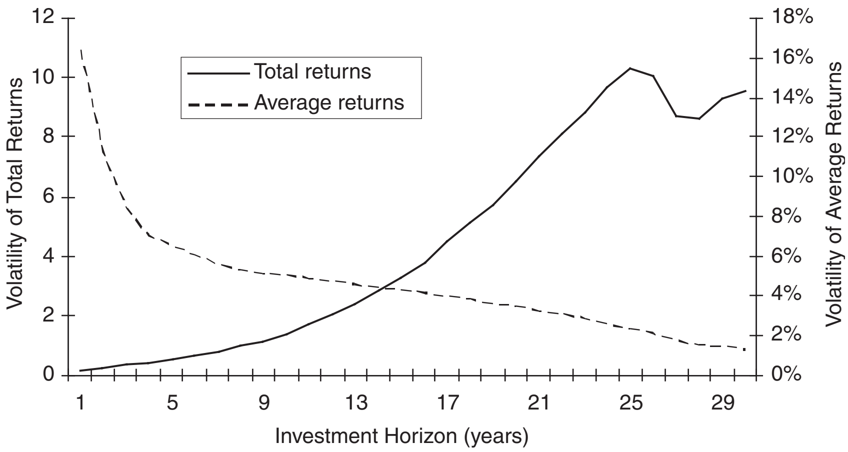

Figure 8-3.

confirms that in the United States, at least, stock returns have historically exceeded bond returns, but with higher volatility. Over the period, the average annual (nominal) return was 14.6 percent on a stock index fund (S&P 500) compared to an average bond fund return of 5.8 percent. The volatility of the stock index over the same period was also higher, at 16.5 percent, compared to bond volatility of 9 percent. (Asset volatility is conventionally measured by the standard deviation of historical returns around the mean). These data illustrate the so-called "equity premium"—that is, because stocks (equities) are seen by the market as more volatile and hence riskier than bonds, purchasers of stocks require an additional risk premium or return in order to hold them.

While all would agree that higher volatility means more risk for short-term holding periods, there is more controversy over returns on assets over longer period. Figure 8-3 illustrates the volatility of stock returns for longer investment horizons, using the same underlying data as Figure 8-2, but now expressing the volatility of total returns over periods between 1 and 30 years. What becomes clear is that the *annualized* stock returns become less volatile over time, but the opposite is true for *compounded* stock returns.[17] Applied to the guarantee context, these findings imply that the volatility of IA accounts should increase over time, because this volatility is affected by compounded, rather than annual, returns. Consequently, guarantee volatility rises over time, rather than being diversified away over longer investment periods.

A closer examination of Figure 8-3 reveals that, for investment periods longer than 25 years, volatility estimates become unstable, due to the paucity of return data for long investment periods. Over the post-WWII period, there are at best two independent observations for the 30-year period returns.[18] Clearly, data limitations weaken confidence regarding the claim that stocks outperform bonds over long-term investment periods. Experts using data from other countries also suggest that the US pattern is an exception, since other countries exhibit much smaller long-term equity premiums.[19] Further, past data may be a rather poor predictor of future performance, so extrapolating the potential costs of a guarantee from this data can be deceptive.

Illustrating Guarantee Costs

To provide a better understanding of the factors determining guarantee costs, this section presents and analyzes several examples. We show how pension guarantee costs depend on three key factors: the relation between the guarantee formula and the benefit structure, the volatility of the investor's portfolio, and the interaction between these two elements and the investor's investment horizon.

Five specific structures for guarantee designs help illustrate the interactions between pension guarantee formulas and benefit structures. The first three IA guarantee designs discussed are examples of a minimum *rate of return* guarantee, differentiated according to the rate of return guaranteed. Example 1 illustrates the cost of providing a *principal* guarantee, one that promises the participant the return of his contributions at retirement (equivalent to a zero nominal interest rate). Example 2 offers a *real principal* guarantee, one that promises the participant the return of his contributions with an adjustment for purchasing power at retirement (equivalent to a zero real interest rate). Example 3 provides the participant a guarantee that his individual account provides his principal plus a minimum interest rate equal to a *10-year Treasury bond* return.

Two additional examples are taken from the minimum *benefit* family of guarantees. Examples 4 and 5 consider a "mixed" system of social security benefits such as the one described in general terms in the second section.[20] In this context, we refer to SS_T as "present law benefits," or the benefits projected according to the formulas in effect under the traditional social security system. The social security benefit formula does not incorporate any minimum or floor benefit on its own. Hence the guarantees provided in Examples 4 and 5 ensure that the retiree receives a total payment equal to the larger of the *present law benefit* or the *poverty line*.[21]

It will be recalled that in a "mixed" system, the retirement benefit consists of the sum of the annuity $SS_T^{RED}$and a payment from the IA_T. In Example 4, as we have constructed it, the plan participant may invest 2 percent of his earnings to an IA, in lieu of paying social security taxes in that amount. Example 5 considers a larger IA system, where the participant can contribute 6 percent of his earnings to an IA. For both examples, in exchange for its participation in the IA, the participant's annuity is obtained by subtracting an "offset" from the present law benefits, that is, $\mathrm{SS}_T^{\mathrm{RED}} = \mathrm{SS}_T - \mathrm{Offset}_T$. This offset is equivalent to the participant's IA contributions accumulated at the 3-month T-bill rate of return.[22]

The social security benefit formula is progressive, providing low earners a higher replacement rate though a lower dollar amount, as compared to higher earners. The guarantee formulas examined here promise different replacement rates by income level, as compared to present law. We illustrate this sensitivity to earnings levels in Example 4 by contrasting the guarantee costs for two hypothetical workers: one at *medium earnings* level corresponding to the Social Security system's Average Wage Index (AWI), and another at a *low earnings* level representing 45 percent of this amount. Guarantee costs are also influenced by how the participant invests his IA account. To show this, we develop guarantee cost estimates for three alternative IA portfolios: one fully invested in equities; a second one invested half in equities and half in bonds; and a third held all in bonds. The role of the investment horizon is depicted through the use of four different contribution periods, with IA contributions occurring over, respectively 10, 20, 30, and 40 years.

For each of the variations just listed, Tables 8-1–8-3 express the cost of providing the guarantee in question, for the specific investment mix, earnings level, and saving horizon illustrated. These costs are computed using the valuation method outlined in the second section of this chapter, and presented in a variety of units: as a percent of assets (Table 8-1), in present value dollars (Table 8-2), and as a percentage of lifetime contributions (Table 8-3). (The Appendix details the assumptions underlying the calculations.) Throughout this section, we refer mainly to Table 4-1's costs expressed in basis points (hundredths of a percent of assets) because it is conventional to refer to costs associated with managing retirement accounts in those terms. However, this measure does not readily reflect changes in costs associated with varying the contribution rate or the investment horizon. Hence, for some purposes, we explore present value dollar costs from Table 8-2.

Guarantee Formula and Benefit Structure

For ease of discussion, we take as the *base case* a participant with a 50/50 stock/bond portfolio and a 40-year investment horizon. For such an investor, Line 8 of Table 8-1 shows that the cost of guaranteeing the 10-year Treasury bond return (Example 3) would be 0.65 percent of assets annually, or 65 basis points (bps). Alternatively, this is worth \$3,406 in present value dollars (Table 8-2), or equivalently, 16 percent of total contributions (Table 8-3). To understand why the guarantee is expensive, it is

TABLE 8-1 Cost Estimates of Alternative Guarantees: Annual Charge as a percentage of IA Assets (in basis points)

Line	Years with Individual Account	Minimum Rate of Return			Minimum Benefit (with a "Mixed"[a] system)		
		Example 1: Principal (i.e. 0%)	*Example 2: Real Principal (i.e. inflation)*	*Example 3: 10-yr Treasury Bond Return*	*Example 4: Present Law Benefit w/ Poverty Line Minimum[b] (Contribution Rate = 2%)*		*Example 5: Present Law Benefit w/ Poverty Line Minimum[b] (Contribution Rate = 6%)*
			Any Earnings		*Low Earnings[c]*	*Medium Earnings[c]*	*Medium Earnings[c]*
I. *Portfolio invested 100% in Equities*							
1	10	60 bp	136 bp	267 bp	2946 bp	265 bp	265 bp
2	20	17	64	184	584	183	183
3	30	7	37	149	193	149	149
4	40	3	24	127	126	126	126
II. *Portfolio invested 50% in Equities, 50% in Treasury 10-yr Bonds*							
5	10	4	33	135	2940	133	133
6	20	0	10	93	523	93	93
7	30	0	4	76	118	76	76
8	40	0	2	65	65	65	65
III. *Portfolio invested 100% in Treasury 10-yr Bonds*							
9	10	0	0	0	2938	11	11
10	20	0	0	0	516	9	9
11	30	0	0	0	66	7	7
12	40	0	0	0	6	6	6

[a] In this example, a participant in a "mixed" system would be allowed to divert part of his social security contribution to an IA and his first-pillar benefit (social security annuity) would be reduced by an offset in return.
[b] This design guarantees that the combination of the first pillar benefits (annuity) and the IA is as least as much as the present law social security benefit. In addition, this benefit is subject to a minimum set equal to the poverty line for those who contribute to the first pillar benefits for at least 30 years.
[c] The income of the low and medium earners represent respectively 45% and 100% of the AWI. In 2000, they would have earned respectively 14,470 and 32,155.
Source: Authors' calculations.

TABLE 8-2 Cost Estimates of Alternative Guarantees (in present value dollars)

Line	Years with Individual Account	Minimum Rate of Return			Minimum Benefit (with a "Mixed"[a] system)		
		Example 1: Principal	*Example 2: Real Principal (Contribution Rate = 2%)*	*Example 3: 10-yr Treasury Bond Return*	*Example 4: Present Law Benefit w/ Poverty Line Minimum[b] (Contribution Rate = 2%)*		*Example 5: Present Law Benefit w/ Poverty Line Minimum[b] (Contribution Rate = 6%)*
			Medium Earnings[c]		*Low Earnings[c]*	*Medium Earnings[c]*	*Medium Earnings[c]*
I. Portfolio invested 100% in Equities							
1	10	$252	$576	$1,127	$16,781	$1,120	$3,361
2	20	258	964	2,782	10,656	2,776	8,329
3	30	214	1,173	4,681	4,150	4,677	14,030
4	40	163	1,240	6,613	2,971	6,602	19,087
II. Portfolio invested 50% in Equities, 50% in Treasury 10-yr Bonds							
5	10	16	141	570	16,775	564	1,692
6	20	4	153	1,408	10,557	1,406	4,219
7	30	1	134	2,390	3,144	2,390	7,171
8	40	0	106	3,406	1,531	3,401	10,204
III. Portfolio invested 100% in Treasury 10-yr Bonds							
9	10	0	0	0	16,771	46	137
10	20	0	0	0	10,560	130	389
11	30	0	0	0	2,804	227	681
12	40	0	0	0	147	328	983

[a] In this example, a participant in a "mixed" system would be allowed to divert part of his social security contribution to an IA and his first-pillar benefit (social security annuity) would be reduced by an offset in return.
[b] his design guarantees that the combination of the first-pillar benefits (annuity) and the IA is as least as much as the present law social security benefit. In addition, this benefit is subject to a minimum set equal to the poverty line for those who contribute to the first-pillar benefits for at least 30 years.
[c] The income of the low and medium earners represent respectively 45% and 100% of the AWI. In 2000, they would have earned respectively $14,470 and $32,155.
Source: Authors' calculations.

TABLE 8-3 Cost Estimates of Alternative Guarantees: as a percentage of Lifetime Contributions

Line	Years with Individual Account	Minimum Rate of Return			Minimum Benefit (with a "Mixed"[a] system)		
		Example 1: Principal (i.e. 0%)	*Example 2: Real Principal (i.e. inflation)*	*Example 3: 10-yr Treasury Bond Return*	*Example 4: Present Law Benefit w/ Poverty Line Minimum[b] (Contribution Rate = 2%)*		*Example 5: Present Law Benefit w/ Poverty Line Minimum[b] (Contribution Rate = 6%)*
		Any Earnings (%)			*Low Earnings[c] (%)*	*Medium Earnings[c] (%)*	*Medium Earnings[c] (%)*
I. Portfolio invested 100% in Equities							
1	10	3.6	8.2	16.1	531.9	16.0	16.0
2	20	2.0	7.6	21.9	186.2	21.8	21.8
3	30	1.2	6.7	27.0	53.2	27.0	27.0
4	40	0.8	5.9	31.3	31.3	31.3	31.3
II. Portfolio invested 50% in Equities, 50% in Treasury 10-yr Bonds							
5	10	0.2	2.0	8.1	531.7	8.1	8.1
6	20	0.0	1.2	11.1	184.5	11.1	11.1
7	30	0.0	0.8	13.8	40.3	13.8	13.8
8	40	0.0	0.5	16.1	16.1	16.1	16.1
III. Portfolio invested 100 in Treasury 10-yr Bonds							
9	10	0.0	0.0	0.0	531.6	0.7	0.7
10	20	0.0	0.0	0.0	184.5	1.0	1.0
11	30	0.0	0.0	0.0	35.9	1.3	1.3
12	40	0.0	0.0	0.0	1.6	1.6	1.6

[a]In this example, a participant in a "mixed" system would be allowed to divert part of his social security contribution to an IA and his first-pillar benefit (social security annuity) would be reduced by an offset in return.
[b]This design guarantees that the combination of the first-pillar benefits (annuity) and the IA is as least as much as the present law social security benefit. In addition, this benefit is subject to a minimum set equal to the poverty line for those who contribute to the first pillar benefits for at least 30 years.
[c]The income of the low and medium earners represent respectively 45% and 100% of the AWI. In 2000, they would have earned respectively $14,470 and $32,155.
Source: Authors' calculations.

helpful to look at the anticipated gap between the value of the guarantee and the benefits provided by the IA.[23] In the base case, the expected values of the IA and the guarantee are equal.[24] Therefore, guarantee payments will be generated as soon as the IA portfolio provides a below-mean return.

In the cases of Examples 1 and 2, the principal and real principal guarantees, the guaranteed amounts represent, respectively, only 30 percent and 52 percent of the expected IA value. Consequently the IA's investment performance would have to be significantly worse than expected before any guarantee would be paid; those payments will also be smaller in size as compared to Example 3. This translates into lower guarantee costs, as illustrated in Line 8 of Table 8-1: the guarantee costs drop to 0 and 2 bp, respectively. Of course guarantee costs this low indicates that such guarantees provide limited protection against investment risk. Although we have ignored administrative costs associated with the guarantee, in this case it is interesting to note that such fees could even exceed the guarantee payments themselves.

Continuing with the base case and moving along Line 8, we next consider the minimum *benefit* guarantees of Examples 4 and 5. Recall that for these examples, the minimum benefit is defined as the US present law benefit, plus a poverty line minimum income. Here the expected gap between the guarantee and the retiree's benefits is influenced by the participant's lifetime earnings level.[25] For the *low* earner, the guarantee represents from 100 percent to 120 percent of expected benefits under the mixed system, whereas this ratio is always 100 percent for the *medium* income earner. It is worth noting that the minimum benefit guarantee in this case introduces benefit improvements unrelated to the provision of investment risk protection. To see this, we note that the guarantee is costly even when the low earner invests his IA entirely in a bond portfolio: as indicated in Table 8-1, providing a minimum benefit for the *low* earner investing only in bonds still costs from 6 bp to 29.38 percent of assets.

Finally, costs are also influenced by the size of the IA. To illustrate this, Examples 4 and 5 compare two different systems, one with an IA contribution rate of 2 percent and the other with a contribution rate of 6 percent. A larger IA introduces more risk, which in turn results in higher costs for the guarantee. The guarantees cost the same amount in basis points (Table 8-1) but these are based on higher contributions and higher assets. To better judge the magnitude of the guarantee cost, dollar figures are presented in Table 4-2. Line 8 shows that as the contribution rate is tripled from 2 to 6 percent, the present value of guarantee costs is also tripled, rising from $3,401 to $10,204. This confirms that a minimum benefit guarantee in the context of a larger investment account is more costly.

Volatility of the Investment Portfolio

The illustrations also reveal that meeting a guarantee threshold is more likely if the IA is invested in more volatile assets; thus boosting the allocation in equities always results in greater guarantee costs. For instance, when we move from the base case with a 50/50 stock/bond mix to a portfolio invested all in equities, the cost of the guarantee doubles from 65 to 127 bp (Table 8-1, Example 3, Lines 4 versus 8). Reducing the fraction in equities to zero eliminates guarantee costs in the Example 3 case, of course, because the IA portfolio cannot do worse than the guaranteed benefit.

This implies that giving IA participants a choice over investment mix could be costly, in that they might boost the guarantee cost by selecting a riskier investment portfolio. In general, it would be dangerous to provide participants with an IA guarantee without placing restrictions on their portfolio mix. However, Table 8-1 reveals that, for some guarantee designs, the impact of the investment portfolio on costs is less than in the base case. When guarantees are either very likely or very unlikely to be exercised, their costs are less sensitive to the portfolio allocation.

Interaction with Investment Horizon

As mentioned above, some observers contend that lengthening the investment horizon might result in lower guarantee costs, because they believe that investment risk decreases over time. Nevertheless, Bodie (1995) showed that a put option guaranteeing the risk-free rate becomes more expensive as the investment horizon widens. In practice, the relation between guarantee costs and investment horizon proves to be fairly complex, as Table 8-2 reveals. This relation is determined by the evolution over time of the two factors defined in this section: the relation between the guarantee formula versus the benefit structure, and the IA volatility.

It will be recalled that, in the base case, the expected value of the guarantee formula and the IA are equal, which implies that the guarantee costs are only driven by volatility. As the investment horizon lengthens, so too does the size of the IA and its volatility. Since guarantee costs increase with volatility, the cost of the guarantee would be expected to rise with the investment horizon. Comparing Lines 5 and 8 of Table 8-2, we see that lengthening the investment horizon from 10 to 40 years in Example 3 results in costs rising more than proportionally, from \$570 to \$3,406. On the other hand, the cost of the principal guarantees (Examples 1 and 2) falls with time, rather than rising. This is because under the principal guarantee, the guarantee cost falls as a percent of the IA from 71 to 30 percent as the time period is extended from 10 to 40 years. The fact that the guarantee becomes less generous over time dominates the volatility effect and explains why the principal guarantee costs fall over time. Similarly, for the low earner in Example 4, the social security annuity grows over time at a faster rate than does the poverty line, which makes it less likely that the guarantee will pay off for the longer holding period.

Financing Pension Accumulation Guarantees

Proposals to include guarantees in an IA model must specify not only their costs, as outlined above, but also how they could be financed. Financing decisions include several aspects:

- Who will bear the guarantee costs? (e.g. participants, taxpayers)
- Who will manage the guarantee and how? (e.g. private sector, government agency)
- What will the price structure be? (e.g. one price for all, prices differentiated by earnings level, portfolio mix, time horizon, etc.)

This section examines several issues related to these three questions.

Guarantee Financing: Pay-as-you-go versus Self-Financed

Feldstein and Liebman (2001) have suggested that the risk associated with guarantees could either be shifted to future taxpayers or transferred to private markets.[26] One way to pay for an IA guarantee is to allow participants to elect self-financed guaranteed choices from a menu of investment options. Financial institutions could offer "guaranteed return accounts" in the set of investment choices for people willing to pay for them. In this case, participants desirous of a guaranteed investment product would pay the premium, irrespective of whether the government or the private sector managed the accounts. (In Germany and Japan, private financial service firms are slated to provide the guaranteed accounts.)

An advantage of the self-financing approach to guarantees is that those who most value the enhanced security would also be those who would pay for it. Less risk-averse people would not have to subsidize the more risk-averse. In addition, since guarantee costs rise with the size of the portfolio being guaranteed in this context, financing would be more expensive for higher earners with larger accounts. Having those who value guarantees most pay for them avoids the poor potentially having to subsidize the risk-averse rich.

A potential disadvantage of self-financing guarantees is that some low-wage earners might value a guarantee more highly, yet they would be least able to afford it. Financing guarantee costs for the poor, in this case, might require subsidizing low-income savers out of general revenue. This might be feasible, but it also might detract from the appeal of guaranteed accounts to the extent that additional revenue would have to be identified to pay for them. Even in this case, however, it is critical to note that guarantee costs do not disappear just because the federal government shoulders them. Failing to report economic costs and benefits of guarantees cannot avoid the reality that economic resources are still at risk under the guarantee, and value is being transferred to participants.

If guarantee costs were passed on to future taxpayers instead of having participants self-finance them, it would mean that future taxes would have to be devoted to the system when guarantees were "in the money." A major problem with this tactic is that the guarantor could be asked to pay out precisely when economic conditions were bleak. This could occur if the stock market and the economy collapsed at the same time, for instance. In such a circumstance, taxpayers might be unable or unwilling to raise taxes on themselves to cover the guarantees, even if promises had been made in the past. In other words, it is incorrect to assume that the federal government has "deep pockets" and can simply raise taxes on future workers to cover shortfalls whenever IA investments perform poorly.

Indeed, one might ask whether such guarantees could be any more reliable than present social security promises. The law has established that traditional social security benefit promises are payable only when revenues are sufficient to cover them (*Fleming v. Nestor,* 1960). A similar point could be made about any form of guarantee: in a massive economic downturn, the promises would be worth no more than could be paid. A related issue is that supporters of IAs often state that these accounts are useful in building wealth and reducing unfunded tax claims on our children and grandchildren. Instituting guarantees without making them self-financed represents a new entitlement likely inconsistent with the reform philosophy.

The Choice of a Guarantee Provider

Although a guarantee resembles an insurance contract, its underlying risk is not diversifiable; hence, it cannot be managed with traditional insurance "pooling" techniques. Figure 8-1 shows that the guarantee payments are asymmetric and this shape is preserved even when guarantee payments of all IA participants are aggregated. This shape cannot be replicated by simply depositing the premiums into an insurance fund. However if these premiums were used to purchase the appropriate financial instruments, it would be possible to obtain the desired structure of payoffs. As an example, Bodie (2001) discusses how investment accumulation products could be guaranteed with the use of a combination of capital market instruments.[27] In the eventuality that these products are not available in the capital markets,[28] their payoffs could be replicated by applying option-pricing techniques to a portfolio of appropriate securities.

When the guarantee payoffs can be replicated by the derivative strategy just described, either the government (or one of its agencies) or private providers would be able to offer the guarantee.[29] In practice, several elements of the guarantee contract cannot be hedged in capital markets (e.g. lifetime earnings, retirement age, etc.) To deal with this issue, one might imagine financial service firms offering contracts that are standardized in terms of earnings, portfolio mix, retirement age, and so forth. This approach has the advantage of reducing moral hazard, but it also subjects the participant to more risk due to the difference between his idiosyncratic situation and the standardized case (the "basis risk," in the options literature). If guarantees were not standardized, it would become more difficult for private providers to manage these contracts and it becomes more likely that the government would provide the guarantee. This is because the government may be better able to transfer losses to future generations, as compared to financial institutions. Such constraints could be mitigated if the private providers had access to reinsurance. Finally, if the guarantees featured some element of subsidy, private providers would be unable to manage the entire program without additional support.

Price Structure

The illustrations in the third section showed that the price of a guarantee is sensitive to the individual investor's characteristics and to his portfolio allocation. Consequently, a well-designed pricing strategy should avoid the creation of opportunities for adverse selection and moral hazard. In this context we have already mentioned the need to have the guarantee linked to a specific IA portfolio mix. Depending on the guarantee structure, providers too can be subject to moral hazard. Making the guarantee provider responsible for asset allocation provides an incentive to invest in safer assets (Jensen and Sorensen, 2000). In Colombia, for instance, the guarantee premium under the IA program is not adjusted for risk; partly as a result, only 0.3 percent of the funds were invested in shares (as of December 1996; Fischer, 1999)).

Discussion and Conclusions

Opponents of IAs tend to understate the problems facing underfunded national pay-as-you-go social security systems, overlooking the fact that reductions in outlays and increases in revenues will be required to close the future financing gap. It is precisely the social security system's looming insolvency that makes current systems politically risky. Including IAs in a national social security reform plan can strengthen old-age economic security. These accounts can reduce the political risk confronting aging Americans when they assess the chances of actually receiving promised benefits under the insolvent social security system. These accounts also afford participants the opportunity to save in a cost-effective manner, and to diversify their investments in ways that they may not be able to at present.

Nonetheless, there may be concern among policymakers that IA participants will face capital market risk, particularly if they concentrate their accounts in stock market investments in the pursuit of higher returns. One approach to this problem is to restrict the extent of equities allowed in workers' accounts; another is to offer guarantees.

This chapter has explored several guarantee designs and assessed their likely costs. It shows that offering guarantees on defined contribution pension accounts could be costly, even when participants are restricted to holding no more than half their portfolio in stock and the rest in bonds. For instance, in this framework our model suggests that a 10-year Treasury bond return guarantee would still require increasing annual contributions by 65 bps, or 16 percent of contributions, for the long-term saver. This would likely be perceived as a substantial cost increase over and above the basic contribution by most plan participants. If these costs were not self-financed, substantial subsidies would be required. Subsidies of this sort must be measured, recognized, and their financing implications spelled out in detail for a full accounting of the economic costs and benefits of guarantees.

These cost estimates might seem high to people accustomed to the argument that stock returns are expected to outperform bond returns over time. We argue, however, that because of the paucity of independent observations in historical data on long holding periods, past returns are noisy predictors of future returns. In addition, guarantee costs are driven by stock and bond volatility rather than their expected returns.

Appendix: An Illustration of Option-Pricing Techniques Applied to Individual Accounts

This Appendix details the modeling assumptions used to derive cost estimates for the illustrative examples discussed in the text. We summarize guarantee costs for four workers who participate in the IA program for, respectively, $T = 10$, 20, 30, and 40 years. It is assumed that the IA starts in 2004 and economic variables are projected accordingly. Sections A and B of this Appendix describe the economic and demographic assumptions. The stochastic processes followed by the bills, bonds, and stocks are modeled separately in Section C. Section D details the elements necessary to compute the IA values as well as the social security annuity. Section E derives the cost of each guarantee formula while Section F shows how to generate numerical values for the guarantee costs using a Monte Carlo simulation.

A. Economic Assumptions

All projections are expressed in nominal values, with the inflation and real processes modeled separately. Assumptions for inflation growth, real wage growth, and real interest rates are taken from the OASDI Annual

Report (2001). In that report, the intermediate scenario assumes that real wage growth is $g = 1.0$ percent, while inflation grows at $i = 3.3$ percent. By combining these two assumptions, the result is a 4.3 percent nominal wage growth assumption. According to the intermediate scenario, the real interest rate assumption is $r^{\text{REAL}} = 3$ percent. This fixed interest rate assumption is used for the annuity calculation in Section B, while the remaining calculations use the stochastic model of Section C.

At the inception of the IAs, earnings levels are denoted by W_0. In subsequent years, earnings W_t are obtained by projecting these initial earnings with a fixed rate of 4.3 percent. Two categories of wages are used in the simulations: the *medium* earner correspond to the Social Security Actuary's AWI while a *low* earner represents 45 percent of this amount. For instance, the low and medium earners would have received \$17,785 and \$32,155, respectively in 2000.[30] Finally, according to the US Census Bureau, the poverty line for singles over 65 years old was \$8,494 in 2001, a level assumed to grow with the Consumer Price Index (CPI) over time.

B. Demographic Assumptions

The four illustrative cases are assumed to be, respectively, 22, 32, 42, and 52 years old at the inception of the IA system. Each participant is assumed to retire at the early retirement age of 62 years. At this age, the value of a \$1 annuity with payments indexed to inflation is denoted by the annuity factor $\ddot{a}_{62}$. To compute this annuity factor, it is necessary to define survival probabilities after retirement. The standard notation ${}_tp_{62}$ is used to denote the probability that an individual retiring at age 62 would still be alive at age 62+t. Post-retirement survival probabilities are derived from the Social Security 1997 period life table[31] (pre-retirement mortality is not included in the model). As for the real interest rate used to discount the annuity payments, it is taken from the Old age survivor and disability insurance (OASDI) intermediate scenario. Letting the last age (radix) of the mortality table be represented by ω, the value of the annuity factor is given by:

$$\ddot{a}_{62} = \sum_{t=0}^{\omega-1-62} (1 + r^{\text{REAL}})_t^{-t} {}_tp_{62}. \tag{8.7}$$

C. Stochastic Processes (risk-adjusted)

Risk-Free Rate

The continuous risk-free rate is defined by Vasicek's (1977) mean reverting model:

$$dr_t = \kappa(\mu - r_t)\,dt + \sigma\,dW_t^r, \tag{8.8}$$

where dW_t^r is a standardized Wiener process and the initial risk-free rate is given by $r_0 = r$. According to Hull (1997), the current term structure should lead directly to the risk-neutral process for interest rates and the necessary risk-adjustments are incorporated in (8). For estimation and simulation purposes, it will be useful to take advantage of the fact that the Vasicek model leads to the following normal representation of the risk-free rate:

$$r_t|r_{t-1} = \mu(1 - e^{-k}) + e^{-k}r_{t-1} + \sigma\sqrt{(1 - e^{-2\kappa})/2\kappa\varepsilon_t}$$
$$\varepsilon_t \sim N(0, 1). \tag{8.9}$$

Equation (8.9) corresponds to a simple regression and its parameters can be estimated by online library system (OLS). For this estimation, the risk-free rate is represented by the 3-month T-Bill annual time series for the period 1980–2001. The period between World War II and October 1979 is excluded due to the Federal Reserve policy of stabilizing interest rates at the time. Using this data, the OLS parameters estimates are respectively $r_0 = 2$ percent, $\hat{\kappa} = 80$ percent, $\hat{\mu} = 3.0$ percent, and $\hat{\sigma} = 2$ percent. The annual risk-free rate values can then be simulated by generating a series of error terms ε_t and substituting them into equation (9).

Bond Returns

To compute the bond portfolio return, we take advantage of the direct relation between the movements of the risk-free rate and bond returns. The bond portfolio is invested in 10-year Treasury zero coupon bonds, assumed to be rebalanced annually. The same assumptions apply to the 10-year Treasury guarantee in Example 3. When it is computed with the Vasicek model, the price at time t of a bond with time to maturity τ can be represented as follows:

$$P(\tau, r_t) = e^{A(\tau)-B(\tau)r_t} \tag{8.10}$$

where,

$$A(\tau) = \big(B(\tau) - \tau\big)\left(\mu - \frac{\sigma^2}{2\kappa^2}\right) - \frac{\sigma^2 B(\tau)^2}{4\kappa}$$
$$B(\tau) = \frac{1 - e^{-\kappa\tau}}{\tau}.$$

Since it is assumed that the 10-year bond fund is rebalanced annually, its annual return B_t is given by the percentage increase in price after 1 year:

$$B_t = \frac{P(9, r_{t+1})}{P(10, r_t)} - 1, \tag{8.11}$$

where r_t and r_{t+1} are generated by (9). Note that there is no relation between the notations B_t and $B(\tau)$.

Stock Returns

Letting S_t^I represent the stock index level at time t, the continuous stock returns are modeled by the following geometric Brownian motion:

$$\frac{dS_t^I}{S_t^I} = r_t dt + \sigma\, dW_t^S, \tag{8.12}$$

where dW_t^S is a standardized Wiener process, which we assumed to be uncorrelated with the one in equation (8.8). Following the risk-neutral valuation technique, the drift of the return process in (8.12) is set equal to the risk-free rate. In addition, let S_t denote the annual stock return in year t. Then S_t is distributed according to a lognormal distribution and can be represented by:

$$\ln(1 + S_t) = r_t - \sigma^2/2 + \sigma\varepsilon_t, \quad \text{where} \qquad \varepsilon_t \sim N(0, 1) \tag{8.13}$$

or, equivalently

$$S_t = \exp(r_t - \sigma^2/2 + \sigma\varepsilon_t) - 1. \tag{8.14}$$

To estimate the parameter σ, we note that equation (8.13) is normally distributed with standard deviation σ. The usual estimator can then be applied to obtain $\hat{\sigma} = 20$ percent. Stock return data for the estimation were taken from the S&P 500 Index (including dividends) during the period 1926–2000. Using this parameter estimate, the annual stock returns are simulated by generating a series of error terms ε_t and substituting them into equation (14).

Investment Returns for Individual Accounts

In this illustrative model, the worker is assumed to allocate his IA investments between two funds: an indexed stock fund and a bond fund (of 10-year Treasuries). Denote by α the proportion invested by the participant in the stock fund. Further, let S_t and B_t represent the total return at time t for each of the funds. It follows that the portfolio investment rate of return in year t is given by:

$$R_t = \alpha S_t + (1 - \alpha)B_t. \tag{8.15}$$

In Tables 4-1–4-3, the results are generated for three alternative portfolios with $\alpha = 0\%, \alpha = 50\%$, and $\alpha = 100\%$.

D. Retirement Benefits Structure: Social Security Benefits and Individual Account Payouts

Social Security Annuity

Denote by SSA_T the annuity payment that a participant would receive if he retired at age 62, under a stylized annuity benefit formula similar to, though not completely identical to, the existing law formulas. The first input to the benefit formula is the Average Indexed Monthly Earnings $(\text{AIME})_T$. Here we assume that the last thirty-five (annual) earnings are used in the AIME_T calculation. These are indexed with wage growth up to age 60, with the exception of the last two earnings values.

$$\text{AIME}_T = \frac{W_{T-1} + W_{T-2} + \sum_{i=T-35}^{T-3} W_t((1+i)(1+g))^{T-2}}{35} \tag{8.16}$$

The social security benefit formula involves the use of two "bendpoints," indexed to the AWI and referred to below as FBP_T and SBP_T. In 2002, the annualized bendpoints were \$7,104 and \$42,804, respectively. Present law benefits are computed by multiplying the AIME_T by 90 percent for the portion below the first bendpoint, and by 32 percent and 15 percent for the portions, respectively, below and above the second bendpoint. Finally, the retiree's benefits are subject to an early retirement reduction factor, denoted by ERR_T. According to SSA, $\text{ERR}_T = 75$ percent for the participant with $T = 10$ and $\text{ERR}_T = 70$ percent for the other participants. The following formula summarizes the benefit calculation.

$$\text{SSA}_T = \text{ERR}_T \left(\begin{array}{ll} 90\% \times \min(\text{AIME}_T, \text{FB}P_T) & \\ + 32\% \times (\min(\text{AIME}_T, \text{SBP}_T) - \text{FBP}_T) & \textit{if } \text{FBP}_T \le \text{AIME}_T \\ + 15\% \times (\text{AIME}_T - \text{SB}P_T) & \textit{if } \text{SBP}_T \le \text{AIME}_T. \end{array} \right. \tag{8.17}$$

The value of the first pillar social security benefit SS_T is then obtained by multiplying (17) by the annuity factor: $\text{SS}_T = \text{SSA}_T \cdot \ddot{a}_{62}$.

Individual Account Payouts

In all models considered but one, system participants are permitted to divert 2 percent of their taxable earnings to an IA. (The exception is Example 5 where participants are allowed to contribute 6% of taxable earnings.) Letting C represent the fixed contribution rate, then the dollar contribution in year t is given by $C_t = C \cdot W_t$. The value of the IA at retirement is represented by IA_T. This value is computed as:

$$\text{IA}_T = \sum_{t=0}^{T-1} C_t \prod_{j=t}^{T-1} (1 + R_j). \tag{8.18}$$

where R_j was defined in Section C.

Social Security Benefits for Individual Account Participants

Those who participate in the IAs reduce their contributions to the Social Security Trust Fund, so their first pillar benefits are offset in exchange. For the present case, we compute the offset by accumulating IA contributions with a stated rate of return R^O,

$$\text{Offset}_T = \sum_{t=0}^{T-1} C_t(1 + R_t^O)^{T-t}. \tag{8.19}$$

For this analysis we set $R_t^O = \exp(r_t) - 1$; i.e. the offset rate is defined as the risk-free rate. It follows that the expected present value of the reduced benefits, denoted by SS_T^{RED}, is given by the following formula:

$$\text{SS}_T^{\text{RED}} = \text{SS}_T - \text{Offset}_T. \tag{8.20}$$

The participant is then assumed to receive the sum of his IA and the reduced social security annuity, or $\text{IA}_T + \text{SS}_T^{\text{RED}}$.

E. Guarantee Formulas

Rate of Return Guarantees

Let R_t^G represent the guaranteed rate of return for any of the rate of return guarantees described in the text. Then G_T, the value of the guarantee at retirement, is given by:

$$G_T = \sum_{t=0}^{T-1} C_t \prod_{j=t}^{T-1} (1 + R_j^G). \tag{8.21}$$

For the principal guarantee and the real principal guarantee, we have $R_t^G =$ 0 percent and $R_t^G = i$ percent, respectively. Letting B_t represent the bond return again, the 10-year Treasury guarantee is modeled using $R_t^G = B_t$. For each of these examples, the guarantee payments are obtained by comparing G_T and IA_T.

Minimum Benefit Guarantees

For Examples 4 and 5, let G_T represent the present value of the guaranteed annuity. Denoting by PL_T the value of a poverty line annuity at retirement, then $G_T = \max(\text{SS}_T, \text{PL}_T)$. The guarantee payments are obtained by comparing this amount to the "mixed" system benefit payment $\text{IA}_T + \text{SS}_T^{\text{RED}}$.

F. Risk-Neutral Valuation and Monte-Carlo Simulations

The results in the text are obtained by simulating the value of equation (8.4) using the appropriate definition of the guarantee payoffs f_T from equations (8.1) and (8.3). Cost estimates are obtained by using 10,000 Monte Carlo simulations. For Tables 4-1 and 4-3, these costs are divided, respectively, by $\sum_{t=0}^{T-1} \hat{E}[e^{-\bar{r}t} C_t]$ and $\sum_{t=0}^{T-1} \hat{E}[e^{-\bar{r}t} IA_t]$ to obtain the appropriate units.

Notes

[1]See President's CSSS 2001.
[2]See Jickling (2002) for an overview of the financial issues surrounding the Enron collapse, and Mitchell and Utkus (Chapter 3, this volume) on company stock in retirement plans.
[3]See, for instance, Benson (2001).
[4]Zarita (1994), Fischer (1999), and Pennachi (1999, 2000) discuss Latin American pension guarantees.
[5]See Clark and Mitchell (2002) for Japan, and Maurer and Schlag (Chapter 9, this volume) for Germany.
[6]Here we focus only on the accumulation phase of IAs, and not the decumulation phase.
[7]The DeMint-Armey plan, for instance, guarantees benefits for those who elect a balanced IA portfolio, but no additional value is assigned to either the guarantee or the cost of providing a minimum benefit under government cost estimates.
[8]Smetters (2001) analyses various alternative specifications for this type of guarantee, in the context of a complete conversion to IAs.
[9]The President's CSSS (2001) identified several alternative ways to handle legacy costs.
[10]This would arise if the provider of a guarantee has different circumstances from that of the recipient (e.g. differential access to the capital market, a broader range of investment choices, etc).
[11]How to value guarantees from the participant's point of view will be discussed in a future chapter. As a related issue, the approach we describe below makes no assumptions regarding any particular IA participant's risk aversion; in particular, workers are not required to be neutral in their preferences for risk for the cost estimates to hold.
[12]Standard references for option-pricing techniques include Duffie (1996) and Hull (1997). Option-pricing techniques require defining an economy, that is a set of market traded securities (e.g. a stock and a bond) along with their stochastic processes. If a consumption plan (in this case, the pension guarantee payments) can be strictly financed by a strategy involving only securities from the available set, then the cost of the guarantee must equal the cost of the replicating strategy in order to rule out "free lunches" (i.e. markets are dynamically complete and each consumption plan can be replicated). Typically, "incompleteness" problems arise when the consumption plan is a function of securities that are not market-traded. In the pension guarantee case, workers' earnings are likely to be problematic since their value cannot be replicated in the markets. Despite the loss of the "no-arbitrage" argument, if these non-replicable factors are relatively unimportant, option-pricing techniques can still provide insight.
[13]Specifically, the risk-free rate, workers' earnings, stock returns, and bond returns have to be modeled. In the appendix, we provide a model that is internally consistent and allows for replication of the guarantee payments. For instance, internal consistency requires that the relation between bond prices and discount rates be taken into account when modeling bond returns. Following Pennachi (1999), we assume that the risk-free rate follows a stochastic process represented by the Vasicek (1977) model. In contrast to prior work, we take advantage of the simple relation

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between bond prices and the risk-free rate (under the Vasicek model) to derive the stochastic process followed by bond returns. Since the stock and bond returns are modeled separately, the total portfolio returns are simply obtained by adding the two processes. This permits an actual replication of the guarantee payments with market securities.

[14]This technique is also referred to as "martingale pricing."

[15]Deriving an analytical solution for (4) in the guarantee case requires an analytical expression for the probability distribution of the IA. In the Black–Scholes framework with a single purchase, the underlying stock growth process is assumed to be lognormal. In the IA framework, each year's contribution is assumed to grow according to a lognormal distribution, but the sum of these contributions is not lognormal nor does it have a meaningful analytical representation. Hence we evaluate equation (4) numerically using Monte Carlo simulation to illustrate the costs of alternative pension guarantee designs.

[16]See for example Siegel (1998) on the relative historical performances of stocks and bonds.

[17]Samuelson (1963) initially referred to this idea as the fallacy of large numbers; see also Bodie (1995).

[18]When analysts use 30-year moving averages over the post-WWII period, these are not independent draws from the underlying distribution. This point has been made by various authors, including Bodie (2001).

[19]For example Jorion and Goetzmann (1999) conclude that the US equity market had "the highest uninterrupted real rate of appreciation of all countries, at 4.3 percent annually from 1921 to 1996. For other countries, the median real appreciation rate was 0.8 percent. The high return premium obtained for US equities therefore appears to be the exception rather than the rule."

[20]This analysis does not incorporate the financing required to move to a fiscally solvent system, since estimates of that cost are available elsewhere. Thus this exercise estimates the marginal cost of providing a guarantee for an IA program, rather than the cost of restoring the first pillar system to solvency. Details of the schematic model used to represent the first pillar system appear in the Appendix. These two cases are examples selected to identify the drivers of guarantee costs; neither coincides with proposals devised by the President's CSSS. In that group's report, the first pillar plan was assumed to be reformed with the advent of IAs, and IA contribution rates as well as offset rates were set to bring fiscal solvency to the system as a whole. Our goal here is not to establish costs of moving to solvency, but rather to outline the magnitude and sensitivity of guarantee costs to different guarantee designs.

[21]To qualify for the poverty line minimum, the participant must contribute at least 30 years to the annuity component of the system.

[22]Neither of these examples corresponds to specific plans outlined by the President's CSSS. In particular, the Commission included no guarantees in its proposed reforms. Our objective here is to describe generic alternatives that help think about guarantees, rather than to cost any specific proposal.

[23]This concept is equivalent to the concept of "moneyness" in option pricing. When the strike price of an option is set equal to the stock price, the option is said to be "at-the-money" and its cost is solely driven by volatility. When the strike price of a put option is larger (smaller) than the stock price, the option is said to be

"in-of-the-money" ("out-the-money"). In those cases, volatility is not the only factor driving value and these options can be very cheap or expensive, respectively.

[24]The term "expected value" refers to the risk-adjusted expected value used to determine the guarantee cost.

[25]In any mixed system, participants' earnings levels will influence guarantee costs; more generally, guarantee formulas and guaranteed benefits are likely to interact nonlinearly with earnings.

[26]As a variant, they also mention the "collar" strategy of Feldstein and Ranguelova (2000). With this strategy, the guarantee is financed by participants who give up some of the upside return potential of the IA's investment return. Smetters (2002) also describes a similar financing strategy.

[27]In this chapter, we describe the guarantee as a put option on the IA. By "put-call parity," the combination of this put option and of the IA is equivalent to Bodie's strategy of investing in bonds and call options.

[28]Alier and Vittas (2001) discuss some alternative strategies to reduce IA risk when it is not possible to manage this risk via the capital markets.

[29]Some private providers already offer guarantees with their investment accumulation products; see Francis (2001).

[30]No adjustment for age is made, since the AWI is an average measure for workers of all ages. Hence, earnings are likely overestimated for younger participants and underestimated for older ones.

[31]Available at <www.ssa.gov/OACT/STATS/table4c6.html>.

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