Appendices

Appendix A

PART I

COST CALCULATIONS FOR SINGLE PREMIUM FUNDING
1937 STANDARD ANNUITY MORTALITY TABLE 3% INTEREST

Age Bracket	*Total Prospective Benefits Upon Which Costs Are Based*		*Single Premium Cost Factors*	*Normal Cost (Col. 7 x Col. 9)*	*Accrued Liability (Col. 8 x Col. 9)*
	*Current Service (1% of Col. 4)**	*Past Service (1% of Col. 4 x Col. 6)**			
(1)	(7)	(8)	(9)	(10)	(11)
15–19	$ 600	$ 600	1.896	$ 1,138	$ 1,138
20–24	2,530	5,060	2.212	5,596	11,193
25–29	4,640	18,560	2.585	11,994	47,978
30–34	4,550	27,300	3.027	13,773	82,637
35–39	4,510	40,590	3.560	16,056	144,500
40–44	4,600	59,800	4.215	19,389	252,057
45–49	4,590	78,030	5.040	23,134	393,271
50–54	4,480	94,080	6.112	27,382	575,017
55–59	4,340	112,840	7.570	32,854	854,199
60–64	3,900	120,900	9.665	37,694	1,168,499
65 and over	....	124,740	10.914	.....	1,361,412
Totals	$38,740	$682,500		$189,010	$4,891,901

* See Table 7, p. 147.

Part II

Cost Calculations for Level Annual Premium and Aggregate Funding

Age Bracket	Prospective Benefits upon Which Costs Are Based [1% of Col. 4 x (Col. 6 + Yrs. to 65)]*	Level Premium Cost Factor for Attained Age	Initial Annual Cost on Attained Age Level Premium Method (Col. 12 x Col. 13)	Level Premium Cost Factor for Entry Age†	Normal Cost on Entry Age Normal Method (Col. 12 x Col. 15)
(1)	(12)	(13)	(14)	(15)	(16)
15–19	$ 29,400	.077	$ 2,264	.074	$ 2,176
20–24	113,850	.095	10,816	.087	9,905
25–29	194,880	.120	23,386	.100	19,488
30–34	177,450	.153	27,150	.114	20,229
35–39	166,870	.200	33,374	.125	20,859
40–44	165,600	.272	45,043	.132	21,859
45–49	160,650	.389	62,493	.138	22,170
50–54	152,320	.606	92,306	.145	22,086
55–59	147,560	1.116	164,677	.145	21,396
60–64	132,600	3.398	450,575	.145	19,227
65 and over	124,740	10.914	1,361,412	...	.....
Totals	$1,565,920		$2,273,496		$179,395

Part II—*continued*

Cost Calculations for Level Annual Premium and Aggregate Funding

Age Bracket	Present Value Factors	Present Value of Prospective Benefits (Col. 12 x Col. 17)	Temporary Annuity Factors	Present Value of Future Normal Costs (Col. 16 x Col. 19)	Present Value of Future Earnings (Col. 4 x Col. 19)*
(1)	(17)	(18)	(19)	(20)	(21)
15–19	1.896	$ 55,742	24.589	$ 53,506	$ 1,475,340
20–24	2.212	251,836	23.204	229,836	5,870,612
25–29	2.585	503,765	21.613	421,194	10,028,432
30–34	3.027	537,141	19.806	400,656	9,011,730
35–39	3.560	594,057	17.775	370,769	8,016,525
40–44	4.215	698,004	15.503	338,880	7,131,380
45–49	5.040	809,676	12.958	287,279	5,947,722
50–54	6.112	930,980	10.087	222,781	4,518,976
55–59	7.570	1,117,029	6.786	145,193	2,945,124
60–64	9.665	1,281,579	2.844	54,682	1,109,160
65 and over	10.914	1,361,412	.....	.....	.....
Totals		$8,141,221		$2,524,776	$56,055,001

* See Table 7, p. 147.

† Based on assumed entry ages: Col. (2)–Col. (6), Table 7, p. 147.

Appendix B

PART I

COST CALCULATIONS FOR SINGLE PREMIUM FUNDING
GA-1951 MORTALITY TABLE, 2½% INTEREST

Age Bracket	*Total Prospective Benefits Upon Which Costs Are Based*		*Single Premium Cost Factors*	*Normal Cost (Col. 7 x Col. 9)*	*Accrued Liability (Col. 8 x Col. 9)*
	*Current Service (1% of Col. 4)**	*Past Service (1% of Col. 4 x Col. 6)**			
(1)	(7)	(8)	(9)	(10)	(11)
15–19	$ 600	$ 600	2.757	$ 1,654	$ 1,654
20–24	2,530	5,060	3.129	7,916	15,833
25–29	4,640	18,560	3.554	16,491	65,962
30–34	4,550	27,300	4.040	18,382	110,292
35–39	4,510	40,590	4.602	20,755	186,795
40–44	4,600	59,800	5.258	24,187	314,428
45–49	4,590	78,030	6.051	27,774	472,160
50–54	4,480	94,080	7.062	31,638	664,393
55–59	4,340	112,840	8.403	36,469	948,195
60–64	3,900	120,900	10.258	40,006	1,240,192
65 and over	....	124,740	11.272	.....	1,406,069
Totals	$38,740	$682,500		$225,272	$5,425,973

* See Table 7, p. 147.

Part II

Cost Calculations for Level Annual Premium and Aggregate Funding

Age Bracket	Prospective Benefits upon Which Costs Are Based [1% of Col. 4 x (Col. 6 + Yrs. to 65)]*	Level Premium Cost Factor for Attained Age	Initial Annual Cost on Attained Age Level Premium Method (Col. 12 x Col. 13)	Level Premium Cost Factor for Entry Age†	Normal Cost on Entry Age Normal Method (Col. 12 x Col. 15)
(1)	(12)	(13)	(14)	(15)	(16)
15–19	$ 29,400	.100	$ 2,940	.097	$ 2,852
20–24	113,850	.121	13,776	.112	12,751
25–29	194,880	.149	29,037	.126	24,555
30–34	177,450	.186	33,006	.143	25,375
35–39	166,870	.238	39,715	.155	25,865
40–44	165,600	.315	52,164	.162	26,827
45–49	160,650	.440	70,686	.170	27,311
50–54	152,320	.669	101,902	.177	26,961
55–59	147,560	1.203	177,515	.177	26,118
60–64	132,600	3.573	473,780	.177	23,470
65 and over	124,740	11.272	1,406,069	...	.....
Totals	$1,565,920		$2,400,590		$222,085

Part II—*continued*

Cost Calculations for Level Annual Premium and Aggregate Funding

Age Bracket	Present Value Factors	Present Value of Prospective Benefits (Col. 12 x Col. 17)	Temporary Annuity Factors	Present Value of Future Normal Costs (Col. 16 x Col. 19)	Present Value of Future Earnings (Col. 4 x Col. 19)*
(1)	(17)	(18)	(19)	(20)	(21)
15–19	2.757	$ 81,056	27.469	$ 78,342	$ 1,648,140
20–24	3.129	356,237	25.776	328,670	6,521,328
25–29	3.554	692,604	23.872	586,177	11,076,608
30–34	4.040	716,898	21.736	551,551	9,889,880
35–39	4.602	767,936	19.346	500,384	8,725,046
40–44	5.258	870,725	16.682	447,528	7,673,720
45–49	6.051	972,093	13.750	375,526	6,311,250
50–54	7.062	1,075,684	10.549	284,412	4,725,952
55–59	8.403	1,239,947	6.987	182,486	3,032,358
60–64	10.258	1,360,211	2.871	67,382	1,119,690
65 and over	11.272	1,406,069	.....	.....	.....
Totals		$9,539,460		$3,402,458	$60,723,972

* See Table 7, p. 147.

† Based on assumed entry ages: Col. (2)–Col. (6), Table 7, p. 147.

Appendix C

Part I

Cost Calculations for Single Premium Funding
1937 Standard Annuity Mortality Table 2½% Interest
Moderate Turnover

Age Bracket	Total Prospective Benefits Upon Which Costs Are Based: Current Service (1% of Col. 4)*	Total Prospective Benefits Upon Which Costs Are Based: Past Service (1% of Col. 4 x Col. 6)*	Single Premium Cost Factors	Normal Cost (Col. 7 x Col. 9)	Accrued Liability (Col. 8 x Col. 9)
(1)	(7)	(8)	(9)	(10)	(11)
15–19	$ 600	$ 600	.192	$ 115	$ 115
20–24	2,530	5,060	.535	1,354	2,707
25–29	4,640	18,560	.879	4,079	16,314
30–34	4,550	27,300	1.339	6,092	36,555
35–39	4,510	40,590	1.926	8,686	78,176
40–44	4,600	59,800	2.665	12,259	159,367
45–49	4,590	78,030	3.629	16,657	283,171
50–54	4,480	94,080	4.909	21,992	461,839
55–59	4,340	112,840	6.597	28,631	744,405
60–64	3,900	120,900	9.388	36,613	1,135,009
65 and over	....	124,740	11.360	.....	1,417,046
Totals	$38,740	$682,500		$136,478	$4,334,704

* See Table 7, p. 147.

Part II

Cost Calculations for Level Annual Premium and Aggregate Funding

Age Bracket	Prospective Benefits upon Which Costs Are Based [1% of Col. 4 x (Col. 6 + Yrs. to 65)]*	Level Premium Cost Factor for Attained Age	Initial Annual Cost on Attained Age Level Premium Method (Col. 12 x Col. 13)	Level Premium Cost Factor for Entry Age†	Normal Cost on Entry Age Normal Method (Col. 12 x Col. 15)
(1)	(12)	(13)	(14)	(15)	(16)
15–19	$ 29,400	.026	$ 764	.021	$ 617
20–24	113,850	.044	5,009	.037	4,212
25–29	194,880	.066	12,862	.048	9,354
30–34	177,450	.097	17,213	.061	10,824
35–39	166,870	.142	23,696	.072	12,015
40–44	165,600	.210	34,776	.077	12,751
45–49	160,650	.323	51,890	.084	13,495
50–54	152,320	.535	81,491	.090	13,709
55–59	147,560	1.040	153,462	.090	13,280
60–64	132,600	3.388	449,249	.090	11,934
65 and over	124,740	11.360	1,417,046	...	.....
Totals	$1,565,920		$2,247,458		$102,191

Part II—*continued*

Cost Calculations for Level Annual Premium and Aggregate Funding

Age Bracket	Present Value Factors	Present Value of Prospective Benefits (Col. 12 x Col. 17)	Temporary Annuity Factors	Present Value of Future Normal Costs (Col. 16 x Col. 19)	Present Value of Future Earnings (Col. 4 x Col. 19)*
(1)	(17)	(18)	(19)	(20)	(21)
15–19	.192	$ 5,645	7.509	$ 4,633	$ 450,540
20–24	.535	60,910	12.181	51,306	3,081,793
25–29	.879	171,300	13.261	124,043	6,153,104
30–34	1.339	237,606	13.760	148,938	6,260,800
35–39	1.926	321,392	13.561	162,935	6,116,011
40–44	2.665	441,324	12.680	161,683	5,832,800
45–49	3.629	582,999	11.229	151,535	5,154,111
50–54	4.909	747,739	9.179	125,835	4,112,192
55–59	6.597	973,453	6.342	84,222	2,752,428
60–64	9.388	1,244,849	2.771	33,069	1,080,690
65 and over	11.360	1,417,046	.....	.....	.....
Totals		$6,204,263		$1,048,199	$40,994,469

* See Table 7, p. 147.

† Based on assumed entry ages: Col. (2)–Col. (6), Table 7, p. 147.

Appendix D

IMPACT OF MORTALITY IMPROVEMENT

The trend toward lower rates of mortality has been described in the text of this volume. This trend has created problems for insurance companies and others engaged in the underwriting of annuities. This Appendix describes the efforts of insurance companies to anticipate the financial effects of continued improvement in annuitant mortality.

The traditional or conventional method of coping with the improvement in annuitant mortality has been the use of a much lower guaranteed rate of interest than is likely to be earned plus, in many cases, a setback in the 1937 Standard Annuity Table. While the use of an extremely conservative rate of interest has provided an effective, and convenient, margin of safety in the past, there is evidence that this device will prove incapable of absorbing the cost of decreasing death rates in the future.[1] Moreover, the use of an unrealistic rate of interest as a margin for mortality improvement is confusing to laymen and may be misunderstood by an employer exploring the cost of a proposed plan. There is a growing feeling that each element entering into the cost structure should rest on reasonable assumptions and be capable of justification to a layman. The efficacy of the setback procedure also appears to be limited inasmuch as the obsolescence of the 1937 Standard Annuity Table varies by age and sex. The device may produce adequate premiums in the aggregate, but it is almost certain to bring about an even greater distortion of equities by age and sex than that inherent in the unmodified form of the Table.

In an effort to provide a more satisfactory basis for the writing of annuities, Wilmer A. Jenkins of the Teachers' Insurance and Annuity Association, and Edward A. Lew, of the Metropolitan Life Insurance Company, undertook to develop a new annuity table. Their objective was to develop a table that would not only accurately reflect current mortality among annuitants but would also make an allowance, on a realistic and equitable basis, for future improvement in mortality. Out of this project emerged an annuity table which has been widely acclaimed as a milestone in the study of annuitant mortality. The development of the table which is now known as the Annuity Table for 1949 is described in the first volume of the *Transactions of the Society of Actuaries*.[2]

1. See Ray M. Peterson, "Group Annuity Mortality," *Transactions of the Society of Actuaries,* Vol. IV, 1952, p. 273.
2. Pp. 369-466.

At the younger ages (55 and below for males, and 50 and below for females) the Table reflects the experience of active lives under group annuity contracts, while at the older ages the experience under individual immediate nonrefund annuities is reflected.[3] The individual annuity experience for the period 1941-46 was utilized, with the experience for calendar years 1939, 1940, 1946, and 1947, centering around 1943, being used for the group annuity contracts. The experience under both sets of contracts was adjusted to bring it up to 1949.

To provide a margin for future improvement in mortality, two sets of projection factors were prepared. One set, known as "projection scale A," assumes that annuitant mortality will continue to decline indefinitely into the future at the same annual rates of decrease that have prevailed in recent decades. The other set, "projection scale B," assumes that the future will produce smaller rates of decrease in mortality at the younger ages, where past reductions have already engendered very low mortality rates, and somewhat higher rates of decrease than those of the past at ages over sixty, which ages should benefit most greatly from current intensive efforts to reduce the toll from cardiovascular-renal diseases and cancer. Projection scale A might be regarded as retrospective in its outlook, while projection scale B is prospective in nature. Both projection scales assume that future mortality rates among annuitants will vary with the year of exposure, or the year passed through, rather than with the year of issue of the annuity or the year of birth.

As has been pointed out, the Annuity Table for 1949 is based on the experience of individual annuitants at those ages represented in the critical period of benefit disbursements. Yet the Jenkins and Lew investigation revealed that the experience of group annuity retired lives possesses demonstrably different characteristics from individual annuity experience. To provide a table that would reflect the special characteristics of group annuities, Ray M. Peterson of the Equitable Life Assurance Society undertook an investigation of group annuitant mortality, the results of which appear in the 1952 *Transactions of the Society of Actuaries*.[4] The table which grew out of this study has been designated the Group Annuity Table for 1951.

This Table was basically derived from the mortality experience of group annuitants—the first published table to be derived in that manner. At ages below 56, the rates are those of the Annuity Table for 1949 adjusted for one year's decrease according to the Jenkins and Lew projection scale B. At ages over 65, the rates are based on the intercompany group annuity retired lives experience for the years 1946-50, with an allowance for three years' decrease in mortality according to projection scale B. Mortality rates for the gap between ages 56 and 65 were derived by extrapolation.

3. The volume of experience under individual immediate nonrefund annuities at the younger ages was too meager to be reliable.

4. Vol. IV, pp. 246-307.

The use of projection factors was designed to adjust the mortality rates to the 1951 level. At that point the basic table was considered to be representative of the *average* actual experience of all occupational groups for the year 1951. On the theory that certain groups of employees, which cannot be identified on an *a priori* basis, will experience lighter mortality than the average, it was deemed necessary to introduce an arbitrary margin of safety. This was accomplished by reducing the mortality rates for males at all ages by 10 per cent and those for females by 12½ per cent. This type of adjustment provides a margin that increases with age which was thought desirable in view of the relative unreliability of the data at the oldest ages.

Peterson then prepared a set of projection factors through which the Table can be kept up-to-date. He designated his set of factors as "projection scale C," in deference to scales A and B prepared by Jenkins and Lew. Projection scale C is 1⅓ times projection scale B, subject to a maximum annual rate of 1.25 per cent. Peterson argues that since current death rates for group annuitants are higher than those of individual annuitants, future progress in medical care, sanitation, and nutrition should exert a slightly stronger influence on group annuitant mortality. In other words, there is more room for improvement. The same reasoning would seem to dictate a higher scale of factors for males than for females, but the desire to avoid undue complexity motivated Peterson to use the same scale for both sexes.

There is divided opinion among actuaries as to the advisability of introducing into annuity premiums and reserves a specific margin for future improvement in mortality.[5] The objections to the use of projection factors are technical in nature and revolve around the administrative complications that would be introduced. The necessity of making provision against future mortality improvement has not been questioned; the issue is the manner in which it should be accomplished. Neither has Peterson's projection scale C been questioned, other than that the rates of decrease at the younger ages may be too small. It would seem advisable, therefore, to take a look at the financial implications of continued improvement in annuitant mortality, as well as that which has occurred in recent years.

With respect to mortality after 65, the 1937 Standard Annuity Table, the most widely used table for pension calculations, shows a life expectancy of 14.40 years for males age 65. In other words, a male upon reaching 65 can expect to live, on the average, another 14.40 years. The Ga-1951 Table with Projection, on the other hand, estimates that a male now age 65 will live 14.86 years longer on the average. This is not a striking difference, of course, but the use of projection factors assumes an ever-lengthening life expectancy. Thus, the Ga-1951 Table with Projection forecasts a life expectancy at age 65 of 15.51 years for a male

5. See, for example, "Discussions on Group Annuity Mortality," *Transactions of the Society of Actuaries,* Vol. IV, 1952, pp. 707-55.

now (1953) age 55. Furthermore, a life expectancy of 16.15 years at age 65 is predicted for a male now 45, with the expectancy rising to 16.75 years for a male now 35. In other words, a two-year extension in the life expectancy of males age 65 is envisioned over the next thirty years. This represents an increase of 16 per cent.

Somewhat smaller increases in life expectancy are predicted for females. The life expectancy of a female age 65 is 17.55 years according to the 1937 Standard Annuity Table. The Ga-1951 Table with Projection, on the other hand, shows a life expectancy of 17.74 years for females now age 65 and projects a life expectancy of 18.30 years in 1963, 18.84 years in 1973, and 19.35 years in 1983.

What does a longer life expectancy mean in terms of dollars? The payment of $1,000 per year for the lifetime of a male employee age 65 requires the accumulation of a sum of $11,555, if the cost of the benefits is calculated on the basis of the 1937 Standard Annuity Table and 2½ per cent interest, with no loading. If the cost is calculated on the more conservative basis of the 1937 Standard Annuity Table with a one year setback and 2¼ per cent interest, the sum required is $12,206. The sum required to provide a life income of $1,000 per year to a male employee reaching 65 in 1953 is $11,920 according to the Ga-1951 Table with Projection and 2½ per cent interest, or somewhat less than the sum required on the most conservative basis of valuation in general use today. Nevertheless, the cost will increase with the passage of time and by 1963, according to the Ga-1951 Table with Projection, it will require a principal sum of $12,376 to provide a life income of $1,000 to a male employee age 65. A male employee reaching 65 in 1973 will represent a commitment of $12,810 for each $1,000 of annual income, whereas the comparable obligation by 1983 will amount to $13,224.

This is only part of the picture. Not only will it cost more in the future to provide an income of a specified amount to an employee who attains the age of 65, but a larger percentage of employees will live to reach 65. According to the 1937 Standard Annuity Table, five males out of every seven alive at age 35 will survive to 65. The Ga-1951 Table with Projection, however, estimates that six out of every seven males alive at age 35 will survive to 65, or 16 per cent more than under the other assumption. The chances of survival from age 45 to 65 are 11 per cent better under the Ga-1951 Table with Projection than under the 1937 Standard Annuity Table. Interestingly enough, the Ga-1951 Table with Projection gives a man age 35 today a slightly better chance of reaching 65 than a man age 45 today!

The picture for the future, then, is one of more employees living to 65 to receive a more expensive benefit. The increased longevity after 65 is the more significant factor and will affect all pension plans in substantially the same manner, the principal mitigating circumstance being the deferment of retirement to an age beyond 65. The impact of the lower rate of mortality before retirement, however, is more difficult to

assess, being dependent upon the vesting provisions of the plan, the death benefits, if any, and the rate of withdrawal. When full or liberal vesting is provided, the improved chances of survival should add significantly to the cost of the plan. If there are no vested rights, or if vesting is provided for only after an extended period of service, the turnover factor will overshadow mortality as a determinant of the plan's cost. A normal rate of turnover would tend to minimize the effect of the greater chance of survival. If the plan provides death benefits other than return of the employee's contributions, if any, the savings in death benefits would be a significant offset against the higher retirement costs.